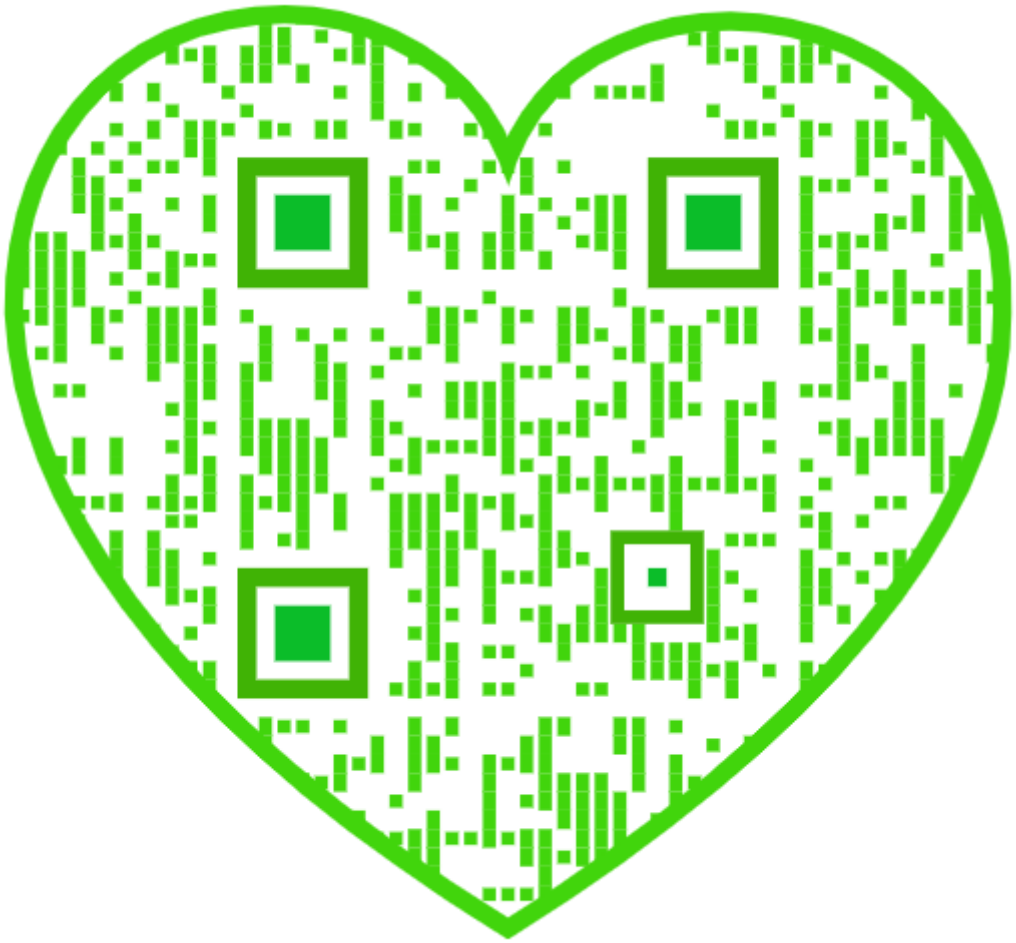

TEXT-BOOK ON NAVIGATION

AND

NAUTICAL ASTRONOMY

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AND

NAUTICAL ASTRONOMY

BY

J. GILL, F.R.A.S.

REVISED AND ENLARGED

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HEAD-MASTER OF THE LIVERPOOL CORPORATION NAUTICAL COLLEGE

NEW EDITION

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PREFACE TO NEW EDITION

THIS work is designed to meet the special requirements of officers of the Merchant Service, and to be a complete Text-Book on Navigation, etc., for the general student.

Candidates for Board of Trade Certificates will find in it all that is needed to cope with the new regulations for the Examinations of Mates and Masters, from the lowest to the highest grades, as regards both practice and principles.

The new features in this edition are—

(1) *Rearrangement*, by which the proofs of the various problems are inserted at the beginning of the separate chapter instead of being collected in one chapter at the end of the book. This gives the student a better chance of comprehending each problem.

(2) All the astronomical examples are fresh, the necessary rules are given for working them, and, where possible, *an example is worked direct from the figure*, without using special formula; this will be found of great assistance to those contemplating going in for the "extra" examination.

(3) The *necessary elements from the "Nautical Almanac"* are embodied in the book in the form found in the "N.A.," and so will afford as good practice in extracting the data as the possession of the Almanac itself.

(4) The chapters on TRIGONOMETRY have been entirely re-written, and the student is shown how to construct both Plane and Spherical Triangles to scale.

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For further information in this important subject he is recommended to consult a *recognised* School Text-Book.

(5) New chapters have been introduced on the Projections, by which any figure may be drawn to scale, and on the Correction of Altitudes.

(6) The chapters on the Tides and Tidal Soundings have been rewritten to suit the recent Tide Tables, extracts from which are found at the end of the book.

The candidate is warned that the abridged "Nautical Almanac" is in use at the examination room, and so must be on his guard when correcting all data, as the entries are arranged in such a manner that they may be extracted at sight.

For the number of page in the new edition of Norie corresponding to the number of table as given in this book, the student is referred to the Index of that work.

Chart work and the Construction of Charts have been more fully dealt with, and it is hoped that the alterations made will ensure the appreciation of the student.

W. V. MERRIFIELD.

LIVERPOOL, 1918.

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PART I.

CHAPTER I.

PROJECTIONS.

A WELL-DRAWN figure is at all times a great help to a student in the solution of a problem, and for a clear understanding of new terms made use of in a fresh subject. For this some knowledge of projection is required.

Projection is the art of representing a solid body on any surface by means of lines drawn according to definite laws. In Nautical Astronomy the solid is a sphere, and the surface a plane. These projections are either Natural or Artificial: Natural when the drawings are such as they would appear to the eye situated at a given point; Artificial when they are not perspective representations. According to the relative positions of the eye, sphere and plane of projection or primitive, the methods employed in Natural Projections receive different names, of which the most important are the *Orthographic*, *Gnomonic*, and *Stereographic*.

In the Orthographic, the eye is supposed so distant from the sphere that the visual rays are parallel to one another, the primitive is perpendicular to this direction.

In the Gnomonic, the eye is supposed to be at the centre of the sphere, and the primitive is a tangent plane to the sphere.

In the Stereographic, the eye is on the circumference of the sphere, and the primitive is a plane cutting the sphere in a great circle, which has the eye as its pole.

Since the primitive may be parallel to the Equator, Meridian, or Horizon, the above projections may be further distinguished as *Equatorial*, *Meridional*, or *Horizontal*.

The one used throughout this work is the Horizontal Stereographic Projection, and its important properties are: (1) all circles on the sphere project into either circles or straight lines; (2) any figure on the sphere projects into a *similar* figure on the primitive.

(*a*) *Projection on the Plane of the Equinoctial.*—Here the primitive is the equinoctial, and the pole P the projecting point. The concentric circles are parallels of declination. Hour circles

are represented by straight lines, since their planes pass through the projecting point. NPS is the celestial meridian, and EPW the six-o'clock hour circle; EEW is the ecliptic (Fig. 1).

A projection of the Earth on the plane of the equator shows

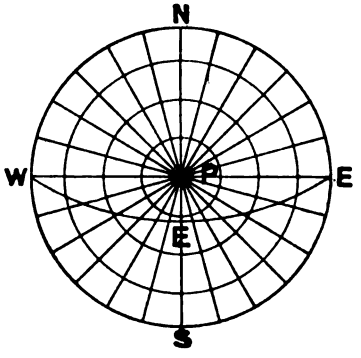


Fig. 1.

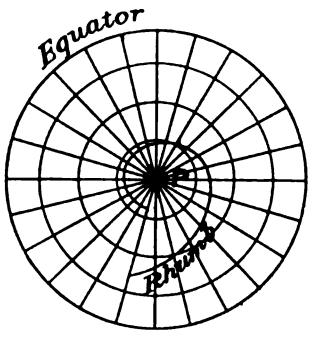


Fig. 2.

the meridians of longitude as straight lines, and the parallels of latitude as concentric circles whose common centre is the pole. This is useful for showing the course of a rhumb line, also for mapping the circumpolar regions (Fig. 2).

(*b*) *Projection on the Plane of the Meridian* (Figs. 3 and

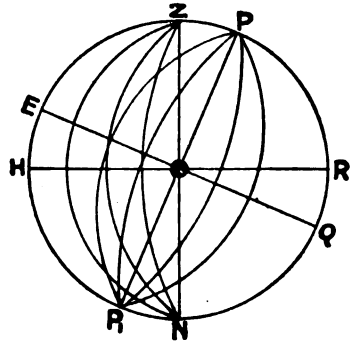


Fig. 3.

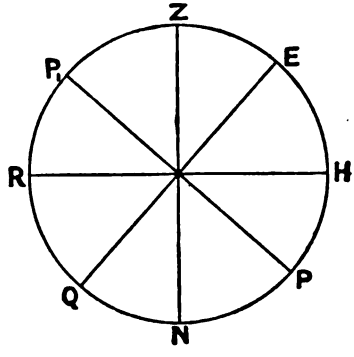


Fig. 4.

4).—The primitive circle represents the celestial meridian, the projecting point O being the east or west point of the horizon; Z and N are the zenith and nadir; P and P_1, the north and south poles respectively; HR (at right angles to ZN) is the horizon; and EQ (at right angles to PP_1) is the equinoctial. The elevation of the upper pole P above the horizon, viz. the arc RP, is equal to the latitude of the observer's position. The arc EZ also equals the

latitude. Supposing R to be the north point of the horizon, then P is the north pole (Fig. 3), and the latitude is north. For south latitude, the south pole P_1 must be above the horizon (Fig. 4).

Hour circles are represented by arcs from P to P_1, and vertical or altitude circles by arcs from Z to N.

(*c*) *Projection on the Plane of the Horizon* (Fig. 5.)—Here the *primitive circle* is the horizon, N., E., S., W. being the cardinal points. The zenith (Z) is the projecting point, hence all *vertical circles* appear as straight lines passing through Z, NZS being the *celestial meridian*, and WZE the *prime vertical*. P is the north pole, at a distance from N. equal to the latitude. The arc ZQ is also equal to the latitude, and PZ is the co-latitude; WQE is the equator, cutting the horizon in the E. and W. points; Pp_1, Pp_2, etc., are hour circles.

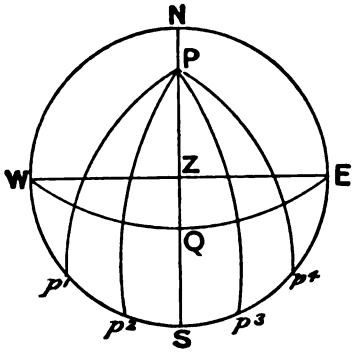


FIG. 5.

The projection on the plane of the horizon has some advantages over that on the plane of the meridian for illustrating most of the problems of nautical astronomy, and is therefore more commonly employed.

To draw a figure to scale, a "Gunter" rule is necessary; the following illustration explains the connection between the lines marked Rum., Cho., Sin., Tan., and S.T. on the rule (Fig. 6).

With centre O, distance OA, describe a semicircle. AOB is

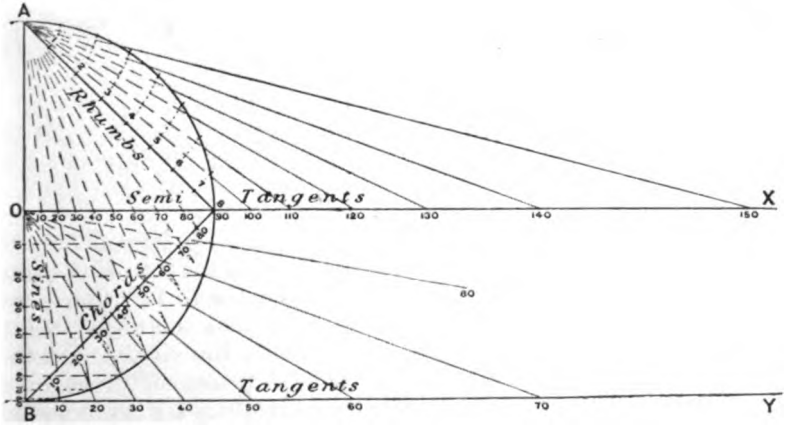


FIG. 6.

a vertical diameter; OX, a horizontal line through centre; BY, a tangent at B. OX divides the figure into two quadrants; the upper one is divided into eight equal parts, and with A as centre and distance the different chords from A to each point; the distances are brought back on the chord of 90°, forming the line of Rhumbs (RUM.). A3 is thus the chord of a circle described with radius OA, which subtends an angle of three points at

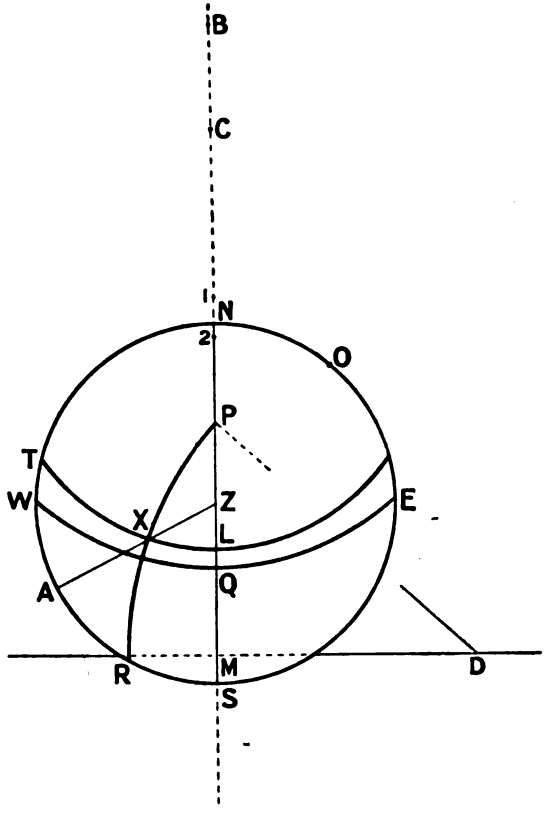


FIG. 7.

centre O. The line of Chords (CHO.) is similarly formed on the lower quadrant, but the quadrant is here divided into nine equal parts instead of eight, and thus gives divisions to 10°, which can be further subdivided. We can now see that the chord of 60° is the radius that must be used to describe the circle. For the line of Tangents (TAN.) through O and points of division in lower quadrant, draw straight lines meeting BY in required points. For the line of Semi-Tangents (S.T.) through A, and

points of division in lower quadrant, and also corresponding points in upper quadrant, draw lines to cut OX in required points.

Example.—Draw a figure from following data : Latitude 40° N. ; western hour angle 2^h 48^m; declination 10° N.; and take off the Amplitude at setting, Azimuth and Altitude (Fig. 7).

With centre Z, and radius the chord of 60°, describe a circle N.E.S.W.; draw SZNB vertically, and lay off E. and W. as the extremities of a horizontal diameter through Z. Make ZM = 40° (Lat.) from scale of tangents, and through M draw a line perpendicular to MZ; in this line are the centres of all hour circles. Make NO = chord of 40° (Lat.), where the line OW cuts the vertical line is the elevated pole P. At P to the right make an angle = complement hour angle that is 48°; this line cuts the line through M in D, and is the centre for describing the hour circle required PXR. For the equinoctial, add the colat to 90° = 140°; subtract the colat from 90° = 40°, and from scale of S.T. make ZB = 140°, and ZQ = 40°; bisect BQ in 1, this is centre for describing equinoctial WQE.

For the declination parallel, add colat to polar distance = 130°, subtract colat from polar distance = 30°; from S.T. scale make ZC = 130°, and ZL = 30°; bisect CL in 2; this is centre for describing the parallel TXL, where the parallel cuts the hour circle is X, the position of the object. Draw ZXA the vertical circle through the object.

Measure WT on the scale of chords for the Amplitude = 15°, or W. 15° N.

Measure SA on the scale of chords for the Azimuth = 63°, or S. 63° W.

Measure ZX on the S.T. scale for the Zenith Distance = 49°, or Altitude = 41°.

For South Latitude set off ZM to the north, and construct the figure downwards instead of upwards.

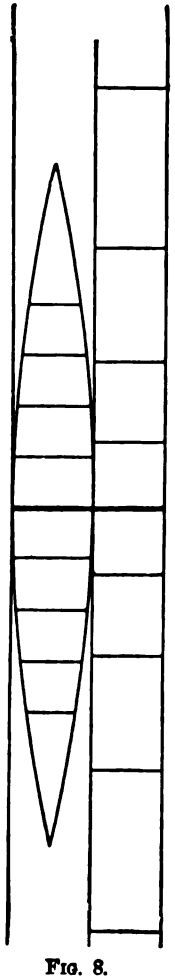


FIG. 8.

The only Artificial Projection to be considered is Mercator's (Fig. 8).

Mercator's Projection.—This is the kind of projection used in the construction of Navigating Charts, and represents the Earth's surface as a plane. The meridians of longitude are drawn as parallel straight lines, and the divisions of latitude are expanded in the same proportion as the meridian distances are lengthened. The effect of this is to make the rhumb course a straight line, which is of great advantage to the navigator.

The distance from the equator of any given parallel of latitude, and the amount of expansion of a degree of latitude, can be found from the table of *Meridional Parts;* for example, the parallel of 30° is 1800 geographical miles from the equator on the globe, but the meridional parts for 30° = 1888,

which means that on a Mercator's chart drawn to the same scale this parallel would be 1888 geographical miles from the equator. Again, the meridional parts for 31° = 1958, and 1958 − 1888 = 70 miles, which would be the length of the expanded degree from 30° to 31°.

The meridional parts for any given latitude may be calculated by adding together the secants of all minutes of latitude up to the given latitude; thus—

$$\sec 0' + \sec 1' + \sec 2' + \sec 3', \text{ etc.}$$

or they may be calculated independently from the formula—

$$\log \text{ of mer. parts} = 3{\cdot}8984895 + \log (\log \cot \tfrac{1}{2} \text{ co-lat.} - 10)$$

CHAPTER II.

DEFINITIONS AND INSTRUMENTS.

1. **Great Circles.**—Circles whose planes pass through the centre of a sphere or globe.

2. **Vertex of a Great Circle.**—That point in it which is farthest from the Equator.

3. **Small Circles.**—Circles whose planes do not pass through the centre of the sphere.

AVB and CV'D are great circles.
V and V' are vertices.
EOF, a small circle.

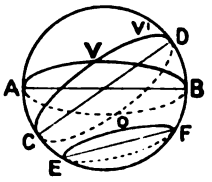


FIG. 9.

4. **Right Angle.**—An angle measuring 90°, or one quarter of a circle.

5. **Oblique Angle.**—An angle not a right angle.

6. **Obtuse Angle.**—An angle greater than a right angle.

7. **Acute Angle.**—An angle less than a right angle.

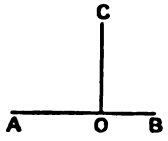


FIG. 10.

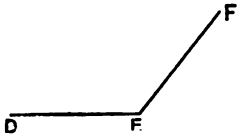


FIG. 11.

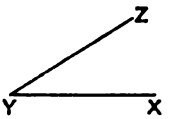


FIG. 12.

AOC and BOC are right angles.
DEF and XYZ are oblique angles.
DEF is an obtuse angle.
XYZ is an acute angle.

8. **Spherical Angle.**—An angle included between two great circles of a sphere (Fig. 13).

AOC and BOC are spherical angles.

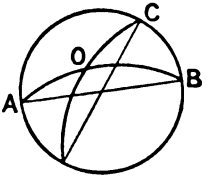


FIG. 13.

9. **Arc.**—A part of the circumference of a circle.

10. **Complement of an Arc or Angle.**—Its difference from 90°.

11. **Supplement of an Arc or Angle.**—Its difference from 180°.

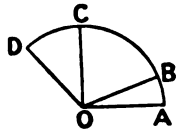


FIG. 14.

AB, BC, CD are arcs.
BOC is complement of BOA.
BC is complement of BA.
COD is complement of AOD.

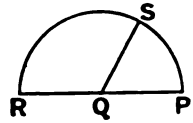


FIG. 15.

PS and SR are arcs.
SR is supplement of PS.
RQS is supplement of SQP.

12. **The Poles.**—The points where the earth's axis meets the surface.

13. **Equator.**—A great circle of the earth 90° distant from the poles.

14. **A Meridian.**—Half a great circle extending from pole to pole of the earth.

15. **Parallels of Latitude.**—Circles whose planes are parallel to the plane of the Equator.

16. **Tropics.**—Two parallels of latitude distant 23° 28′ on each side of the Equator: Cancer on the north, and Capricorn on the South.

17. **Difference of Latitude.**—The arc of a meridian intercepted between the parallels of latitude on which the places stand.

18. **Prime Meridian.**—The meridian from which longitudes are measured; in this country it is the meridian of Greenwich.

19. **Longitude.**—Distance in arc east or west of the meridian of Greenwich. It is measured on the Equator, or on a graduated parallel. vp to 180°

20. **Difference of Longitude.**—An arc of the Equator intercepted between two meridians of longitude (Fig. 16).

P and P′ are the poles.
EQ is the equator.
PAP′ and PBP′ are meridians.
POP′ the prime meridian.
LL′, TT′, RR′ are parallels of latitude.
TT′ is tropic of Cancer.
RR′ is tropic of Capricorn.
XY is difference of latitude when places are on same side of equator, on parallels LL′ and TT′.
XZ is difference of latitude when places are on opposite sides of equator, on parallels LL′ and RR′.
OA and OB are Longitudes of places on meridians PAP′ and PBP′.
AB is difference of longitude, between places on meridians PAP′ and PBP′

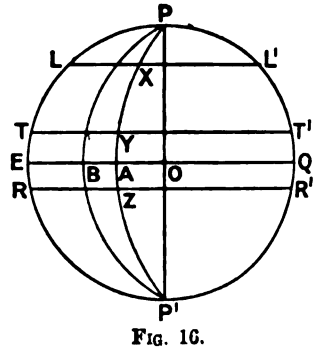


FIG. 16.

21. **Meridional Parts.**—The distance from the Equator in geographical miles of a given latitude on Mercator's Chart (Fig. 17).

ab = parallel on globe.
cd = same parallel on chart.
ec = meridional parts for lat. *ea*.

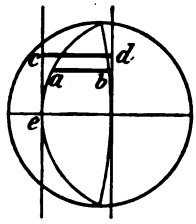


FIG. 17.

22. **Departure.**—The distance in geographical miles between two places in the same latitude. In the traverse table, it is the number of miles of *easting* or *westing* made on a given course and distance.

23. **Nautical Mile.**—The one-sixtieth part of a degree measured on a meridian. It varies slightly with the latitude, and its mean value is taken at 6080 feet.

24. **Rhumb Line.**—A line which makes equal angles with all the meridians it crosses.

25. **Zenith.**—The point in the heavens vertically over the observer's head.

26. **Nadir.**—The point in the heavens diametrically opposite to the Zenith.

27. **Equinoctial.**—A great circle in the heavens 90° from the celestial poles.

28. **Ecliptic.**—A great circle in the heavens representing the sun's annual apparent path.

29. **First Point of Aries.**—The point where the sun crosses the Equinoctial about the 21st of March.

30. **Celestial Meridian of Observer.**—The vertical circle which cuts the Horizon in the true North and South points (Fig. 18).

NESW., Plane of horizon.
Z is zenith.
EQW is equinoctial.
C♈C′ is ecliptic.
♈ is first point of Aries.
NPZS is celestial meridian of observer.

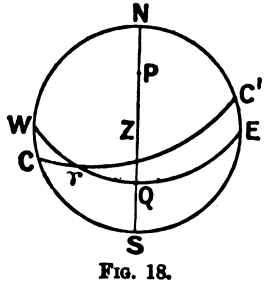


FIG. 18.

31. **Azimuth.**—The bearing of an elevated object reckoned from the N. or S. point.

32. **Amplitude.**—The bearing of an object when rising or setting, reckoned from the E. or W. point.

33. **Vertical Circles.**—Circles passing through the Zenith and Nadir, and therefore perpendicular to the Horizon. They are also called "altitude circles" and "azimuth circles."

34. **Prime Vertical.**—The vertical circle which cuts the Horizon in the true East and West points.

35. **Declination.**—Distance in arc north or south from the Equinoctial measured on an Hour Circle.

36. **Polar Distance.**—The distance in arc from the elevated pole, and therefore the complement of the Declination, measured on an Hour Circle.

37. **Hour Circles.**—Halves of great circles which pass through the celestial poles, one hour being equal to 15° (Fig. 19).

NESW, Plane of horizon.
NPZS, Celestial meridian.
P, Elevated pole.
X, a heavenly body on parallel of declination LL'.
NZA or arc NA is Azimuth of X.
WL, Amplitude of X at setting.
ZXA, Vertical circle through X.
WZE, Prime vertical.
XD, Declination of X.
PX, Polar distance of X.
PXD, Hour circle through X.

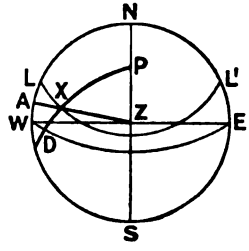


FIG. 19.

38. **Right Ascension.**—Distance eastward from the first point of Aries measured on the Equinoctial and expressed in Time.

39. **Hour Angle.**—The angle at the pole between the observer's Celestial Meridian and an Hour Circle.

40. **Civil Time.**—The time used in civil life. The day begins at midnight, and is divided into two parts of twelve hours each, a.m. from midnight to noon, and p.m. from noon to midnight.

41. **Astronomical Time.**—Time used for astronomical calculations. The day begins at noon, and the hours count from 0 to 24.

42. **Mean Time.**—The Westerly hour angle of the Mean Sun, it is the time shown by a correctly going timepiece.

The Mean Sun is an imaginary body which is supposed to move along the equinoctial with the average velocity of the True Sun in the ecliptic.

43. **Apparent Time.**—The Westerly hour angle of the true Sun.

44. **Equation of Time.**—The difference between Mean and Apparent Times; and therefore used to convert one time to the other.

The Equation of Time is least about April 15th, June 15th, August 31st, and December 24th; and greatest about February 10th, May 14th, July 26th, and November 2nd in each year.

45. **Sidereal Time.**—The Westerly hour angle of the first point of Aries (Fig. 20).

Figure as before on horizon plane.
ϒC' is Ecliptic.
ϒ is first point of Aries.
M is mean Sun.
T is true Sun.
X is any heavenly object.
Then
ϒA is right ascension of X.
QPA is easterly hour angle of X.
QPM is westerly hour angle of mean sun, or Mean Time.
QPT is westerly hour angle of true sun, or Apparent Time.
QPϒ is westerly hour angle of ϒ, or Sidereal Time.
TPM is Equation of Time.

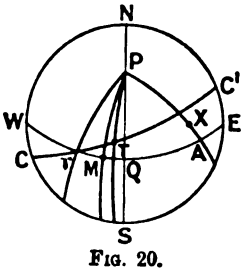


FIG. 20.

46. **Visible Horizon.**—The circle which bounds the observer's view at sea.

47. **Sensible Horizon.**—A plane touching the Earth at the observer's position and extended to the Heavens.

48. **Rational Horizon.**—A plane passing through the Earth's Centre parallel to the Sensible Horizon and extended to the Heavens.

49. **Artificial Horizon.**—A horizontal reflecting surface used for observing altitudes in the absence of a good sea-horizon.

50. **Dip.**—The angular depression of the Visible horizon below the Sensible, due to the observer's elevated position.

51. **Observed Altitude.**—The angular height of an object above the Visible horizon as measured by a Sextant.

52. **Apparent Altitude.**—The angular height of an object above the Sensible horizon, found by correcting the observed altitude for Index Error and Dip.

53. **True Altitude.**—The angular height of an object above the Rational Horizon, found by correcting the Apparent Altitude for Refraction and Parallax (Fig. 21).

AB is visible horizon.
MOA = angle OA makes with sensible horizon is Dip.
AOX = observed altitude.
MOX = apparent altitude.
XCN = true altitude.

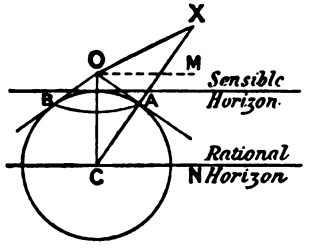


FIG. 21.

54. **Refraction.**—The error of altitude caused by the bending of rays of light in passing through the atmosphere (Fig. 22).

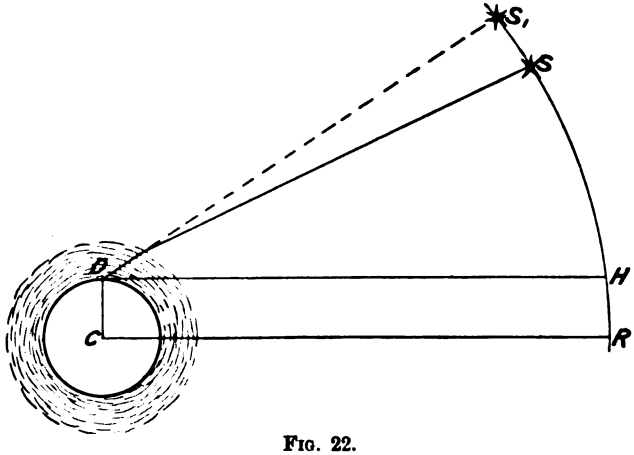
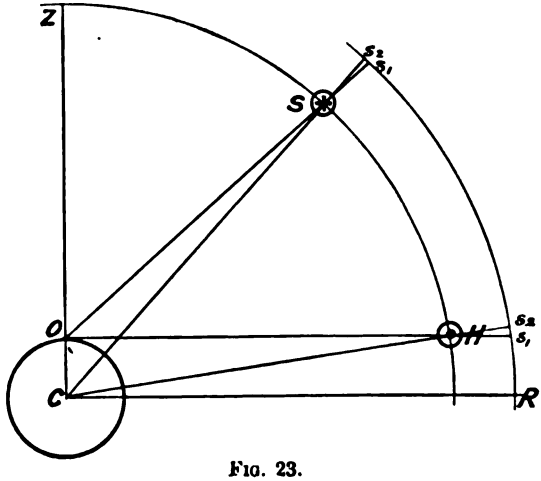


FIG. 22.

O, Observer's position.
S, True position of star.
S_1, Apparent position of star.
SS_1, Refraction.
OH, Horizon.
HS_1, Apparent altitude of a star.
RS, True altitude of a star.

55. **Parallax** is the angle at the centre of the object subtended by the earth's radius at the position of the observer. It is a correction for reducing an observation to the earth's centre (Fig. 23).



O, Observer's position.
⊙ H, Sun's position on horizon.
⊕ S, Sun's position in altitude.
∠ OHC, Horizontal parallax.
∠ OSC, Parallax in altitude.
Z, The zenith of observer.
Rs_2, True altitude of sun.

FIG. 23.

56. **Semidiameter.**—The angle subtended at the earth's centre by the radius of the sun, moon, or planet.

57. **Augmentation of Moon's Semidiameter.** — An apparent increase in the moon's semidiameter due to the fact that her distance from the observer decreases as her altitude increases (Fig. 24).

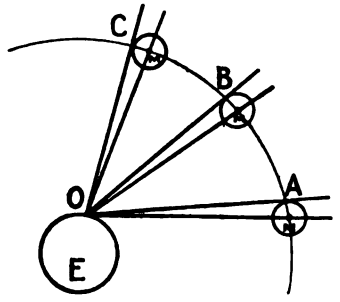


FIG. 24.

E, Centre of earth.
O, Observer's position.
MA, Position of moon on horizon.
MB, MC, Other positions of moon in altitude: her centre being at fixed distance from E. As she rises her distance from O decreases.
MOA, Semidiameter of moon on horizon.
MOB and MOC, Semidiameters when in altitude.
MOB–MOA and MOC–MOA, Augmentation of semidiameter when in altitude.

58. **Reduction of Horizontal Parallax.**—A correction necessary on account of the shortening of the earth's radii towards the poles (Fig. 25).

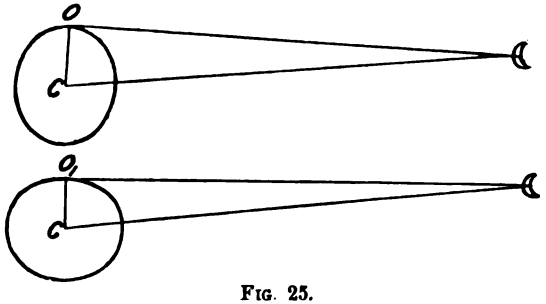


FIG. 25.

O, Observer's position on longest diameter of earth.
O_1, Position on a shorter diameter.
O ☾ C, Equatorial horizontal parallax.
O_1, ☾ C, Horizontal parallax for latitude of observer, which is less because subtended by a shorter radius.

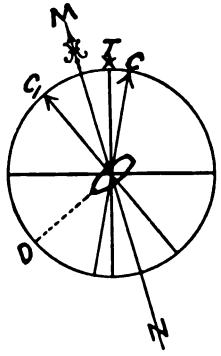
59. **Zenith Distance.**—The angular distance from the Zenith, or the complement of the true altitude measured on a vertical circle.

60. **Magnetic Meridian** at a place is a great circle in whose plane a compass-needle lies when under the earth's influence only.

61. **Variation.**—The angle between the True and Magnetic meridians, measured from True North.

62. **Deviation.**—The angle between the Compass-needle and the Magnetic meridian.

63. **Error of the Compass.**—The angle between the Compass-needle and the True meridian (Fig. 26).



T, True north.
MN, Magnetic meridian.
C and C_1, Different directions of compass north.
TM, Variation (west).
MC, Deviation (east).
MC_1, „ (west).
TC, Error of compass (east).
TC_1, „ „ (west).
CD or C_1D, Compass course from north.
TD, True course from north.
MD, Magnetic course from north.

FIG. 26.

64. **Leeway.**—The angle between the ship's keel and her "wake" (Fig. 27).

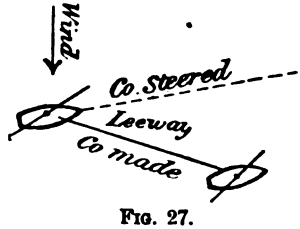


FIG. 27.

NOTE.—*The foregoing comprise the definitions required by the Board of Trade Regulations for the examination of mates and masters.*

65. **Latitude.**—Distance in arc north or south from the Equator measured on a Meridian.

66. **Arctic Circle.**—The parallel of 66° 32′ N. lat.

67. **Antarctic Circle.**—The parallel of 66° 32′ S. lat.

68. **Obliquity of Ecliptic.**—The angle between the planes of the Ecliptic and Equinoctial.

69. **Equinoxes.**—The points where the Ecliptic intersects the Equinoctial. The Vernal Equinox is where the sun crosses the Equinoctial in March, and the Autumnal where it crosses in September.

70. **Precession.**—A slow motion of the Equinoxes along the Equinoctial westwards, amounting to about 50″ annually.

71. **Solstices.**—The points on the Ecliptic farthest from the Equinoctial.

N.B.—The terms "Equinoxes" (Spring and Autumn) and "Solstices" (Summer and Winter) are also applied to the times of the year when the sun is on the Equinoctial or at the Solstices.

72. **Celestial Latitude.**—Distance in arc north or south from the Ecliptic.

73. **Celestial Longitude.**—Distance in arc from the "first point of Aries" measured on the Ecliptic.

74. **Circumpolar Stars.**—Stars whose polar distance is less than the latitude of the place, and consequently never set.

Questions on Variation and Deviation, from the Board of Trade Regulations.

1. *Q.*—Does the variation change with time?
   A.—Yes; it slowly changes in amount. At London it now decreases about 9′ annually.
2. *Q.*—Is the variation the same all over the world?
   A.—No; all amounts from 0° to 180° may be found.
3. *Q.*—Where do you find the variation?
   A.—On a variation chart.
4. *Q.*—Does the deviation change? if so, when?
   A.—It changes with a change of course; also, for the same course with lapse of time, because the magnetic condition of the ship is liable to change, and from change of cargo.
5. *Q.*—Where is the north magnetic pole situated?
   A.—In about 70° N. lat. and 92° W. long.
6. *Q.*—Where is the south magnetic pole situated?
   A.—In about lat. 75° S., and long. 150° E.
7. *Q.*—Should the compass-needle point to the magnetic or the true pole of the earth?
   A.—To the magnetic pole.
8. *Q.*—When is the altitude of an object most seriously affected by refraction?
   A.—When nearest to the horizon.
9. *Q.*—Where is the pole star situated?
   A.—At the distance of 1° 12′ from the true celestial pole.
10. *Q.*—Which is the most favourable time for determining the hour angle of a celestial body, and thence the longitude, and state the reason why?
    A.—When it is on the prime vertical, because the hour angle is then least affected by errors in altitude or latitude.

INSTRUMENTS.

Mariner's Compass.—The Mariner's Compass consists of three parts:—Bowl, Card, and Needles. The Bowl is a hollow hemisphere of brass, weighted at its lowest part to keep it horizontal. On its inside a vertical line is painted called the "lubber" line, which indicates on the card the Course of the ship. From the centre of the bottom of the bowl springs a support which holds a sharp pivot tipped with iridium to render it hard and non-rustable. The pivot fits into a brass cap having an agate or ruby centre to work on. From the cap by means of silk threads the card is suspended.

The card is circular, made of thin paper with the centre part cut out to lessen the weight, and having a band of aluminium round its circumference to strengthen it, and is graduated to quarter-points and degrees.

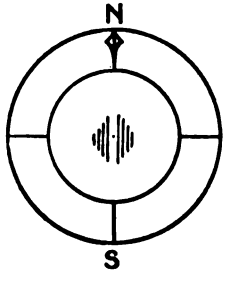


FIG. 28.

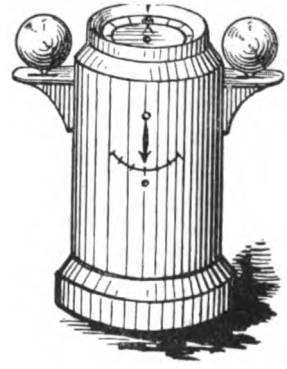


FIG. 29.

From the card hang the magnetised needles, which are generally 8 in number, arranged symmetrically, parallel to one another, and to the N. and S. points of the card (Fig. 28). The needles are about the size of a large knitting-needle, the longest pair with a 10″ compass card being $3\frac{1}{4}$″ long.

The whole instrument is mounted on gimbals, with strong wire supports to take off the vibration of the ship as much as possible, and is placed in a binnacle (Fig. 29). Two brackets at the side support cast-iron correctors, and provision is made inside the binnacle for compensating magnets. A brass cylindrical box on the outside is provided for a Flinders Bar. A Clinometer shows the angle of "heeling."

Azimuth Compass.—The Azimuth Compass is like an ordinary Steering-compass in the mounting of the needles and card. There is a movable ring on the bowl, provided with sight-vanes at opposite ends of a diameter. One of these should have

a reflecting prism for reading the graduations on the card, and the other a mirror for reflecting elevated objects (Fig. 30).

It is mounted in a box or on a tripod, and in using it the observer looks through the narrow slit, and moves the ring round until the object is bisected by the wire in the other sight-vane, and then reads off the bearing by means of the prism.

Sometimes a vertical style is fixed on the centre of the glass cover of an ordinary compass, for showing the sun's compass bearing by means of the shadow.

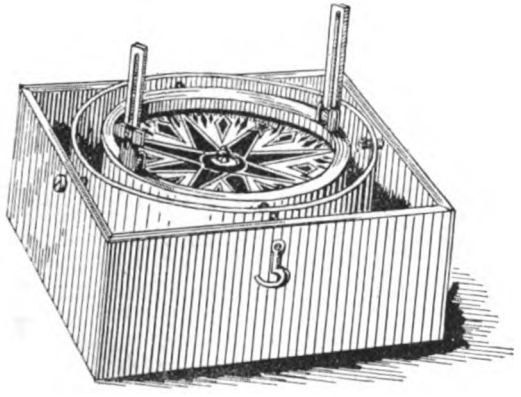


FIG. 30.

Pelorus.—The "Pelorus" resembles an Azimuth Compass, but has neither magnets nor bowl. The "card," usually of metal, is not suspended, but is free to move round its centre, and is mounted on gimbals and weighted to keep it horizontal. A bar carrying the two sight-vanes is pivoted at the centre of the card, and may be clamped in any position. Four "lubber-points," 90° distant, are usually marked outside the edge of the "card" (Fig. 31).

The uses of the "Pelorus" are—

(*a*) To find the Deviation on any course. The sight-vane being clamped to the known *correct magnetic* bearing of a distant object, the card is turned round until, on looking through the slit, the object is seen bisected by the wire. The *correct magnetic* course is shown at the lubber-point, and the difference between this course and the course by compass is the Deviation.

(*b*) To place the ship's head in any desired direction for the purpose of compensating the compasses.

The sight-vanes being clamped to a known *correct magnetic* bearing, and the card turned round until the desired direction of ship's head is at the lubber-point, then the ship is swung until

the object is seen bisected by the wire. Her head is now on the desired point, and the magnets or correctors may be placed in position so that the Compass shall show the same course as the Pelorus.

(*c*) For taking Bearings when the Standard Compass is not available.

Care must be taken that the ship is upright, and that a line

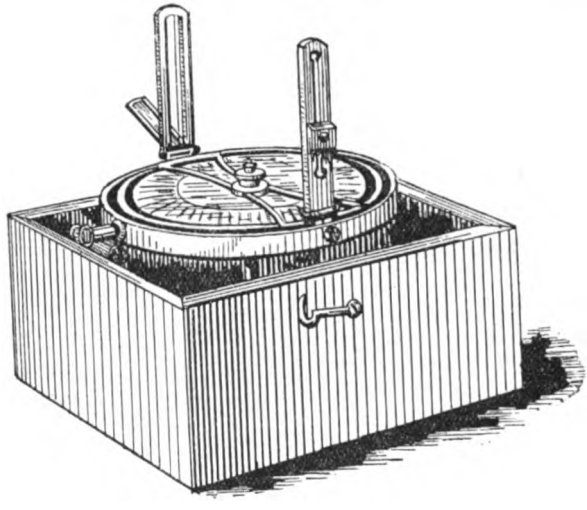


FIG. 31.

through the centre of the card and the lubber-point shall be parallel to the line of the keel.

Barometers.—The atmosphere surrounds the earth to a height of two hundred miles or more, and presses on the surface with an average weight of nearly 15 lbs. on a square inch, and its average pressure at sea-level is nearly 1000 millibars. A cubic foot of atmospheric air under ordinary circumstances weighs approximately an ounce and a quarter and its density increases one per cent. with an increase of ten millibars of pressure, and decreases one per cent. with an increase of temperature of 5° F. (see "Barometer Manual"). The pressure varies with changes of temperature and moisture and from other causes, and decreases from the sea-level upwards. A Barometer is an instrument for measuring this pressure.

Barometers are generally of two kinds, viz. "Aneroid," meaning without liquid, and "Mercurial."

The **Aneroid Barometer** has a shallow metal cylinder with a flexible cover, and partially exhausted of air. The tendency to collapse in consequence of the partial vacuum is balanced by a

strong spring attached to the cover and fastened to the bottom of the containing box. As the pressure of the atmosphere varies, the movements of the flexible cover inwards and outwards are, by a mechanical contrivance, conveyed to a pointer which shows on a dial the pressure of the atmosphere (Fig. 32).

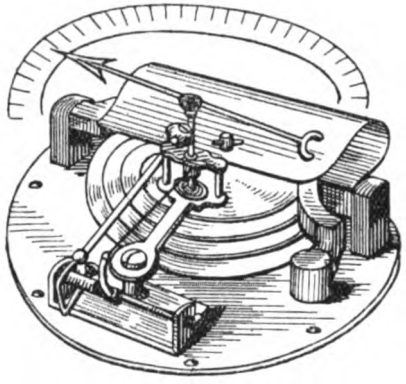


FIG. 32.

The readings from an aneroid cannot be depended upon, and should be compared from time to time with those taken from a mercurial barometer, and adjusted by means of a screw at the back of the instrument, and index error recorded.

It is very portable, convenient for reading, and the readings recorded require no correction for temperature or latitude, but must be corrected for height above sea-level and index error.

The dial of the barometer is graduated on three concentric circles, the outer showing the reading in millibars, the centre in inches and tenths, and the inner in millimetres.

The converters for the three scales may be obtained from the fact that 100 centibars or 1000 millibars correspond to 29·53 inches of mercury at 32° Fahr., and 45° lat. and to 750 millimetres at 0° C. and 45° lat. A table for this purpose is found in the "Seaman's Hand Book on Meteorology," and may be obtained by the student as follows:—

One inch of mercury corresponds to 33·86 millibars.

Hence to convert "inches" to "mbars." multiply reading in inches by 33·86.

To convert "millibars" to "inches," divide reading in "mbars." by 33·86.

Again ·03937 inch equals 1 millimetre.

Hence to convert "inches" to "mms." divide reading in inches by ·03937.

To convert "mms." to inches, multiply reading in "mms." by ·03937.

And since 1000 mbars. correspond to 750 mms.,

To convert mbars. to "mms.," subtract one-quarter of reading in mbars. for corresponding "mms."

To convert "mms." to mbars. add one-third of reading in "mms." for corresponding "mbars."

The **Mercurial Barometer** consists of a glass tube about 33 in.

long (whose open end dips into a quantity of mercury contained in a trough or "cistern"), enclosing a column of mercury of varying height. The height of this column measured from the surface of the mercury in the cistern at any moment is the measure of the pressure of the atmosphere on a surface equal to the section of the tube. This pressure is indicated in inches of mercury and mbars. on scales near the top. Between the top of the column, and the top of the tube there is a space called the Torricellian vacuum, because of the experiments made in 1643 by an Italian called Torricelli (*v*, Fig. 34).

FIG. 33.

FIG. 34.

The portable barometers (Fig. 33) used at sea have the glass tube protected by a casing of wood or iron, and the mercury in the cistern is prevented from escaping by a cover or bag of washleather, which, however, does not interfere with the air-pressure. In this form of barometer the "inches" on the scale are shortened, so as to compensate for the changing level of the mercury in the cistern as the column rises or falls.

For scientific purposes, corrections of the barometer-readings are made for temperature height above sea-level, and latitude.

Thermometer.—A Thermometer is an instrument for measuring temperatures. The common Thermometer is a glass tube of exceedingly small bore, blown out at one end into a bulb, which is filled with mercury, the other end being hermetically sealed after the air is driven out. As the mercury in the bulb expands with heat and contracts with cold to a greater degree than the glass envelope, this causes the column in the tube to advance or retire. The degree of the scale indicated by the end of the column is the temperature.

Three different scales of temperature are in use.

(*a*) The **Fahrenheit** thermometer[1] (Fig. 35, *a*), which is the one generally used in this country, marks the freezing-point of water 32° and the boiling-point 212°, when the barometer reads 29·905 inches at 32° Fahr., a difference of 180°. Zero (0) on this scale is the temperature of a mixture of salt and snow.

[1] So called because Fahrenheit, a German, in 1721, first suggested this method of graduation.

(*b*) The **Centigrade** or **Celsius**[1] thermometer (Fig. 35, *b*) marks the freezing-point zero (0) and the boiling-point 100°.

(*c*) The **Réaumur** thermometer[2] (Fig. 35, *c*) marks freezing-point zero (0) and boiling-point 80°.

The attached thermometer in the barometer (T, Fig. 33) has two scales, one graduated to give the temperature Fahrenheit, the other graduated in Centigrade degrees, showing the temperature reckoned from absolute zero, which is −273° C. or −459° Fahr., and is the temperature at which permanent gases following the ordinary rate of contraction on decrease of temperature would occupy no volume and meet no pressure.

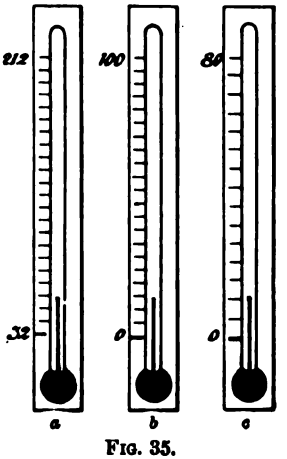


FIG. 35.

Mercury is the liquid generally in use in thermometers because of its low freezing-point (−40° C.), high boiling-point (358° C.), and its steady rate of expansion through long ranges of temperature; other advantages are its opacity and the fact that it does not wet the glass envelope. For lower temperatures alcohol is used instead of mercury, as it solidifies at −130° C. and boils at 78° C., the alcohol being coloured to render it visible.

The number of divisions on the three scales between freezing and boiling-points are therefore in the ratio of 180, 100, and 80, or 9, 5, 4, from which it is easy to compare the readings of any one of these instruments with each of the other two.

Examples.—1. What are the readings of the Centigrade and the Réaumur scales corresponding to 72°, 18° and −40° Fahr.?

72°	40	40
Freezing 32°	5	4
Above freezing 40°	9)200	9)160
	22·2° Cent.	17·7° Réau.
	273	
	295·2° A.	
18° Fahr.	14	14
32	5	4
Below freezing 14°	9)70	9)56
	−7·7 Cent.	−6°·2 Réau.
	273	
	265°·3 A.	

[1] Named after Celsius, a Swede, who, about 1743, suggested this method of graduation.

[2] Called after its proposer, Réaumur, a Frenchman, about 1743.

	−40° Fahr.	72	72
	32	5	4
Below freezing	−72	9)360	9)288
		−40° Cent.	−32° Réau.
		273	
		233° A.	

The amounts marked "A." being the readings when the thermometer graduations commence at absolute zero.

2. Convert 32° Cent. and 20° Réaumur into the corresponding Fahrenheit temperatures.

32° Cent.	20° Réau.
9	9
5)288	4)180
57·6 above freezing.	45 above freezing.
+ 32	+ 32
89·6 Fahr.	77 Fahr.

3. Convert −25° Cent. and 268° A. to Fahr.

−25° Cent.	268° A.
9	273
5)225	−5° Cent.
	9
−45 below freezing.	5)45
+32	
−13° Fahr.	−9 below freezing.
	+32
	23° Fahr.

Chronometer (Fig. 36).—A chronometer is an accurately made and compensated timepiece with its balance so constructed that any change of temperature produces the smallest change in its error from day to day; this difference of error is called its *daily rate*. Most commonly it is made to work for two days, and on its face is an index hand showing how long since the instrument was wound. The winding should take place every day at the same hour in order that the operation may become a habit, and as a guarantee that a constant interval elapses between the operations, and is performed by six or seven half-turns of the key.

The keyhole at the back is protected by a sliding cap. The instrument is mounted on gimbals in a mahogany case having a glass roof and outer cover. In the case is a clamp by means of which it can be prevented from swinging in the gimbals when conveyed from ship to shore.

Its use is to obtain correct mean time at Greenwich (the error and rate being known), and thence the longitude.

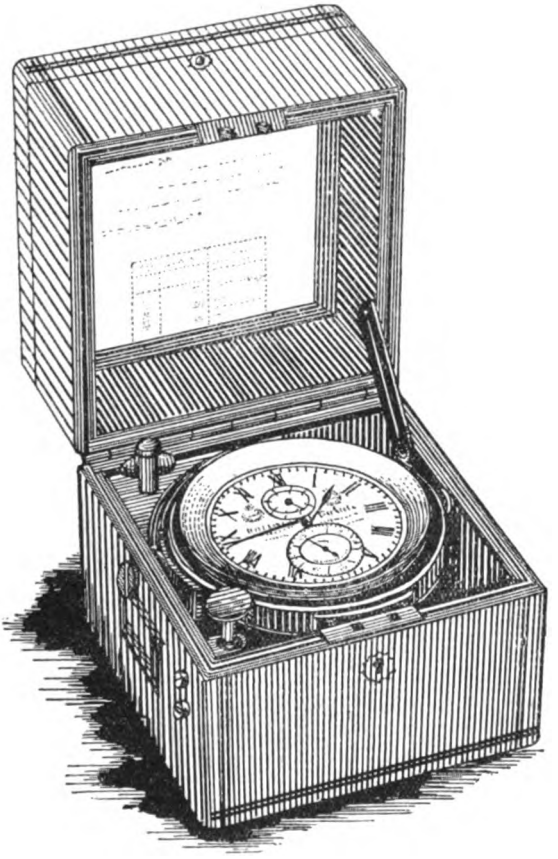


FIG. 36.

Hydrometer.—The Hydrometer is an instrument for measuring Density or Specific Gravity (Fig. 37).

Specific gravity of a solid or liquid is its weight compared with the weight of an equal bulk or volume of distilled water at the temperature of 62° Fahr., or, as some scientists prefer, 39°·2 Fahr., which is the point of maximum density.

The hydrometer used on board ship for obtaining the density of sea-water, and the water in docks or rivers, is a glass or metal stem with two bulbs, the lower of which is weighted with mercury or small shot to make the instrument float upright. The scale on the stem reads from 0 downwards. It is seen that

in using this hydrometer we displace *constant* weights of *different* volumes: the instrument is graduated so that the S.G. may be read directly from the scale, which is effected by shortening the divisions from 0 downwards.

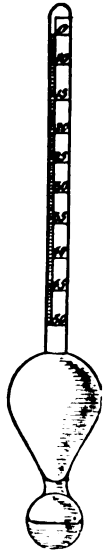


FIG. 37.

When in distilled water (temp. 62°), the level of the water is at 0, but in sea-water at about 26. In brackish water it will be somewhere between those marks, according to the degree of saltness.

One thousand must be added to the figures on the scale when stating the S.G.

Hence the S.G. of distilled water (temp. 62° Fahr.) is 1000, and the S.G. of sea-water 1025 or 1026, whilst the S.G. of the water in a river or estuary or dock where fresh water enters might be anything between these two limits.

These facts have an important bearing on the *loading of ships*, since the *law* fixes the *load-line* for seagoing vessels.

It is obvious that a ship may be loaded deeper than her "load-mark" in fresh or brackish water, for she will "rise to her marks" when she goes to sea.

The S.G. of the water at the loading-place may be obtained by a Hydrometer, and, her *draught* being known, a simple proportion will give the "sea draught" nearly. Also when the "draught" at sea is known, the "draught" in dock or river can be found.

Examples.—1. The S.G. of the water in the dock is 1015, and the ship's draught 23 ft. 6 in. : required her draught at sea. Taking S.G. of sea-water as 1025—

ft. ft.
1025 : 1015 :: 23·5 : 23·3 nearly

23·5

5075
3045
2030

1025)23852·5(23·27 ft. nearly
2050

3352
3075

2775
2050

7250
7175

75

Answer, 23·3 ft. nearly.

2. At sea a ship draws 26 ft. 9 in. : what will her draught be in dock, where the S.G. is 1010?

ft. in. ft. in.
1010 : 1025 : : 26 9 : 27 2
12

321 in.
1025

1605
642
321

1010)329025(326 inches nearly
3030 = 27 ft. 2 in. Answer.

2602
2020

5825

3. The number of inches between the F.W. mark and centre of disc is given on the load-line certificate as 7·5 : how much may the Plimsoll mark be submerged in dock when Hydrometer reads 9?

In diagram density at F.W. is 0 at centre of disc 25 : vessel floats in 9. Hence for a difference in saltness of 25, the mark is put down 7·5 inches : how much will it be put down for a difference in saltness of 25 − 9 = 16?

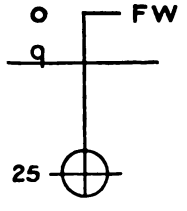


FIG. 37A.

$$= \frac{7{\cdot}5 \times 16}{25} \text{ inches}$$

= 4·8 inches,

and F.W. mark would be out of water 2·7 inches.

4. The tons per inch in sea water is 40 : what would it be in water where Hydrometer reads 16?

As the water is lighter, fewer tons will sink her one inch.

$$= \frac{40 \times 1016}{1025}$$

= 39·65 tons per inch.

The Station Pointer.—The Station Pointer is a graduated circle having three *radial arms*, one of which is fixed, and the other two moveable, the latter being provided with verniers and clamp and tangent screws (Fig. 38).

The use of the Station Pointer is to locate the ship's position on a chart from the angles subtended by three lighthouses or other fixed objects on shore.

With a sextant, the horizontal angles between the middle object and each of the other two are measured, and the moveable arms are set to these angles.

Then the instrument is laid on the chart with the bevelled edge of the fixed arm on the middle object, and moved about

until the bevelled edges of the other two arms fall on the other two objects. The centre of the circle gives the ship's position.

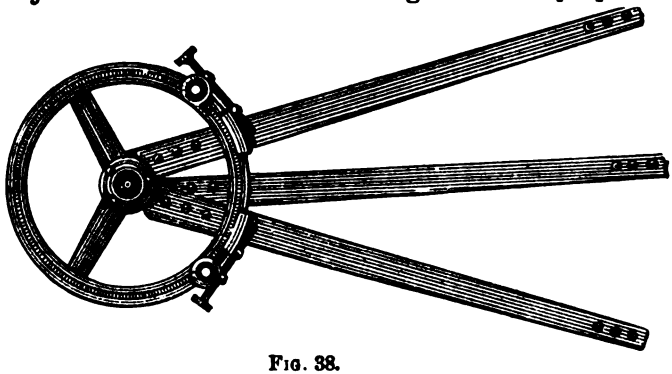


FIG. 38.

The Sextant (Fig. 39).—The "full" Sextant for sea use consists of a metal frame in the form of a Sector of about 60°, with an *arc* or limb graduated to degrees and 10′ spaces. An index bar, pivoted at the centre of the circle, carries the silvered *Index Glass*, and at the other end a Vernier or Nonius for subdividing to 10″. The index bar is provided with clamp and tangent screws, and carries a reading microscope. The *Horizon Glass*—half of which is silvered—stands on the frame. Both glasses are provided with screens for taking off the glare of a strong light when the instrument is used. A handle at the back of the frame is convenient for holding, and a collar, which is moveable towards or from the frame by means of a milled-head screw, serves to hold a plain tube or a telescope.

When in correct adjustment, the index and horizon glasses are *perpendicular* to the plane of the instrument (the upper surface of the frame), and *parallel* to each other when the pointer on the vernier indicates 0 on the arc. The axis of the telescope when screwed into the collar is *parallel* to the plane of the instrument.

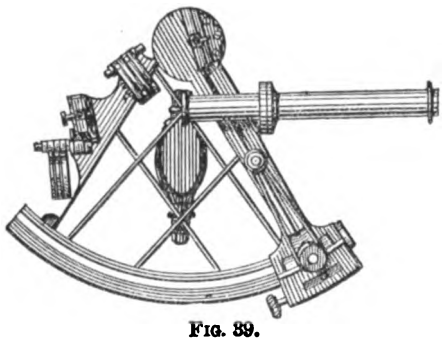


FIG. 39.

The first two adjustments are essential, but if the glasses are slightly out of parallelism, it is of little consequence, for the "index error" can be easily determined and applied. Screws are provided for making all the adjustments.

QUESTIONS ON THE ADJUSTMENTS OF THE SEXTANT IN BOARD OF TRADE EXAMINATIONS FOR MATES AND MASTERS.

1. What is the first adjustment of the sextant?
2. How do you make that adjustment?
3. What is the second adjustment?
4. Describe how to make that adjustment.
5. What is the third adjustment?
6. How would you make the third adjustment?
7. In the absence of a screw, how would you proceed?
8. How would you find the index error by the horizon?
9. How is it to be applied?
10. Place the index at error of minutes to be added, clamp it and leave it

NOTE.—The examiner will see that it is correct.

11. The examiner will then place the zero of the vernier on the arc, not near any of the marked divisions, and the candidate will read it.

NOTE.—In all cases the applicant will name or otherwise point out the screws used in the various adjustments.

12. How do you find the index error by the sun?
13. How is the same applied?

Suppose the reading on the arc to be , and the reading off the arc to be , find the index error.

14. What proof have you that those measurements or angles have been taken with tolerable accuracy?
15. Describe the fourth adjustment.

ANSWERS TO THE QUESTIONS ON THE SEXTANT.

1. To set the **Index Glass** *perpendicular* to the plane of the sextant.

2. The index should be clamped at about 60°, and the sextant held face upwards with the limb from the observer; then, on looking into the index glass towards the limb, if the reflected and real arcs do not coincide, turn the adjusting screws at the back of the glass until they do coincide.

3. To set the **Horizon Glass** *perpendicular* to the plane of the sextant.

4. The index being clamped at 0, the sextant should be held *horizontally*, face upwards—then, on looking through the telescope collar towards the horizon, if the reflected and real horizons do not coincide, turn the upper adjusting screw at the back of the glass until they do coincide. By night, look at a star and see if its reflection passes up and down through the star, if it does the horizon glass is truly perpendicular, if not, use the upper screw to make it do so.

5. To set the **Horizon Glass** *parallel* to the **Index Glass** when the index is at 0 on the arc.

6. The index being clamped at 0, the sextant should be held *vertically*; then, on looking through the tube or telescope at the horizon, if the reflected and real horizons do not coincide, turn the lower adjusting screw at the back of the glass until they do coincide.

7. Find the **Index Error**.

8. Placing the index at 0, and holding the sextant as for the third adjustment, make the reflected and real horizons coincide by means of the *Tangent Screw*. By night, if when the index is at 0, the true and reflected stars do not coincide, make them do so by means of the tangent screw. The reading "On" or "Off" is the Index Error.

9. It is *subtractive* if "on," and *additive* if "off."

10 and 11 *are practical and require no written answer.*

12. Clamping the index at 40′ "on" the arc and looking through the sight-tube or telescope at the sun, make the limbs of the reflected and real suns touch by means of the tangent screw, and note the reading; then clamp at 40′ "off," and again make the contact and note the reading. Half the difference of the readings is the Index Error.

13. If the greater reading is "on," the error is *subtractive*; if "off," it is *additive*.

14. One-fourth the sum of the readings should be the Sun's Semi-diameter, as in the Nautical Almanac for the given day.

15. The Fourth Adjustment is to set the line of collimation or axis of the telescope parallel to the plane of the instrument. The adjustment is made as follows: Screw on the *Inverting Telescope*, and, getting it in correct focus, turn the eye-piece round until two parallel wires in the tube are parallel to the plane of the sextant. Then select two celestial objects not less than 90° apart, and holding the sextant with its face in the plane of the objects, move the index forward until the reflected image of one object comes into view in the centre of the field of the telescope with the direct image of the other. Bring both images in contact on the wire nearest the frame, and by slightly moving the instrument get them on the other wire: if they remain in contact, the adjustment is perfect; if they separate, the object end inclines towards the frame; but if they overlap, it declines from it. The adjustment is made by means of two screws in the collar which holds the telescope.

The effect of the collimation error is to make the observed angle *too great*.

Any error of centering is an essential defect of the instrument, and may be detected by comparing the distance of two stars as measured by the sextant, and as measured by a standard instrument, or deduced from calculation.

Any error due to eccentricity of the limb may be determined by Lord Ellenborough's method.

Select two stars of nearly equal zenith distances on opposite sides of the zenith, and from their altitudes calculate the latitudes. If the latitudes agree there is no error, otherwise half the difference of the latitudes is the error, *additive* when the latitude from the star on the equatorial side is greater, and *subtractive* if less.

For suitable stars to observe see "Stars and Sextants," by J. A. Sprigge and others.

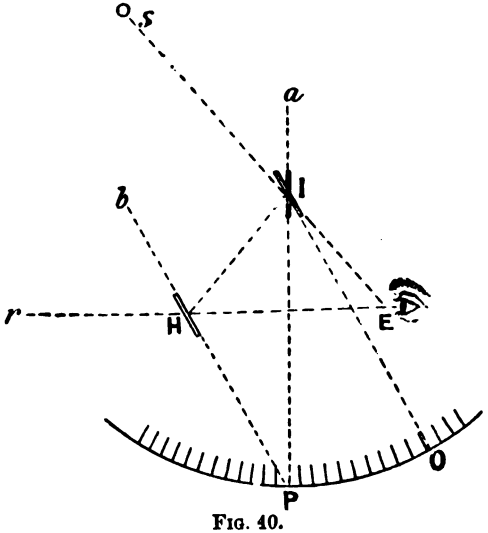


FIG. 40.

Principle of the Sextant.—The principle of the sextant is based on the optical fact that the angle of incidence of a ray of light upon a mirror is equal to the angle of reflection, and hence "The angle between the first and last directions of a ray which has suffered two reflections in one plane is equal to twice the inclination of the reflecting surfaces to one another."

Let I and H represent the index and horizon glasses of a sextant. Light from S falling on I is reflected at an equal angle to H,

and thence reflected to the eye at E, so that the object S would be seen by reflection in the direction E*r*, and if *r* were the horizon, the object's image would be seen on the horizon.

IO represents the index bar when the mirrors are parallel, and IP when it has been moved to P. Produce PI to *a*, PH to *b*, and EH to *r*.

Now, since $\angle$ HIP = $\angle$ SI*a* = $\angle$ EIP (Euc. I. 15), the $\angle$ HIE is bisected by IP, making $\angle$ HIP = $\angle$ EIP.

Also, since $\angle$ *b*HI = $\angle$ EHP = $\angle$ *b*H*r* (Euc. I. 15), the $\angle$ *r*HI is bisected by *b*H, making $\angle$ *r*H*b* = $\angle$ *b*HI.

The $\angle$ *r*HI = $\angle$ HIE + $\angle$ HEI (Euc. I. 32); therefore $\angle$ *r*H*b* = $\frac{1}{2}\angle$ HIE + $\frac{1}{2}\angle$ HEI = $\angle$ HIP + $\frac{1}{2}\angle$ HEI.

Again, $\angle$ *r*H*b* (= *b*HI) = $\angle$ HIP + $\angle$ HPI; therefore HIP + HPI = HIP + $\frac{1}{2}$ HEI.

Taking away the common angle HIP, there remains $\angle$ HPI = $\frac{1}{2}\angle$ HEI; and as $\angle$ OIP = $\angle$ HPI, because IO and HP are parallel, it follows that the angle through which the idex bar IP has moved from *zero* is half the angle described by the image of the object. For this reason an arc of $\frac{1}{2}°$ on the limb is read as 1°, and an arc of 60° will measure 120°.

ART. 87. **Principle of the Vernier, or Nonius.**—The value of the smallest division on an ordinary sextant is 10, but the

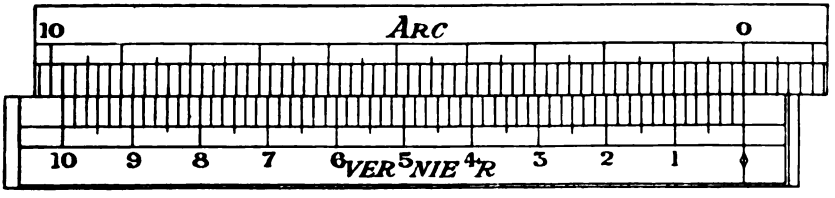


FIG. 41.

vernier attached to the index bar subdivides into single minutes and 10, 20, 30, etc. seconds.

This is accomplished by taking 59 divisions of the limb for the length of the vernier, and dividing it into 60 equal parts;

$$\text{Thus, } 60\text{V} = 59\text{L}$$

$$\therefore \text{IV} = \frac{59}{60}\text{L} = \text{L} - \frac{\text{L}}{60}$$

that is, one division on the vernier is $\frac{1}{60}$ of 10′ = 10″ shorter than one on the limb. Six of these vernier divisions are marked 1, and the whole length reads 10′.

When the pointer or 0 on the vernier coincides with a cut on the limb, the next vernier cut falls short of the one above it on the limb by 10″, the second one by 20″, the third by 30″, etc. So that if the index is moved forward until the first, second,

third, etc., cut coincides with the one above it on the limb, the pointer will have moved 10″, 20″, 30″, etc., and so on to the end of the vernier scale. When the 10′ cut coincides, the pointer will have moved a whole division of the limb, *i.e.* 10′. Therefore, to find how far the pointer or 0 has moved from the last limb division, it is only necessary to see what cut on the vernier coincides with a cut on the limb.

The same rule holds good when the reading is "off" the arc, only in this case both scales must be read from left to right.

Star Maps and Planispheres.—A knowledge of the Star-groups or Constellations and the names of the principal bright Stars is essential to the Navigator who would make use of these objects to find the Latitude and Longitude at sea.

Many Star-maps and "Planispheres" are in use, but it is unnecessary to mention here more than the following:—

Proctor's Star Atlas.
Philip's Planisphere.
Poole's Celestial Planisphere.

On a Star-map or a Celestial Globe, the position of a star can be found from its R.A. and Dec. in the same manner as a place is located on a globe or chart by the Latitude and Longitude.

Planispheres are designed to show the principal star-groups above the Horizon at a given hour. Philip's Planisphere consists of a moveable disc turning in a leather frame about the pole as centre. On the disc is a map of the heavens, hour circles being drawn for every two hours, the equinoctial, ecliptic, and circles of declination are also shown. On the circumference of the disc the days of the month are printed. Out of the leather frame an oval is cut, and around the edge of the frame are printed the hours of the day and night. To find the stars visible at any time, turn the disc until the date is abreast the hour; the stars visible in the oval are those above the horizon at the time, and in the place for which the planisphere is constructed.

A wire stretched across the middle of the ellipse represents the Meridian.

The Planisphere gives by inspection the approximate Time of Meridian Passage of a star as well as the times of Rising and Setting. It can also be used to find by inspection what bright stars are within a given time from the Meridian at any given hour of Greenwich or local Time.

Tables of Meridian Passage of Stars.—Table XLIV. of Norie gives the apparent times of Mer. Pass. of the principal stars at intervals of five days, and as a star crosses the Meridian about 4^m *earlier* on each successive day, it is easy to find the time for intermediate days.

Table 27 of Raper gives the time of Mer. Pas. of fifty-five bright stars on the first day of each month, and by subtract-

ing 4^m for each day elapsed, as before, the required time is approximately found for any day, or correction found in Table 27A.

The Meridian Altitude of a selected star being computed from its Dec. and the *Approximate* Latitude, the sextant can be set to this angle; and as the approximate time of Mer. Pass. is known, the observer can be in readiness to measure the altitude at the moment the star is on the Meridian, and so obtain the *Correct* Latitude.

Exercises.

1. The meridional parts for lat. 38° 25′ are 2500·08. Calculate those for 38° 31′. *Ans.* 2507·6.

2. The meridional parts for lat. 57° 46′ are 4267·97. Calculate those for 57° 50′. *Ans.* 4275·5.

3. The vernier of a sextant, the limb of which is divided to 10′, has 40 divisions, coinciding with 119 divisions on the limb. With what accuracy may the instrument be read? *Ans.* 15″.

4. The limb of a sextant is divided to 15′, and 59 divisions on the limb are formed into 60 divisions on the vernier. With what accuracy may it be read? *Ans.* 15″.

5. If every inch on a barometer scale be divided into 10 parts, show how to form a vernier such that the reading of the instrument may be taken to ·005 of an inch. *Ans.* Take 9 divisions and divide into 20 equal parts.

6. Describe Philip's Planisphere.

7. Where may the Meridian Passage of a star be found? Is it given in mean or apparent time?

8. On the arc readings, 34′ 50″, 34′ 20″, 34′ 20″; off the arc readings, 29′ 40″, 29′ 0″, 29′ 20″; find the Index Error and Semidiameter. *Ans.* − 2′ 35″ : 15′ 57·5″.

9. On the arc reading, 28′ 50″; off the arc, 34′ 40″; find Index Error and Semidiameter. *Ans.* + 2′ 55″ : 15′ 52·5″.

10. When the hydrometer marks 16, a ship's draught is 25 ft. 8 in.: find her draught at sea. *Ans.* 25 ft. 5 in.

11. At sea a ship draws 18 ft. 10 in.: what will her draught be in a dock where the reading on the hydrometer is 12? *Ans.* 19 ft. 1 in.

12. Between the F.W. mark and centre of disc, the distance is 6 inches. How much can the Plimsoll mark be submerged in dock, where hydrometer reads 9? How much is F.W. mark above water level? *Ans.* 3·8 in. : 2·2 in.

13. Ship's displacement is 10,000 tons. Her tons per inch in salt water 40. What will be her change in draught on entering dock where hydrometer reads 10? *Ans.* 3·7 ins.

14. The area of a ship at load water level is 15,120 sq. ft. How many tons will sink her 1 inch in salt water? *Ans.* 36 tons.

15. Barometer reads 29·92 in.: find what pressure per sq. in. this is equivalent to, taking the Specific Gravity of mercury to be 13·598. *Ans.* 14·72 lbs.

16. Barometer reads 29·65 inches: find corresponding readings in mm. and mbars. *Ans.* 753·1 mm.; 1003·9 mbars.

17. Express a barometrical reading of 745·6 mm. in inches and mbars. *Ans.* 29·35 ins.; 994·1 mbars.

18. Convert a reading of 983·4 mbars. to mm. and ins.
Ans. 29·04 ins.; 737·6 mm.

19. The temperature of the maximum density of water is 39·4° F., and at which salt water freezes is 26° F. Express these as Cent. and Absolute readings. *Ans.* 4·1° C. and 277·1° A.; $-3\frac{1}{3}$° C., $269\frac{2}{3}$° A.

20. Alcohol boils at 68° C.; absolute zero is − 273 C. What are the corresponding temperatures Fahr.? *Ans.* 154·4° F.; − 459·4° F.

CHAPTER III.

LOGARITHMS.

ART. 1.—In the expression $N = a^x$; "N" is the number obtained by raising "a" to the power of "x"; keeping "a" fixed in value, and finding all the values of "N" obtained by performing the operation a^x: where "x" may be anything fractional or integral, results in a table of logarithms: the fixed, or constant, number is called the "Base," the power chosen the "Logarithm": thus a Logarithm is the index to which the Base must be raised to give the number. In common logs the base is always 10, hence the logs used in our problems represent the power to which 10 must be raised to equal the number. Logs are chiefly fractional, since very few numbers are exact powers of 10: the whole number, or integral part of the log., is called the *index*, and the decimal part the *mantissa*. It is the decimal part that is found on pages 2–16 in Norie's Tables, or any other nautical tables. The index is added by the computer, and is found by subtracting unity, "1," from the number of figures in the integral part of the quantity; this is evident from the following considerations:—

$$10^0 = 1,\ 10^1 = 1,\ 10^2 = 100,\ 10^3 = 1000.$$

Thus from definition—

$$\begin{aligned} \log 1 &= 0 \\ \log 10 &= 1 \\ \log 100 &= 2 \\ \log 1000 &= 3; \quad \text{and so on.} \end{aligned}$$

Hence the logs of all numbers between 1 and 10, those which only have *one* digit in the whole number part, fall between 0 and 1, and so are wholly decimal, having 0 for index. Again, the logs of all numbers between 100 and 1000, which are expressed by *three* digits in the whole number part, fall between 2 and 3, and so always have 2 for the index.

The index of a wholly decimal number is negative, and is found by adding unity, "1," to the number of cyphers immediately after the decimal point, and before the first significant figure, and putting a minus sign ($-$) over it.

Logs of all numbers which are expressed by the same figures, but differ only in the position of the decimal point, have the same mantissa.

A minus index is made positive by adding 10 to it.

ART 2.—**Finding the Logarithms of Numbers.**—In Table, pp. 2–16, Norie, the first column is headed "No."; here are entered all numbers from 100 to 999. The other columns are headed 0 to 9; and the end one "Diff.": the first three figures of the number whose log is required are found under "No."; the fourth figure represents the column to be used; the number found in this column abreast the figures under "No." is the mantissa of log required. The index must be placed as indicated above. Should there be fewer than four figures in the number, add cyphers to make up four and take out mantissa, using index corresponding to original number.

Examples.—

Numbers.			Logarithms.
1. 6	...	...	0·778151
2. 34	...	...	1·531479
3. 420	...	...	2·623249
4. 7·856	...	...	0·895201
5. 38270	...	...	4·582858
6. ·5	...	...	$\bar{1}$·698970
7. ·065	...	...	$\bar{2}$·812913
8. ·0013	...	...	$\bar{3}$·113943
9. ·00000671	...	...	$\bar{6}$·826723

NOTE.—The negative index is generally changed into a positive for Nautical Computations by adding 10, so that $\bar{1}$ becomes 9, $\bar{2}$ becomes 8, etc.

When the number contains more than four figures, take out the log. of the first four; then find the correction for the remaining figures by multiplying the "Diff." by these figures, and cutting off as many to the right. The remaining quantity is the correction to be added.

Examples.—

Number.			Logarithms.	
1. 2·340401	...	...	0·369216	Diff. 185
			74	401
				185
				740
			0·369290	74,185
2. 55300·017	...	...	4·742725	Diff. 78
			0	0017
				546
				78
			4·742725	1326
3. 86430000	...	...	7·936664	

NOTE.—There is no correction for 0's at the end of a number.

EXERCISES.

1. Find the Logarithms of the following numbers:—

3	3·409
47	7·5
50	163·45
100	704·009
275	32636·04
8360	·5
57·29	·049
365·74	·0036
1·0009	·001204
75230	·006728
62·7014	·00028041
175·008.	·0000753
23002·7	·0346009
164·3019	·075604
21550·007	·3670007
23460	

ART. 3.—**Finding the Numbers corresponding to Given Logarithms.**—When searching for the number corresponding to a log., no account is taken of the index, the mantissa is looked out in the tables, and will be found to either correspond exactly with an entry there, *or* fall between two entries. In the event of exact coincidence, the figures in the "No." column and at the top of the column where the log. is found, give the first four figures of the Number required: the place of the decimal point depends on the Index, and is found by counting off from the left one more than the number of figures in the index. Should the index call for more than four digits in the Number, cyphers are added to make up the required quantity.

Should the log. not agree, as is more often the case, the next *less* entry is taken from the tables and subtracted from the given log., cyphers are added to this difference and then divided by the number abreast in the "Diff." column. The figures in the dividend are extra ones to be placed to the right after the four figures taken from the tables corresponding to the *less* entry used.

Examples.—

Given logs.	Numbers.
1. 0·113943	1·3
2. 1·791480	61·87
3. 2·724931	530·8

When the Index exceeds three, and more than four figures are required, the log. *next less* in the tables is taken out, and the remainder divided (as in Division of Decimals) by the "Diff." gives the additional figures required.

Examples.—

Given logs.			Numbers.
1. 6·490379	...	...	3092993

490239

Diff. 141)140·0(993
1269

1310
1269

410
423

2. 5·820006	...	...	660703

820004

Diff. 66)2·00(03
198

2

3. 4·875003	...	...	74990

When the index is negative, and the number therefore a decimal, the position of the first figure (of value) is indicated by the index; thus, $\bar{1}$ shows it will occupy the *first* place after the decimal point, $\bar{2}$ requires the first figure (of value) in the *second* place, etc.

Examples.—

Logs.			Numbers.
$\bar{1}$·432488	...	...	·2707
$\bar{2}$·715339	...	...	·05192
$\bar{4}$·934550	...	...	·0008601.

NOTE.—These index numbers when increased by 10 become 9, 8, 6.

EXERCISES.

2. Find the numbers of which the following are the Logarithms :—

3·203577	5·175801
2·145818	6·531481
1·700271	7·000435
0·715920	5·361917
0·004382	4·041393
4·681484	6·049606
5·690196	0·000434
6·148900	3·670154
7·040203	7·276003
4·806044	5·380032

NOTE.—Since logarithms are merely the powers of "10," which would make the numbers, to multiply quantities, we do as in Algebra, *add* the powers.

To divide quantities, *subtract* the powers.
To perform Involution, *multiply* the log. by the power.
To perform Evolution, *divide* the log. by the root.

ART. 4. **Multiplication by Logarithms.**

Rule.—Add together the logarithms of the numbers, and find the number corresponding to the sum of logarithms.

Examples.—

1. Multiply 234 by 1·76 by common logarithms.

234	log 2·369216
1·76	log 0·245513
Ans. 411·8	log 2·614729

2. Multiply 32640 by ·00876.

32640	log 4·513750
·00876	log 7·942504
Ans. 285·9	log 2·456254

3. Multiply together 3·17, 24·781, and ·067.

3·17	log 0·501059
24·781	log 1·394119
·067	log 8·826075
Ans. 5·263	log 0·721253

4. Multiply 760·56 by ·0001877.

760·56	log 2·881133
·0001877	log 6·273464
Ans. ·1427	log 9·154597

5. Multiply ·89472 by ·003708.

·89472	log 9·951687
·003708	log 7·569140
·003318	log 7·520827

Remark.—It is evident that the product of a whole number by a whole number is a *whole number*, and the product of a decimal by a decimal is a *decimal;* but when a whole number and a decimal are multiplied together, the answer contains a *whole number* if the sum of indexes amounts to ten or more (in which case ten is rejected); but if the sum is less than ten, the answer is a *decimal.*

EXERCISES.

1. Multiply 479 by 41.
2. ,, 4672 by 5.
3. ,, 1037·5 by 269.
4. ,, 35890 by 10.
5. ,, 16·536 by 42·90.
6. ,, 12·45 by 3·84.
7. ,, 7·5642 by 279·008.
8. ,, 2·3467 by 123·006.
9. ,, 164·3019 by 21550·007.

10. Multiply 356·8 by ·0167.
11. ,, ·001204 by ·006728.
12. ,, 32415·5 by ·0002.
13. ,, ·046009 by ·00028041.
14. ,, ·1000012 by ·00259.
15. ,, 100 by ·005.
16. ,, 7642·005 by 3·26507.
17. ,, 999999 by ·1.
18. ,, ·0867 by 235·6004.
19. ,, ·98421 by ·234.
20. ,, ·00765 by ·23461.
21. ,, 28550 by ·0444.
22. ,, 30000000 by ·00765.
23. Find the product of 2·36 × 19·742 × ·3897.
24. ,, ,, 104·26 × ·018 × 95·41 × ·8.

ART. 5. **Division by Logarithms.**

Rule.—Subtract the log of the Divisor from the log of the Dividend, and then find the corresponding number.

Examples.—

1. Divide 37640 by 4705.

37640	log 4·575650
4705	log 3·672560
Ans. 8·000	log 0·903090

2. Divide 246·78 by ·0272.

246·78	log 2·392310
·0272	log 8·434569
Ans. 9073	log 3·957741

3. Divide ·14 by ·09876.

·14	log 9·146128
·09876	log 8·994581
Ans. 1·418	log 0·151547

4. Divide 207·4 by 3709·1.

207·4	log 2·316809
3709·1	log 3·569269
Ans. ·05592	log 8·747540

5. Divide ·0076438 by ·92.

·0076438	log 7·883310
·92	log 9·963788
Ans. ·008308	log 7·919522

Remark.—The answer always contains whole numbers (one more than the index) when the Divisor is *less* than the Dividend, but is all decimals when the Divisor is the *greater*.

EXERCISES.

1. Divide 32636·04 by 704·009.
2. ,, 687·31 by 2·34.
3. ,, 45·2 by ·005.
4. ,, 985·712 by ·0056.
5. ,, 9·23 by 2·001.
6. ,, ·64238 by ·026.
7. ,, 99 by ·0000745.
8. ,, 35161·9 by 34·24.
9. ,, ·746202 by ·52496.
10. ,, 197·65773 by ·00139.
11. ,, 35 by 34·24063.
12. ,, 9 by 874303.
13. ,, 16192·2 by ·01086.
14. ,, 1·3 by ·0000000290000.
15. ,, ·001 by 1000.
16. ,, ·000876 by ·0000438.
17. ,, ·012 by ·009847.
18. ,, ·76591 by ·13.
19. Find the value of $\frac{198·79}{3·46}$.
20. ,, ,, $\frac{2·13 \times 856·4}{·087}$.
21. ,, ,, $\frac{·00175 \times 3872·4}{7·69 \times ·0432}$.
22. ,, ,, $\frac{8·32 \times ·096 \times 248·109}{1·74 \times 23·42 \times 148·6}$.
23. ,, ,, $\frac{100·108 \times ·36749}{·00984 \times 468·06}$.
24. ,, ,, $\frac{·0756 \times ·00998 \times ·247}{·0001 \times ·0137 \times ·142}$.

ART. 6. **Involution by Logarithms.**

Rule.—Multiply the log of the number by the required Power, and then find the corresponding number.

Example.—Required the 7th power of 1·09.

1·09 log 0·037426

7

Ans. 1·828 log 0·261982

EXERCISES.

1. Required the square or 2nd power of 23·4.
2. ,, cube or 3rd power of 1·876.
3. ,, 5th power of ·08967.
4. ,, 9th power of ·1039.
5. ,, 13th power of ·367.

ART. 7. **Evolution by Logarithms.**

Rule.—Divide the log of the given number by the required Root, and then find the corresponding number.

Examples.—

1. Required the square root of 10816.

10816 log 2)4·034066

Ans. 104 log 2·017033

2. Required the cube root of 42875.

42875 log 3)4·632204

Ans. 35 log 1·544068

3. Required the 5th root of 9·618.

9·618 log 5)0·983085

Ans. 1·573 log 0·196617

4. Required the 7th root of 234·68.

234·68 log 7)2·370476

Ans. 2·181 log 0·338639

5. Required the cube (or 3rd) root of ·004913.

·004913 log 3)$\bar{3}$·691347

Ans. ·17 log $\bar{1}$·230449

6. Required the cube root of 0·06174.

·06174 log $\bar{2}$·790567 or $-3+1{\cdot}790567$

$3)\ -3+1{\cdot}790567$

Ans. ·3952 log $\bar{1}$·596856

7. Required the 5th root of ·007652.

·007652 log 5)$\bar{3}$·883775 ($\bar{3}=\bar{5}+2$)

Ans. ·3774 log $\bar{1}$·576755

EXERCISES.

1. Required the square root of 306·25.
2. ,, cube ,, 46·656.
3. ,, 5th ,, ·002435.
4. ,, cube ,, 12167000.
5. ,, 7th ,, ·00000002097152.

Remark.—With a Table of Logarithms containing only six places of figures, such as is used for Navigation, the answers in the preceding exercises cannot be expected to be correct to more than four or five places.

Of course, it must be understood that whenever logs are *added* in the calculations of problems in Navigation and Nautical Astronomy, it means that the terms are *multiplied* together, and when logs are *subtracted*, it means division. This is important to remember when one of the terms is zero (or 0).

ADDITIONAL EXERCISES.

Find by logs the values of the following:—

1. $\sqrt{20675}$; $\sqrt{769{\cdot}52}$; $\sqrt[3]{46976482}$.
2. $(29{\cdot}865)^2$ and $(98{\cdot}7652)^2$.
3. $(91{\cdot}543)^2 \times \sqrt[3]{8794{\cdot}027}$; $\sqrt{\dfrac{4682}{99 \times 37987}}$.
4. $\dfrac{\sqrt{8794{\cdot}08}}{\sqrt[3]{276509}}$.

ART. 8. **To find the Logarithm of a Given Ratio** (Sin, Cos, Tan, etc.).—In Table XXV., pp. 264–353, Norie, are entered the logarithms of the Sines, Cosines, Tangents, etc., of the various angles from 0° to 90°, given to every half-minute, two pages being reserved for each degree: on the top of the page the degrees from 0° to 44° in consecutive order, the minutes and half-minutes being entered in the column on the left-hand side, the names of the ratios at the top; and in columns between two ratios the differences for each second of arc to 30″. At the bottom of the page the degrees from 45° to 89° are entered, the names of the ratios, and on the right-hand side reading upwards the minutes and half-minutes of arc. In order that there shall be no minus indices, 10 is added to each index before tabulation. Pages 240–263 at the beginning of the table are devoted to the Sines of small angles from 0° to 4° to each second of arc, and to the Cosines of large angles from 86° to 90°. To find a log, turn up the number of degrees on the top of the page if the angle is less than 44°, at the bottom of the page if the angle exceeds 44°; find the minutes on the left if the degrees are at the top, and on the right if the degrees are at the bottom, and abreast the minutes in the column corresponding to the ratio required is the log.

Examples.—

L sin	23° 25′	=	9·599244	L cos	40° 36′	=	9·880397
L sin	58° 36′	=	9·931229	L tan	47° 28′	=	10·037440
L sin	27° 43½′	=	9·667666	L cos	65° 22½′	=	9·619800
L sin	64° 14½′	=	9·954549	L sec	14° 19′	=	10·013701
L cosec	75° 56′	=	10·013222	L cosec	51° 40′	=	10·105454
L cot	37° 28½′	=	10·115412	L tan	13° 47′	=	9·389724

If the angle exceeds 90°, subtract the amount from 180° and take out the *same* ratio of the remainder; or take 90° from the angle and take out the complementary ratio of the remainder.

Examples.—

L sin	102° 47′	= L sin	77° 13′	= L cos	12° 47′	=	9·989100
L cosec	111° 15′	= L cosec	68° 45′	= L sec	21° 15′	=	10·030580
L cot	98° 32′	= L cot	81° 28′	= L tan	8° 32′	=	9·176224

To correct for odd seconds in an angle, find the amount in

the proper "Diff." column corresponding to the seconds required, add to the log for sines, tangents, and secants, and subtract for cosines, cotangents, and cosecants.

Remark.—The beginning part of this table gives the L Sines of small angles (0° to 4°) to seconds of arc: or the L Cos of large angles (86° to 90°): seeing that sines and cosecants are complementary, and that the sines and tangents, also the cosecants and cotangents of small angles are approximately equal, by subtracting the L Sines from 20 we have the corresponding L Cosec, and when the angle is very small (0° to 0° 30′) the L Cot.; this avoids using the very large differences tabulated for 30″.

Examples.—

L sin	56° 37′ 25″ = 9·921726	corr. for 25″	= +35
L cos	37° 26′ 47″ = 9·899779	,, 17″	= −27
L sec	55° 14′ 38″ = 10·244061	,, 8″	= +24
L cosec	104° 17′ 20″ = 10·013648	,, 10″	= − 5
L cos	88° 12′ 26″ = 8·495333	(large "cos" table).	

ART. 9. **To find the Angle corresponding to a Logarithm.**—Reverse the above process, looking out the log in its proper column, and taking the next less log to subtract from the given log in the case of sines, secs, and tans, and the next greater one in the case of cosines, cosecs, and cots, to subtract the given log from; this difference referred to the corresponding "Diff." column gives the extra seconds.

Examples.—

9·764318 = L sin	35° 32′ 3″	diff. of 10	gives 3″
10·124397 = L tan	53° 5′ 45″	,, 65	,, 15″
9·914386 = L cos	34° 48′ 25″	,, 36	,, 25″
8·452450 = L sin	1° 37′ 27″	(small "sine" table)	
9·796824 = L tan	32° 3′ 41″	diff. of 51	gives 11″
10·495387 = L cot	17° 43′ 27″	,, 195	,, 27″
9·918645 = L sin	56° 0′ 50″	,, 28	,, 20″

CHAPTER IV.

SOLUTION OF PLANE TRIANGLES.

ART. 10. **Trigonometrical Ratios.**—Comparisons can only be made between two quantities which refer to the same kind of thing. A Ratio is the comparison of two quantities, and in Trigonometry the quantities compared are the lengths of the sides, or functions of the angles. A Ratio may be expressed by writing the two quantities with two dots (:) between them, or by forming a vulgar or decimal fraction of them. In every triangle there are three sides, and since two are used to form the ratio, it follows that these can be arranged in "six" different ways; and hence there are "six" Trigonometrical Ratios. Each one has received a definite name; thus there are sines, cosines, tangents, cotangents, secants, and cosecants.

In any right-angled triangle the side opposite the right angle is always called the HYPOTENUSE. The side which with the hypotenuse forms the angle is called the BASE. The side opposite the angle under consideration is called the PERPENDICULAR.

The ratio	$\frac{\text{perp}}{\text{hyp}}$	is called	"sine"	or shortly	"sin."
"	$\frac{\text{base}}{\text{hyp}}$	"	"cosine"	"	"cos."
"	$\frac{\text{perp}}{\text{base}}$	"	"tangent"	"	"tan."
"	$\frac{\text{base}}{\text{perp}}$	"	"cotangent"	"	"cot."
"	$\frac{\text{hyp}}{\text{base}}$	"	"secant"	"	"sec."
"	$\frac{\text{hyp}}{\text{perp}}$	"	"cosecant"	"	"cosec."

Thus in triangle PQR, right angled at Q, the *hypotenuse* is *always* PR; and when the ratios of P are considered,

PQ is the base,

RQ is the *perpendicular*.

But when the ratios of R are under consideration, then—

RQ is the *base*,

PQ is the *perpendicular*.

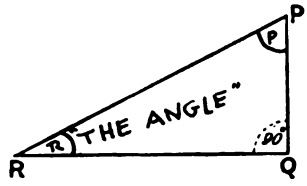


FIG. 42.

Thus from definition—

$$\sin P = \frac{RQ}{PR} \quad \text{and} \quad \sin R = \frac{PQ}{PR}$$
$$\cos P = \frac{PQ}{PR} \quad \text{,,} \quad \cos R = \frac{RQ}{PR}$$
$$\tan P = \frac{RQ}{PQ} \quad \text{,,} \quad \tan R = \frac{PQ}{RQ}$$
$$\cot P = \frac{PQ}{RQ} \quad \text{,,} \quad \cot R = \frac{RQ}{PQ}$$
$$\sec P = \frac{PR}{PQ} \quad \text{,,} \quad \sec R = \frac{PR}{RQ}$$
$$\operatorname{cosec} P = \frac{PR}{RQ} \quad \text{,,} \quad \operatorname{cosec} R = \frac{PR}{PQ}$$

Hence— $\sin P = \cos R$
$\tan P = \cot R$
$\sec P = \operatorname{cosec} R$

and since P and R are complementary, this is expressed by saying that—

The sine of an angle is equal to the cosine of its complement, and *vice versa*.

The cosine of an angle is equal to the sine of its complement.

And so with tan and cot : sec and cosec. It will be seen that since—

$$\operatorname{cosec} = \frac{1}{\sin} \qquad \sec = \frac{1}{\cos} \qquad \cot = \frac{1}{\tan}$$

the cosec is the reciprocal of the sin, sec of the cos, and cot of the tan. This fact is useful, as it enables a divisor to be replaced by a multiplier in a formula; the same result being obtained whether we divide by sin, cos, or tan; or multiply by their reciprocals cosec, sec, or cot.

ART. 11.—**Changes in Sign and Magnitude of Angles.**—Consider two lines POQ and ROT meeting in O at right angles. It is necessary, in order to know in what direction a line is drawn, that some difference should be made when a line is drawn perpendicularly to TR up or down, or when perpendicularly to PQ towards R or T; this is done by the use of the signs + and −. Lines drawn perpendicularly up from TR are considered +, down −. Lines drawn perpendicularly to PQ to the right are considered +, to the left −.

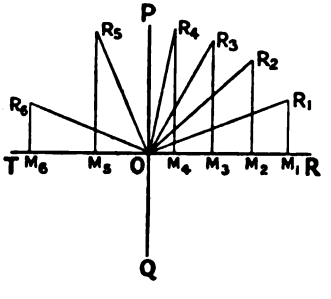


FIG. 43.

Let a line with centre O revolve from the position OR, in a direction against the hands of a clock. In going from OR to OP, it traces out all angles from 0° to 90°, and in going from OP to OT all angles from 90° to 180°; larger values need not be considered, as the angles and arcs dealt with in this book do not exceed 180°.

Suppose the revolving line to take up positions indicated by R_1, R_2, R_3, R_4, R_5, R_6. Draw perpendiculars from these points meeting TR in M_1, M_2, M_3, M_4, M_5, M_6;

Then—

$$\sin R_1OM_1 = \frac{R_1M_1}{OR_1}$$

$$\sin R_2OM_2 = \frac{R_2M_2}{OR_2}$$

$$\sin R_6OM_6 = \frac{R_6M_6}{OR_6}$$

Since $OR_1 = OR_2 = \ldots OR_6$, it follows that the above fractions, which represent the values the sin has as the angle increases, also increase as the line revolves from OR to OP, and decrease as the line revolves from OP to OT. In the position OR or OT the perpendicular vanishes, consequently—

$$\sin 0° = 0$$
$$\sin 180° = 0$$

Also, when the revolving line coincides with OP, the perpendicular coincides with and is equal to it, or—

$$\sin 90° = 1$$

All the time, however, the perpendicular has been above TR and is reckoned +.

We arrive at the following conclusions:—

As the angle increases from 0° to 90°, the sine takes all values from 0 to 1.

As the angle increases from 90° to 180°, the sine takes all values from 1 to 0.

In both the above cases the sine is positive.

Again—

$$\cos R_1OM_1 = \frac{OM_1}{OR_1}$$

$$\cos R_4OM_4 = \frac{OM_4}{OR_4}$$

$$\cos R_6OM_6 = \frac{OM_6}{OR_6}$$

The value of the cosine depends on the base, and it is seen from the figure that as the angle increases from 0° to 90° the base gradually diminishes, and the cosine with it; also when the angle increases from 90° to 180° the base increases, but is now

drawn in a contrary direction, and so of opposite sign. Hence in this case—

As the angle increases from 0° to 90°, the cosine takes all values from 1 to 0.

As the angle increases from 90° to 180°, the cosine takes all values from 0 to −1.

The tangent depending on the ratio of perpendicular to base follows the same rule as the sine in increasing as the angle increases; and follows the cosine by becoming a negative value in the quadrant from 90° to 180°. Reciprocals of ratios follow the same rule as the ratios themselves, only as regards sign.

ART. 12. **Supplementary Angles.**—Make angle TOS = angle ROM, where OR = OS; ST and RM perpendiculars.

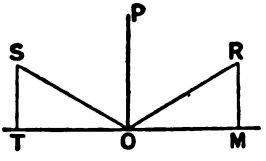


FIG. 44.

The angles ROM and SOM are supplementary.

$$\text{Sin ROM} = \frac{\text{RM}}{\text{OR}},\ \sin \text{SOM} = \frac{\text{ST}}{\text{OS}} = \frac{\text{RM}}{\text{OR}}$$

$$\therefore \sin \text{ROM} = \sin \text{SOM}.$$

Again—

$$\cos \text{ROM} = \frac{\text{OM}}{\text{OR}},\ \cos \text{SOM} = \frac{\text{OT}}{\text{OS}} = \frac{-\text{OM}}{\text{OS}}$$

$$\therefore \cos \text{ROM} = -\cos \text{SOM};$$

or sine of an angle equals the sine of its supplement; but cosine of an angle equals the cosine of its supplement with − sign prefixed.

ART. 13.—**Summary.**—As angles increase, the sines, tangents, and secants increase.

As angles increase, the cosines, cotangents, and cosecants decrease.

All ratios are positive (+) when angles fall between 0° and 90°.

When angles fall between 90° and 180° all ratios *except* sines and cosecants are negative (−); these two are positive (+).

$$\text{Sin A} = \cos \text{(comp. A)}$$
$$\text{Cos A} = \sin \text{(comp. A)}$$
$$\text{Tan A} = \cot \text{(comp. A)}$$
$$\text{Cot A} = \tan \text{(comp. A)}$$
$$\text{Sin A} = \sin \text{(supp. A)}$$
$$\text{Cos A} = -\cos \text{(supp. A)}$$
$$\text{Cosec A} = \frac{1}{\sin \text{A}}$$
$$\text{Sec A} = \frac{1}{\cos \text{A}}$$
$$\text{Cot A} = \frac{1}{\tan \text{A}}$$

ART. 14. **Solution of Right-angled Triangles.**—In all triangles there are six parts (three sides and three angles). In a plane triangle the sum of the three angles is 180°; if three parts be given (one given part *must* be a side), the other parts may be determined. In right-angled triangles a knowledge of the ratios enables any one to effect the solution; one of the given parts being a right angle, the other two parts which are given can only be (1) two sides, or (2) one side and an angle. The tables are needed, because in them are tabulated the logarithms of all ratios of angles from 0° to 90°; consequently, when an angle is given, the log of any ratio of that angle may be found; or if an angle is required, knowing the log of the ratio, the angle may be looked up.

Considering the first case where two sides are given. These sides put in the form of a fraction will make four different ratios of the two acute angles, and there are four different ways of starting the solution, all equally correct.

Remark.—All plane triangles should be drawn to scale. To do this the student requires a pen or pencil bow and a six-inch boxwood protractor. The advantage is that the problem may be solved graphically, and the answer obtained is a useful check on the numerical calculation, and so serious mistakes avoided. Angles are laid off or measured as with a Field's parallel rule, by placing the plain edge on the line, and the centre point at the point at which the angle with the line is to be made or measured. On the space at disposal depends the scale of the figure, and this scale should be entered at the side. Thus if we wish to represent a length of 215 inches, and choose a scale of $\frac{1}{10}$ inch to 10 inches, the total length of the line would be $21\frac{1}{2}$ tenths of inches, or 2·15 inches. Again, to represent a length of 425 miles on a scale of $\frac{1}{10}$ inch to 25 miles, the length would be $\frac{425}{25} \times \frac{1}{10}$ inches = 1·7 inches: or if in such a figure the length of a side is 2·6 inches, its actual size would be $\frac{2{\cdot}6 \times 25}{\frac{1}{10}}$ miles = 650 miles.

Example 1.—In triangle ABC, B = 90°, AB = 368, BC = 215. Find the angles, and AC.

Graphical Solution.—Draw horizontal line AB and make it $\frac{368}{10}$ mm. = 36·8 mm. in length: at B draw BC perpendicular to AB and make it $\frac{215}{10}$ = 21·5 mm. in length. Join AC.

Measure AC = 42·5 mm. ∴ AC = 425
,, $B\hat{A}C = 30°$
,, $B\hat{C}A = 60°$

Trigonometrical Solution.—Since AB and BC are known, we may say—

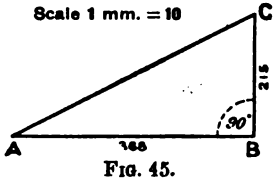


FIG. 45.

$$\frac{AB}{BC} = \cot A \text{ or } \frac{AB}{BC} = \tan C$$
$$\text{or } \frac{BC}{AB} = \tan A \text{ or } \frac{BC}{AB} = \cot C.$$

Using the first ratio—

$$\cot A = \frac{AB}{BC} = \frac{368}{215}$$

Apply logs, and adding 10 to top index, since tabular cotangent log has been increased by 10—

AB	...	368	log = 12·565848
BC	...	215	log = 2·332438
30° 17′ 42″	L cot =		10·233410

$\therefore$ A = 30° 17′ 42″

and C = 59 42 18

To find AC, we form a fraction, putting side wanted in the numerator and side known in the denominator, and writing down the name of ratio so formed. This gives rise to several ways of solution, as we may say—

$$\frac{AC}{AB} = \sec A \text{ or } \frac{AC}{BC} = \operatorname{cosec} A \text{ or } \frac{AC}{AB} = \operatorname{cosec} C \text{ or } \frac{AC}{BC} = \sec C$$

$\therefore$ AC = AB × sec A

Apply logs, rejecting 10 from index, added to make a tabular secant.

AB	...	...	368	log = 2·565848
A	...	30° 17′ 42″	L sec =	10·063768
AC	...	...	426·2	log = 12·629616

Ans. A = 30° 17′ 42″; C = 59° 42′ 18″; AC = 426·2.

Example 2.—In triangle PQR, Q = 90°, PR = 17·8, PQ = 15·5. Solve.

Graphical Solution.—Draw a horizontal line RQ; at Q make QP perpendicular to RQ and $\frac{15·5}{16}$ of an inch in length. Take $\frac{17·8}{16}$ of an inch as a radius, and with P as centre, cut RQ in R. Then RPQ is triangle.

Measure RQ = 8·8 sixteenths $\therefore$ RQ = 8·8

,, $R\hat{P}Q$ = 30°

,, $P\hat{R}Q$ = 60.

Trigonometrical Solution.—

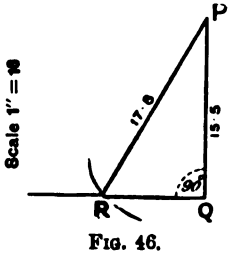


FIG. 46.

$$\text{Sin R} = \frac{PQ}{PR}$$

Apply logs.

PQ	...	...	15·5	log = 11·190332
PR	...	...	17·8	log = 1·250420
R	...	60° 33′ 1″	L sin =	9·939912

P = 90 − R = 29° 26′ 59″

Again—

$$\frac{RQ}{PQ} = \cot R$$

$\therefore$ RQ = PQ × cot R

Apply logs.

PQ	... ...	15·5	log = 1·190332
R	...	60° 33′ 1″ L cot	= 9·751752
RQ	... ...	8·751	log = $\overline{1}$0·942084

Ans. P = 29° 26′ 59″; R = 60° 33′ 1″; RQ = 8·751.

The second case, where one side and an angle are given, is solved by remembering that the ratio to work from is formed by placing the side wanted in the numerator and the side known in the denominator. As before, there are four different ways, all equally right, of starting.

Example 3.—In triangle LMN, M = 90°, N = 55° 15′, LM = 1342. Solve.

Graphical Solution.—Draw a horizontal line NM: at M draw ML perpendicular to MN and make it equal to $\frac{1342}{1000}$ inch = 1·342 inch. At any point O in MN make an angle = 55° 15′: through L draw LN parallel to this line. Then $L\hat{N}M$ = 55° 15′.

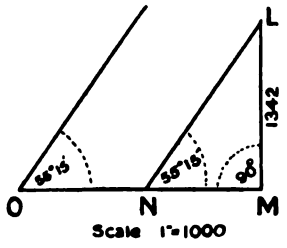


Fig. 47.

Measure NL = 1·62 inch ∴ NL = 1620
„ MN = ·92 „ ∴ MN = 920
„ $N\hat{L}M$ = 35°

Trigonometrical Solution —

$$\frac{LN}{LM} = \text{cosec } N, \text{ and } \frac{MN}{LM} = \cot N$$

$$\therefore LN = LM \times \text{cosec } N, \text{ and } MN = LM \times \cot N$$

Apply logs.

LM ... 1342	log = 3·127752		log = 3·127752
N 55° 15′ L cosec	= 10·085315	L cot	= 9·841187
1633	log = $\overline{1}$3·213067	931	log = $\overline{1}$2·968939

L = 90° − N = 34° 45′

Ans. L = 34° 45′; LN = 1633; MN = 931.

EXERCISES.—RIGHT-ANGLED TRIANGLES (PLANE).

1. In triangle ABC, AB = 503, AC = 357, C = 90°. Find A, B, and BC.
2. In triangle PQR, PQ = 371, QR = 204, Q = 90°. Find P, R, and PR.
3. In triangle OPB, OP = 125·7, PB = 695, O = 90°. Find P, B, and OB.
4. In triangle ABC, A = 64° 41′, C = 90°, AB = 1037. Find AC and BC.
5. In triangle ABC, B = 34° 17′, C = 90°, BC = 905. Find AB and AC.
6. In triangle ABC, AB = 147, BC = 116, C = 90°. Find A, B, and AC.
7. In triangle PQR, P = 33° 14′, PQ = 4372, Q = 90°. Find PR and RQ.
8. In triangle ABC, C = 90°, AC = 706, BC = 981. Find A, B, and AB.
9. At 145 ft. from the base of a flagstaff its angle of elevation is observed to be 32° 16′. Find the height.
10. The difference of latitude made good is 236·5 miles south, the departure 215·7 miles west. Find the course, and distance sailed.
11. The length of the shadow cast by a vertical post is 39 ft. 6 in. when the sun has an elevation of 21° 32′. Find the height of the post.
12. From a ship a promontory was observed to be 3½ points on the bow,

and after making 7 miles on her course, it was found to be abeam. What distance was the vessel from the point at the two bearings?

13. A lighthouse, whose top is 200 ft. above the sea-level, marks a danger situated at three cables' length from the shore. Required the vertical "danger angle."

14. From a reef awash at low water, the angle of elevation to the summit of a hill distant 2050 yards in a horizontal direction is 1° 12'; height of the eye, 6 ft.; tidal rise, 21 ft. Required the height of the hill above high water.

ART. 15. **Solution of Oblique-angled Plane Triangles.**—Triangles which do not contain a right angle are called oblique-angled. Facts to be remembered in connection with triangles are—

The sum of the angles is 180°.

Equal angles are subtended by equal sides.

The greater side is opposite the greater angle.

Any two sides are together greater than the third side.

According to the parts given the solution falls into three different cases.

1. Given two sides and an angle opposite to one of the known sides.

Or, given two angles and a side.

2. Given three sides.

3. Given two sides and the angle contained by the known sides.

CASE I.—For the first case, the following property is used: *Sides are proportional to the sines of the opposite angles.*

ACB is a triangle: draw CD at right angles to AB.

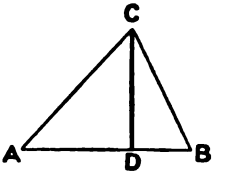


FIG. 48.

Then— $\frac{CD}{AC} = \sin A$ . . . . . I.

and— $\frac{CD}{BC} = \sin B$ . . . . . II.

Divide I. by II.

$$\therefore \frac{CD}{AC} \div \frac{CD}{BC} = \frac{\sin A}{\sin B}$$

that—

$$\frac{\cancel{CD}}{AC} \times \frac{BC}{\cancel{CD}} = \frac{\sin A}{\sin B}$$

or, cancelling CD—

$$\frac{BC}{AC} = \frac{\sin A}{\sin B}$$

Now, if a perpendicular were dropped from A on BC, it might be proved in a similar manner that—

$$\frac{AC}{AB} = \frac{\sin B}{\sin C}$$

Hence the proposition is established.

Example 1.—Given PQ = 350, PR = 367, and Q = 54° 30′. Solve.

Draw a horizontal line RQ, and at Q make angle 54° 30′, also on a scale of 1·1 inch to 350 make PQ 1·1 inch long. Find length of PR on our scale $= \frac{1{\cdot}1 \times 367}{350} = 1{\cdot}15$ inch, take this length in compass, and with P as centre draw an arc cutting QR in R. Then PQR is triangle.

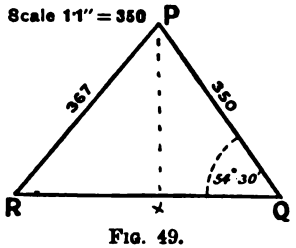


Fig. 49.

PRQ = 51°
RPQ = 75°
RQ = 1·38 inch ∴ RQ = 439.

Two sides and an angle opposite to a given side; hence the part to be found is the angle opposite to the other given side; here we form a fraction having for its numerator the sine of the angle wanted, and for its denominator the sine of the given angle. This fraction is equal to one formed from preceding proof.

Thus—

$$\frac{\sin R}{\sin Q} = \frac{PQ}{PR}$$

$$\therefore \sin R = \frac{PQ \times \sin Q}{PR}$$

Apply logs.

PQ	...	... 350	log =	2·544068
Q	...	54° 30′ L sin	=	9·910686
				12·454754
PR	...	... 367	log =	2·564666
R	...	50° 55′ 57″ L sin	=	9 890088

Q = 54° 30′ 0″
R = 50 55 57

Q + R = 105 25 57
180

P = 74 34 3

Again, to find the third side RQ: form a fraction with the side wanted (RQ) as numerator, and one of the given sides (PR) as denominator; this fraction equals one formed by placing sine of angle opposite RQ (sin P) in numerator and sine of angle opposite PR (sin Q) in denominator.

$$\frac{RQ}{PR} = \frac{\sin P}{\sin Q}$$

$$\therefore RQ = \frac{PR \times \sin P}{\sin Q} = PR \times \sin P \times \text{cosec } Q$$

In order to make all the logs added, sin Q as a divisor is changed into cosec Q as a multiplier—

$$\text{since cosec } Q = \frac{1}{\sin Q}$$

PR	...	...	367	log = 2·564666
P	...		74° 34′ 3″ L sin	= 9·984052
Q	...		54° 30′ L cosec	= 10·089314
RQ	...	...	434·5	log = $\overline{2}$2·638032

Ans. P = 74° 34′ 3″ ; R = 50° 55′ 57″ ; RQ = 434·5.

Example 2.—Given C = 118° 30′, B = 21° 22′, and AC = 67·5. Solve.

Graphical Solution.—Draw horizontal line AC of length $\frac{67{\cdot}5}{100}$ inches = ·68 inch. At C draw CB making an angle of 118° 30′ with AC, and through A draw AD parallel to CB. At A draw AB making an angle 21° 22′ with AD, to meet CB in B. Then ABC is triangle.

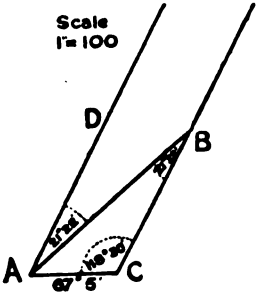


Fig. 50.

Measure A = 40°
,, AB = 1·6 inch ∴ AB = 160
,, BC = 1·2 ,, ∴ BC = 120

Trigonometrical Solution.—

$$A + B + C = 180°$$
$$\therefore A = 180° - (B + C)$$
$$= 180° - 139° 52'$$
$$= 40° 8'$$

To find BC—

$$\frac{BC}{AC} = \frac{\sin A}{\sin B}$$

$$\therefore BC = \frac{AC \times \sin A}{\sin B} = AC \times \sin A \times \operatorname{cosec} B$$

AC	...	...	67·5	log = 1·829304
A	...		40° 8′ L sin	= 9·809269
B	...		21° 22′ L cosec	= 10·438499
BC	...	...	119·4	log = $\overline{2}$2·077072

To find AB—

$$\frac{AB}{AC} = \frac{\sin C}{\sin B}$$

$$\therefore AB = \frac{AC \times \sin C}{\sin B} = AC \times \sin C \times \operatorname{cosec} B$$

AC	...	...	67·5	log = 1·829304
C	...		118° 30′ L sin	= 9·943899 ... (Use sin 61° 30′, the supplement.)
B	...		21° 22′ L cosec	= 10·438499
AB	...		162·8	log = $\overline{2}$2·211702

Ans. A = 40° 8′ ; AB = 162·8 ; BC = 119·4.

Example 3.—Given AP = 1134 ; PM = 875 ; A = 35° 20′. Solve.

Here is given the angle opposite to the smaller side, and when the triangle is drawn to scale, it is found that *two* triangles can be drawn from the data; there are thus two solutions, and it is known as the *ambiguous* case.

Graphical Solution.—Lay off a horizontal line of unlimited length, and at one end A make an angle equal to the given angle, viz. MAP = 35° 20′.

Along AP measure the longer side = 1134, that is 1·13 inch; and with centre P and distance equal to the shorter side 875, that is ·87 inch; describe an arc cutting AM in M and M'. Thus both PM and PM' = 875. Hence there are two triangles, PAM and PAM', which satisfy the given conditions.

Measure AM = 1·5 inch, that is 1504
AM' = ·35 inch, that is 350.

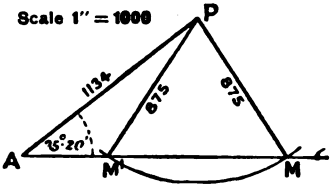


Fig. 51.

Trigonometrical Solution.—To find M in triangle APM—

$$\frac{\sin M}{\sin A} = \frac{AP}{PM}$$

$$\therefore \sin M = \frac{AP \times \sin A}{PM}$$

AP	...	...	1134	log =	3·054613
A	...	...	35° 20′	L sin =	9·762177
					12·816790
PM	...	...	875	log =	2·942008
M	..		48° 32′ 55″	L sin =	9·874782
A	...		35 20 0		
A + M	...		83 52 55		
			180		
P	...		96 7 5		

To find AM in triangle AMP—

$$\frac{AM}{PM} = \frac{\sin P}{\sin A}$$

$$\therefore AM = \frac{PM \times \sin P}{\sin A} = PM \times \sin P \times \operatorname{cosec} A$$

PM	...	...	875	log =	2·942008
P	...		96° 7′ 5″	L sin =	9·997519
A	...		35° 20′	L cosec =	10·237823
AM	...	...	1504	log =	3·177350

To find M' in APM'—

Since PM = PM', angle PMM' = angle PM'M
Hence M' = 180 − M
= 131° 27′ 5″

A =	35	20	
M' + A =	166	47	5
	180		
P =	13	12	55

To find AM' in triangle APM'—

$$\frac{AM'}{PM'} = \frac{\sin P}{\sin A}$$

$$\therefore AM' = \frac{PM' \times \sin P}{\sin A} = PM' \times \sin P \times \operatorname{cosec} A$$

PM'	...	...	875 log =	2·942008
P	...	13° 12′ 55″ L sin =		9·359096
A	...	35° 20′ L cosec =		10·237823
AM'	...	...	345·9 log =	2·538927

Ans. M = 48° 32′ 55″, P = 96° 7′ 5″, AM = 1504
or 131 27 5, or 13 12 55, or 345·9

CASE II.—Given three sides, to find the angles. The half sum of the three sides is denoted by the letter s. If ABC is a triangle, the formula for finding an angle is—

$$\cos \frac{A}{2} = \sqrt{\frac{s \,.\, (s - BC)}{AB \,.\, AC}}$$

where $s = \frac{1}{2}(AB + AC + BC)$

This is proved in any "Trigonometry," and may be expressed in writing thus—

Cosine of half an angle is equal to the square root of the amount obtained by dividing the product of the half sum of sides and the *remainder* by the product of the sides which contain the angles.

The *remainder* is obtained by subtracting the side opposite the angle required from the half sum of the sides.

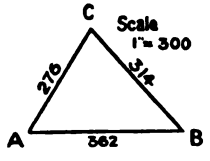


FIG. 52.

Example 4.—Given AB = 362, BC = 314, AC = 276. Solve.

Graphical Solution.—Draw a horizontal line AB in length = $\frac{362}{300}$ inch = 1·21 inch. With centre A, radius = $\frac{276}{300}$ = ·92 inch, describe an arc, and with centre B, radius = $\frac{314}{300}$ = 1·05 inch, describe another arc cutting first arc in C. Then ABC is the triangle.

A measures 57½°
B ,, 47½°
C ,, 74°

Trigonometrical Solution.—To find C—

$$\cos \frac{C}{2} = \sqrt{\frac{s \,.\, (s - AB)}{AC \,.\, BC}}$$

AB = 362			
AC = 276	...	... log	2·440909
BC = 314	...	... log	2·496930
2)952			
s = 476	log	2·677607	
s − AB = 114	log	2·056905	
		24·734512	4·937839
		4·937839	
		2)19·796673	
37° 41′ 37″ L cos =		9·898336	
2			
C = 75 23 14			

Again, to find B—

$$\cos\frac{B}{2} = \sqrt{\frac{s.(s-AC)}{AB.BC}}$$

AB = 362	...	...	log = 2·558709
BC = 314	...	...	log = 2·496930
s = 476	log =	2·677607	
s − AC = 200	log =	2·301030	
		24·978637	5·055639
		5·055639	
		2)19·922998	
23° 46′ 15″	L cos =	9·961499	
2			
B = 47 32 30			

A = 180° − (B + C) = 180° − 122° 55′ 44″ = 57° 4′ 16″

Ans. A = 57° 4′ 16″; B = 47° 32′ 30″; C = 75° 23′ 14″.

Case III.—Given two sides and the contained angle, to solve the triangle. The solution depends on the following property. In any triangle, the tangent of half the difference of two angles bears the same ratio to the tangent of half the sum of the same two angles as the difference of the sides opposite the angles bears to the sum of the same sides.

This is expressed as a formula, thus—

$$\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)} = \frac{BC-AC}{BC+AC}$$

$$\therefore \tan\frac{A-B}{2} = \frac{BC-AC}{BC+AC}\cdot\tan\frac{A+B}{2}$$

and since—

$$\frac{A+B}{2} = 90 - \frac{C}{2}$$

$$\therefore \tan\frac{A+B}{2} = \cot\frac{C}{2}$$

and the formula used for the solution reduces to—

$$\tan\frac{A-B}{2} = \frac{BC-AC}{BC+AC}\cdot\cot\frac{C}{2}$$

Example 5.—Given BC = 145·5, AC = 98·1, and C = 87° 20′. Solve the triangle.

Graphical Solution.—Draw a horizontal line BC = 1·46 inch; at C make angle = 87° 20′ by drawing CA = $\frac{98\cdot1}{100}$ = ·98 inch long.

Measure A = 58°
,, B = 35°
,, AB = 1·71 inch = 171

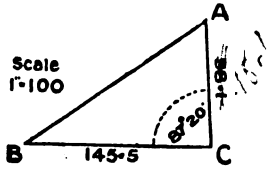


FIG. 53.

Trigonometrical Solution.—

$$\frac{A+B}{2} = 90° - \frac{C}{2}$$
$$= 90° - 43° 40'$$
$$= 46° 20'$$
$$\tan\frac{A-B}{2} = \frac{BC-AC}{BC+AC}\cdot\cot\frac{C}{2}$$
$$= \frac{145{\cdot}5 - 98{\cdot}1}{145{\cdot}5 + 98{\cdot}1}\cdot\cot 43° 40'$$
$$= \frac{47{\cdot}4}{243{\cdot}6}\cdot\cot 43° 40'$$

$BC - AC$	... ...	47·4	log = 1·675778
$\frac{C}{2}$	... ...	43° 40′	L cot = 10·020220
			11·695998
$BC + AC$	... ...	243·6	log = 2·386677
$\frac{A-B}{2}$	... ...	11° 31′ 20″	L tan = 9·309321
$\frac{A+B}{2}$	... ...	46 20 0	
A	... ...	57 51 20	
B	... ...	34 48 40	

To find AB—

$$\frac{AB}{AC} = \frac{\sin C}{\sin B}$$
$$\therefore AB = \frac{AC \times \sin C}{\sin B} = AC \times \sin C \times \operatorname{cosec} B$$

AC	... ...	98·1	log = 1·991669
C	... ...	87° 20′	L sin = 9·999529
B	...	34° 48′ 40″	L cosec = 10·243461
AB	... ...	171·7	log = 2·234659

Ans. A = 57° 51′ 20″; B = 34° 48′ 40″; AB = 171·7.

ART. 16.—The proofs of the rules used in the two last cases are inserted here for the benefit of any student who may not have a "Trigonometry" handy.

To express the cosine of an angle in terms of the sides.—

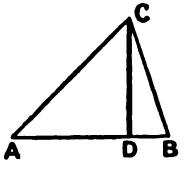


FIG. 54.

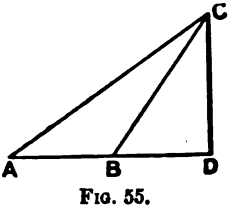


FIG. 55.

Let ABC be a triangle; one figure represents B acute, the other B obtuse. Draw CD at right angles to AB or AB produced.

By Euclid II. 13 and 12—

$AC^2 = AB^2 + BC^2 - 2AB.BD$ or $AC^2 = AB^2 + BC^2 + 2AB.BD$ using small letters for sides and observing that—

$$\frac{BD}{BC} = \cos B \qquad \text{also } \frac{BD}{BC} = \cos CBD$$

$$\therefore BD = BC.\cos B = a.\cos B \qquad \therefore BD = BC.\cos CBD = -a.\cos B$$

Insert these values for BD—

$$\therefore b^2 = c^2 + a^2 - 2ca\cos B \quad \text{or } b^2 = c^2 + a^2 - 2c.a\cos B$$

Hence— $$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

Similarly for cos A and cos C.

To show that $\cos\frac{A}{2} = \sqrt{\frac{s.(s-a)}{bc}}$

Now— $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$1 + \cos A = 1 + \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{2bc + b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(b+c)^2 - a^2}{2bc}$$

$$= \frac{(b+c+a)(b+c-a)}{2bc}$$

Let $s = \frac{1}{2}(b + c + a)$;

then $s - a = \frac{1}{2}(b + c - a)$

Hence— $$1 + \cos A = \frac{2s.2(s-a)}{2bc}$$

or— $$2\cos^2\frac{A}{2} = \frac{2s(s-a)}{bc}$$

$$\therefore \cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

Again—

To show that $\tan\frac{A-B}{2} = \frac{a-b}{a+b}.\tan\frac{A+B}{2}.$

Since— $$\frac{\sin A}{\sin B} = \frac{a}{b}$$

$$\therefore \frac{\sin A}{\sin B} + 1 = \frac{a}{b} + 1 \quad \ldots\ldots\ldots \text{I.}$$

or— $$\frac{\sin A + \sin B}{\sin B} = \frac{a+b}{b}$$

and— $$\frac{\sin A}{\sin B} - 1 = \frac{a}{b} - 1$$

or— $$\frac{\sin A - \sin B}{\sin B} = \frac{a-b}{b} \quad \ldots\ldots\ldots \text{II.}$$

Divide II. by I.

Hence— $$\frac{\sin A - \sin B}{\sin A + \sin B} = \frac{a-b}{a+b}$$

$$\frac{2\cos\frac{A+B}{2}.\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}} = \frac{a-b}{a+b}$$

$$\therefore \tan\frac{A-B}{2} = \frac{a-b}{a+b}.\tan\frac{A+B}{2}$$

Exercises.—Oblique-angled Plane Triangles.

If ABC be any triangle, complete the solution in the following cases :—

1. $a = 1056$, $b = 359$, $c = 1267$.
2. $a = 364$, $b = 217$, $c = 494$.
3. $a = 96$, $b = 80$, $c = 63$.
4. A = 37° 54′, B = 74° 25′, $a = 104{\cdot}6$.
5. B = 14° 20′, C = 101° 13′, $b = 296$.
6. A = 97° 34′, C = 50° 16′, $c = 16$.
7. $a = 6954$, $b = 4865$, C = 39° 40′.
8. $a = 267$, $c = 341$, B = 68° 46′.
9. $b = 61$, $c = 49$, A = 97° 18′.
10. $a = 865$, $b = 742$, A = 52° 26′.
11. $b = 469$, $c = 348$, C = 39° 4′.
12. $a = 1065$, $c = 897$, A = 49° 36′.
13. $a = 587$, $b = 925$, A = 21° 23′.
14. $a = 7$, $b = 8$, $c = 9$.
15. $a = 600$, $b = 500$, $c = 1000$.
16. $a = 50$, $b = 60$, $c = 70$.
17. A = 18° 20′, B = 150°, $b = 10{,}000$.
18. A = 78° 40′, C = 39° 55′, $c = 135$.
19. $b = 237{\cdot}09$, $c = 130{\cdot}96$, A = 57° 59′.
20. $c = 2265{\cdot}4$, $a = 1779$, B = 58° 17′.
21. $a = 1300$, $b = 500$, A − B = 30° 22′.
22. $a = 1586{\cdot}6$, $b = 5374{\cdot}5$, A = 15° 11′.
23. $a = 200$, $b = 300$, A = 35° 15′.

24. The angular elevation of a tower at a place A due south of it is 45°, and at a place B, due west of A, the elevation is 30° ; AB = 300 ft. Find height of tower.

25. AB is a line 250 ft. long, in the same horizontal plane as the foot D of a tower CD ; the angles DAB and DBA are respectively 61° 23′ and 47° 14′ ; the angle of elevation CAD is 34° 50′. Find height of tower.

Miscellaneous Problems.

1. The difference between the base and perpendicular of a right-angled triangle is one-fifth of their sum. Find the angles.

2. P and Q are two stations 1000 yards apart on a straight stretch of sea-shore which bears east and west. At P a rock bears S. 42° W., at Q it bears S. 35° E. Find distance of rock from shore.

3. A person on a ship, which is sailing on a straight course observes two lighthouses, one of which, L, is due north of the other, M. When M is due

east, L bears 45° to the east of north, and when L is due east, M bears 60° to the east of south. Find direction of ship's course.

4. AB is a vertical pole 50 feet high, A being above B; BC is inclined upward at an angle of 20° to the horizontal line, BD, so that ABC is an angle of 70°; the shadow of AB on BC is 50 feet long. If the shadow fell on BD, what would be its length?

5. From the top of a vertical cliff 100 ft. high, forming one bank of a river, the angles of depression of the top and bottom of a vertical cliff, which forms the opposite bank of the river, are 28° 40′ and 64° 30′ respectively. Find height of opposite cliff and width of river.

6. The summit of a spire is vertically over the middle point of a horizontal square enclosure, side = 1000 ft. When the shadow of the spire just reaches a corner of the square the altitude of the sun is 27° 29′ 50″. Find height of spire.

7. A tower near the coast is 197 feet high. From a rock the angle of elevation of the top of the tower is 8° 15′. A ship steams so as to clear the rock by 1·5 cables. What is the angle of elevation of tower from ship?

8. At two boats 600 yds. apart, the angles of elevation of a hydroplane flying directly between the boats are observed to be 57° and 65° 30′. Find the height of the hydroplane.

9. Two ships, A and B, leave harbour together. A sails S. 12° W. at a rate of 10·5 knots, and B S. 67° E. at 8 knots. How far apart will they be in 2½ hours, and what will be the bearing of A from B?

10. From a ship the bearings of two objects, A and B, are N. 30° E., and N. 20° W. After the ship has sailed 5 miles due N., the bearings are N. 60° E. and N. 40° W. Find distance and bearing of A from B.

11. The foot, Q, of a tower and two stations, A and B, are in the same horizontal plane. The angular elevation of the tower at A is 60°, and at B is 45°, the distance from A to B is 100 feet, and the angle AQB is 60°. Find height of tower.

12. From P the bearing and distance of A is NE. 5 miles, and of B, S. 70° E. From Q, due east of P, the bearing of A is N. 25° E. If B is due south of A, what is the bearing of B from Q?

13. The hypotenuse of a right-angled triangle is 1000 feet long, and the difference between the other two sides is 240 feet: calculate the other sides and angles.

14. The height of a tower is 67·375 yds. At P, which is 5·35 yds. above the level of its base, the angle subtended by the tower is 74° 40′. Find the angle of elevation of the top of the tower as seen from P, and horizontal distance of P from tower.

CHAPTER V.

SOLUTION OF SPHERICAL TRIANGLES.

ART. 17.—A spherical triangle is a portion of the surface of a sphere bounded by arcs of three great circles, and, like a plane triangle, has three sides and three angles. The sides, being arcs of circles, are measured like angles, in degrees, minutes, and seconds (° ′ ″).

When one of the angles is 90°, the triangle is *right-angled;* when one side is 90°, it is *quadrantal;* and when no side or angle is 90°, it is an oblique-angled triangle.

Unlike a plane triangle, a spherical triangle may have two or three right angles, or one, two, or three sides of 90°, but no angle or side can be as much as 180°.

The sum of any two sides is greater than the third side.

The sum of the three angles of a spherical triangle is always more than 180°, but less than 540°, and the sum of the three sides is less than 360°.

The greater angle is opposite to the greater side, as in plane triangles.

The angles at the base of an isosceles triangle are equal, as in plane triangles.

ART. 18. **Solution of Right-angled Spherical Triangles.**—Here one of the given parts is the right angle; any other two parts being given, the triangle can be solved. In order to simplify the solution, instead of using the five remaining parts of the triangle, Napier devised rules which refer only to his "Circular Parts."

Napier's Circular Parts.—These are five in number, and are the two sides which contain the right angle, the complement of the hypotenuse, and the complements of the two angles. Thus in triangle ABC, right-angled at A, the circular parts are—

FIG. 56.

AB; AC; Co. C; Co. BC; Co. B

These are split into three groups, consisting of—

1 middle,
2 adjacents,
2 opposites.

Any part may be taken as middle.

Thus—

Middle.	Adjacents.	Opposites.
AC	AB; co. C	Co. B; co. BC
Co. C	AC; co. BC	Co. B; AB
Co. BC	Co. B; co. C	AB; AC

Napier's Rules.—

I. Sine mid part = tangents of adjacents multiplied.

II. Sine mid part = cosines of opposites multiplied.

Each of these rules connects three parts. When used it must be arranged that two are given parts; also that the given parts are the only ones used to find the others. To do this, always start by considering whether the given parts are adjacents or opposites; then take as middle each given part in succession, and use the Rule which will connect the given parts.

Hence $\sin AC = \tan AB \cdot \tan \text{co. } C$
$= \tan AB \cdot \cot C$
and $\sin AC = \cos \text{co. } B \cdot \cos \text{co. } BC$
$= \sin B \cdot \sin BC$

Again $\sin \text{co. } BC = \tan \text{co. } B \cdot \tan \text{co. } C$
or $\cos BC = \cot B \cdot \cot C$
and $\sin \text{co. } BC = \cos AB \cdot \cos AC$
or $\cos BC = \cos AB \cdot \cos AC$

ART 19.—Hints on construction of Spherical Triangles.

Draw a circle as a primitive, with radius of sphere, we shall use one inch; but the larger the radius the better the result will be; with the Gunter rule it would be two inches.

Let C be the centre, then any diameter drawn will represent a great circle at right angles to the primitive: thus ACB (Fig. 57) is a right circle.

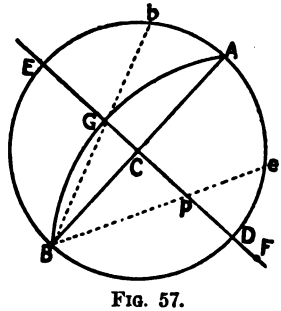


FIG. 57.

Draw DCE at right angles to AB. Then D and E are the **Poles** of ACB. The centres of all oblique circles through A and B lie on the line DCE. Thus with any centre F and radius FA describe an arc of a circle, then AGB is an oblique circle. To find its pole, join BG and produce to *b*, make *be* equal to EA (90°), join *e*B cutting DE in P: then P is the pole of AGB.

N.B.—Great circles at right angles to AGB must pass through its pole P.

Hence great circles can be drawn through a point *on* or *within* the primitive, or through two points, one on, the other

inside the primitive. Thus in Fig. 58 let A and B be two points, A on, and B inside. Draw diameter ACD, and a line through C, as CO, at right angles. Join AB, and bisect it by a line cutting CO in X. Then X is centre of great circle through A and B.

If an oblique circle is to be drawn through any two points *inside* the primitive as through A and B, Fig. 59, proceed

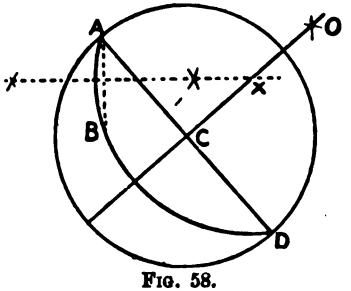


FIG. 58.

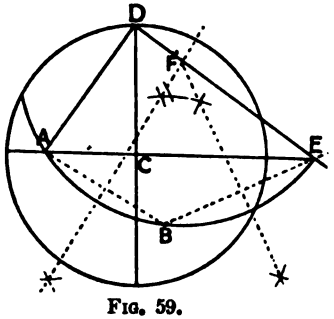


FIG. 59.

thus—Draw diameter AC, and CD through C at right angles to AC. Join AD, make ADE a right angle, cutting AC in E: join AB, BE, bisect these by lines at right angles intersecting in F. F will be the centre for describing oblique circle required.

To draw a great circle at right angles to a given oblique circle through a given point, first find the pole of the given oblique circle; then by the preceding examples draw a great circle through the given point and this pole; this will be at right angles to the given oblique circle, the given point may be either on the primitive, or on the oblique circle.

A great circle is drawn making a given angle with the primitive (say 35°) through a given point as follows:—

From centre with radius = tan 35° describe a small arc, and with given point as centre and radius = sec 35°: draw another arc, this intersects the first arc at the centre from which the great circle is to be drawn with sec 35° as radius.

N.B.—With a radius of one inch, the tan and sec of any angle may be obtained from the traverse table thus—Take angle (say 35°) as course and radius (1·00) in lat. col., then sec (1·22) is found in dist. col. and tan (0·7) in dep. col., the semi-tangent = tangent of half the angle = 0·31. In Fig. 60, A is given point, and *o* is centre of required great circle.

If *o*C be drawn as a diameter, the arc *ab* measures the angle: C*a* = 0·52; in traverse table find 1 in lat. 0·52 in dep.; angle = 27½°, hence C*a* on scale of semi-tangents = 55°, or *ab* = 35°.

A small circle parallel to the primitive and at any distance from it is drawn from the centre of primitive, with radius = semi-tangent of complement of required distance.

A small circle through any point parallel to a right circle at a given distance is drawn thus: let A be given point, and 35° given distance; then with sec (Co. 35°), that is sec 55°, as radius sweep an arc from centre C, where this arc intersects an arc drawn from A as centre and tan 55° as radius is the centre for describing the small circle (Fig. 61).

On scale—

Sec 55° = 1·74″

Tan 55° = 1·43″.

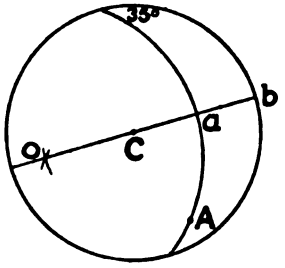


FIG. 60.

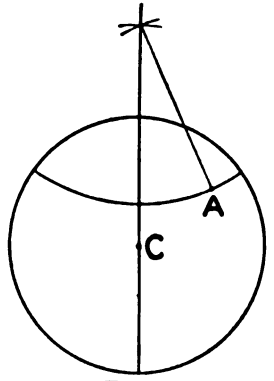


FIG. 61.

To measure any arc of an oblique circle, say arc AB (Fig. 62). Find pole P of oblique circle, join PA, PB, and produce to meet primitive in *a* and *b*, then *ab* measured on the scale of chords is the length of AB.

Similarly, to measure an angle at the intersection of two great circles, first find their poles, join the point of intersection of the

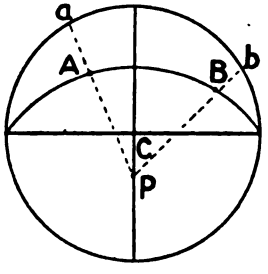


FIG. 62.

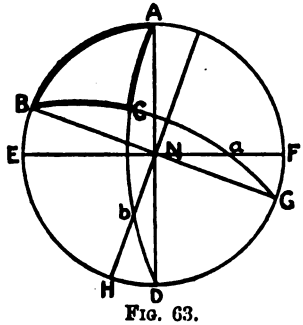


FIG. 63.

circles by lines passing through the poles, and produced to cut the primitive. The measure of this arc of the primitive on the scale of chords is the measure of the angle.

Example 1.—Given A = 64° 30′, B = 56° 26′, C = 90°. Solve.

Construction (Fig. 63).—Draw primitive, AD and EF, diameters at right angles. At A make angle of 64° 30′ with primitive, centre of circle is on EF produced, radius = sec 64° 30′ = 2·32. Find pole *a* of circle ACD. Then

with N as centre and radius = semi-tangent of angle B 56° 25′ = 0·53 describe an arc cutting ACD in *b*, this point is pole of circle required. Draw diameter *b*N and BG at right angles. Since C is to be a right angle, the circle drawn through B*a*G is one required ; this cuts ACD at C. ABC is triangle.

By measurement AB = 71°
BC = 60°
AC = 51°

Calculation.—Here the given parts are opposites. We therefore use co. A with its opposites, and then co. B with its opposites. Finally, to find AB, we use adjacents.

$$\text{Sin (co. A)} = \cos \text{(co. B)} \,.\, \cos BC$$
$$\text{or } \cos A = \sin B \,.\, \cos BC$$
$$\therefore \cos BC = \frac{\cos A}{\sin B} = \cos A \,.\, \operatorname{cosec} B \quad . \; . \; . \; \text{I.}$$
$$\text{Again, sin (co. B)} = \cos \text{(co. A)} \,.\, \cos AC$$
$$\text{or } \cos B = \sin A \,.\, \cos AC$$
$$\therefore \cos AC = \frac{\cos B}{\sin A} = \cos B \,.\, \operatorname{cosec} A \quad . \; . \; . \; \text{II.}$$
$$\text{Now, sin (co. AB)} = \tan \text{(co. A)} \,.\, \tan \text{(co. B)}$$
$$\text{or } \cos AB = \cot A \,.\, \cot B \; . \; . \; . \; . \; . \; . \; . \; \text{III.}$$

A	64° 30′ cos	= 9·633984		cosec = 10·044512		cot = 9·678496
B	56 25 cosec	= 10·079312		cos = 9·742842		cot = 9·822154
58° 53′ 3″ cos	= 9·713296		52° 12′ 15″ cos = 9·787354		71° 32′ 11″ cos = 9·500650	

Ans. BC = 58° 53′ 3″, AC = 52° 12′ 15″, AB = 71° 32′ 11″.

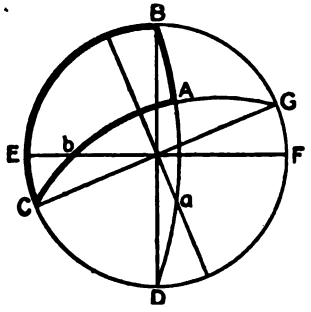


FIG. 64.

Example 2.—Given A = 90°, B = 117° 20′, BC = 113° 40′. Solve.

Construction (Fig. 64).—Draw primitive, diameters BD and EF at right angles. At B make an angle 117° 20′ with primitive ; sec = 2·21 as radius, draw circle BAD, find its pole *b*. Make BC = 113° 40′, through C draw diameter CG and through C*b*G describe a circle cutting BAD in A. Then ABC is triangle. Pole of C*b*G is at *a*.

By measurement
radius for C*b*G = 1·63 = sec. 52° C = 52°
AB = 45°
AC = 127°

Calculation.—Here the given parts are adjacents. We therefore use co. B with its adjacents ; then co. BC with its adjacents. Lastly, to find AC, we use opposites.

$$\text{Sin (co. B)} = \tan AB \,.\, \tan \text{(co. BC)}$$
$$\text{or } \cos B = \tan AB \,.\, \cot BC$$
$$\therefore \overset{+}{\tan} AB = \frac{\cos B}{\cot BC} = \overset{-}{\cos} B \,.\, \overset{-}{\tan} BC$$
$$\text{Again, sin (co. BC)} = \tan \text{(co. B)} \,.\, \tan \text{(co. C)}$$
$$\text{or } \cos BC = \cot B \,.\, \cot C$$
$$\therefore \overset{+}{\cot} C = \frac{\cos BC}{\cot B} = \overset{-}{\cos} BC \,.\, \overset{-}{\tan} B.$$
$$\text{And } \overset{+}{\sin} AC = \cos \text{(co. B)} \,.\, \cos \text{(co. BC)}$$
$$= \overset{+}{\sin} B \,.\, \overset{+}{\sin} BC.$$

B = 117° 20′	cos = 9·661970	tan = 10·286614	sin = 9·948584
BC = 113 40	tan = 10·358253	cos = 9·603594	sin = 9·961846
46° 20′ 1″	tan = 10·020223		
	52° 9′ 59″	cot = 9·890208	
		54° 27′ 10″	sin = 9·910430

In the case of AB and C, notice that they are marked +, so these angles are in the first quadrant, and the values are those obtained direct from the tables; in the case of AC, which is marked +, the value might be the tabular value or its supplement. As angle B opposite to it exceeds 90°, use the supplement.

Ans. AB = 46° 20′ 1″; C = 52° 9′ 59″; AC = 125° 32′ 50″.

ART 20.—In solving these triangles, observe—

1. Angles and opposite sides are of the same affection, that is, are both over, or both under, 90°.

2. The hypotenuse is under 90° when the angles *or* sides are of the same affection.

The hypotenuse is over 90° when the angles *or* sides are of contrary affection.

3. When an angle and opposite side are given, the triangle admits of *two* solutions; this is called the *Ambiguous* case, and may be understood from the following figure. Let AOB and AMB be two arcs of great circles intersecting at A and B. The angles at A and B are equal, and each arc is a semi-circle. Now let OM, another great circle arc, be drawn at right angles to AMB. Thus we have *two* right-angled triangles AOM and BOM, in which angle A and arc OM are equal to angle B and arc OM. Hence with angle = A and arc = OM given, there is nothing to indicate which of the triangles AOM or BOM is the required one.

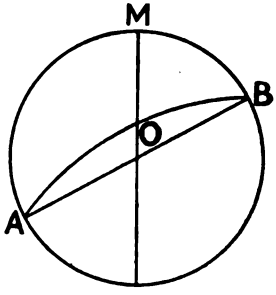


FIG. 65.

EXERCISES.—RIGHT-ANGLED SPHERICAL TRIANGLES.

Solve the following triangles:—

1. C = 90°, AC = 65° 12′, BC = 51° 36′.
2. A = 78° 11′, B = 39° 49′, C = 90°.
3. AB = 84° 13′, BC = 61° 44, C = 90°.
4. AB = 79° 27′, A = 51° 30′, C = 90°.
5. BC = 32° 51′, B = 56° 17′, C = 90°.
6. BC = 54° 37′, A = 67° 53′, C = 90°.
7. B = 90°, AB = 70° 29′, BC = 102° 38′.
8. B = 90°, AB = 110° 17′, BC = 98° 46′.
9. B = 90°, C = 114° 51′, A = 67° 32′.
10. B = 90°, C = 92° 31′, A = 118° 44′.
11. B = 90°, AC = 107° 19′, AB = 69° 8′.
12. B = 90°, AC = 88° 36′, BC = 101° 25′.
13. A = 90°, AB = 97° 20′, BC = 94° 13′.
14. A = 90°, BC = 113° 10′, C = 74° 32′.

15. A = 90°, BC = 83° 12′, C = 98° 15′.
16. A = 90°, AB = 95° 52′, B = 76° 13′.
17. A = 90°, AC = 106° 27′, C = 94° 8′.
18. A = 90°, AB = 114° 22′, C = 108° 19′.

ART. 21. **Solution of Oblique-angled Spherical Triangles.**—The object of this section is to give the student an insight into the solution of those triangles which refer particularly to the ordinary problems of Nautical Astronomy. Two cases only will be considered.

CASE I.—Given the three sides, to find the angles.

If ABC be a triangle, where the sides AB, AC, and BC are known.

Let $s = \frac{1}{2}(AB + AC + BC)$.

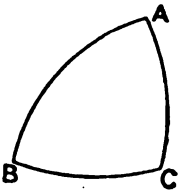


FIG. 66.

The formulæ used are—

$$\cos\frac{A}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - CB)}{\sin AB \,.\, \sin AC}}$$

$$\cos\frac{B}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - AC)}{\sin AB \,.\, \sin BC}}$$

$$\cos\frac{C}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - AB)}{\sin AC \,.\, \sin BC}}$$

Example 1.—In PQR, given PQ = 84° 30′, QR = 108° 44′, PR = 72° 20′. Find the angles.

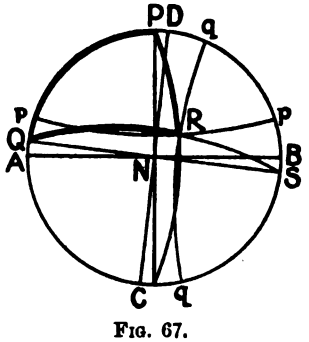


FIG. 67.

Construction (Fig. 67).—Draw primitive, lay off PQ = 84° 30′. Through P draw diameter, and another AB at right angles. Around P as pole describe small circle *pRp* distant 72° 20′ from it or 17° 40′ from AB: the centre for this small circle is on NP produced at distance sec 72° 20′ = 3·24″ from N, radius = tan 72° 20′ = 3·08″. Through Q draw diameter, and another CD at right angles. Around S as pole describe small circle *qRq* distant supplement 108° 44′ or 71° 16′ from it or 18° 44′ from CD: the centre for this small circle is on NS produced at distance sec 71° 16′ from N = 3·07″, and radius is tan 71° 16′ = 2·9″. Where these two small circles intersect is R. Through PR and QR draw oblique circles. PQR is triangle required.

By measurement QPR = 112° semi-tangent = 0·19 or 22°.
" PQR = 70° semi-tangent = 0·18 or 20°.
" PRQ = 78°.

Calculation.—

$$\cos\frac{P}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - QR)}{\sin PQ \,.\, \sin PR}}$$

$$\cos\frac{Q}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - PR)}{\sin PQ \,.\, \sin QR}}$$

$$\cos\frac{R}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - PQ)}{\sin PR \,.\, \sin QR}}$$

To make all the logs added, "sines" in denominators are changed to their reciprocals "cosecs."

QR = 108° 44′	...	...	...	cosec = 10·023639		cosec = 10·023639		
PQ = 84 30	cosec = 10·002004			cosec = 10·002004				
PR = 72 20	cosec = 10·020981			...	...	...	cosec = 10·020981	
2)265 34								
s = 132 47	sin	=	9·865653	sin	=	9·865653	sin	= 9·865653
s − QR = 24 3	sin	=	9·610164					
s − PR = 60 27	...	...	...	sin	=	9·939482		
s − PQ = 48 17	...	...	...	...	...	...	sin	= 9·872998
		2)19·498802			2)19·830778			2)19·783271
55° 50′ 9″	cos =	9·749401						
2		34° 36′ 53″	cos =	9·915389				
P = 111 40 18		2		38° 48′ 53″	cos =	9·891635		
		Q = 69 13 46		2				
				R = 77 37 46				

Ans. P = 111° 40′ 18″; Q = 69° 13′ 46″; R = 77° 37′ 46″.

CASE II.—Given two sides and the contained angle. In the above triangle ABC, let AB, BC be known and angle B; suppose AB greater than BC. The solution depends on Napier's Analogies.

$$\tan \tfrac{1}{2}(C + A) = \frac{\cos \frac{1}{2}(AB - BC)}{\cos \frac{1}{2}(AB + BC)} \cdot \cot \frac{B}{2}$$

$$\tan \tfrac{1}{2}(C - A) = \frac{\sin \frac{1}{2}(AB - BC)}{\sin \frac{1}{2}(AB + BC)} \cdot \cot \frac{B}{2}$$

These by addition and subtraction determine C and A; the third side is obtained from Gauss' theorem.

$$\cos \frac{AC}{2} = \frac{\cos \frac{1}{2}(AB + BC)}{\cos \frac{1}{2}(C + A)} \cdot \sin \frac{B}{2}$$

Note that the reciprocals of the ratios in the denominators are used to make the logs added, and that the *half sum of angles is of the same affection as the half sum of sides.*

Example 2.—In XYZ, given XY = 88° 20′, YZ = 60° 50′, angle Y = 75° 30′. Solve.

Construction.—Draw primitive, measure on it YX = 88° 20′. Draw the diameter through Y: and AB at right angles to it. At Y make angle with primitive = 75° 30′ by drawing oblique circle YZ. Around Y as pole, draw a small circle *aZa* distant 60° 50′ from it at distance sec = 2·06″, and radius tan = 1·8″: this cuts the oblique circle in Z.

XYZ is triangle required.

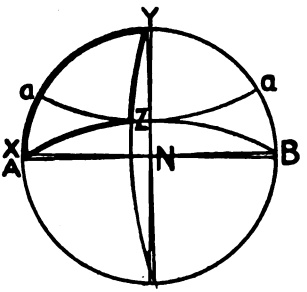


FIG. 68.

By measurement angle X = 60°
,, semi-tangent = 0·26 or 30°
,, angle Z = 96°
,, XZ = 77°

Calculation.—

$$\tan \tfrac{1}{2}(Z + X) = \frac{\cos \frac{1}{2}(XY - YZ)}{\cos \frac{1}{2}(XY + YZ)} \cdot \cot \frac{Y}{2}$$

$$\tan \tfrac{1}{2}(Z - X) = \frac{\sin \frac{1}{2}(XY - YZ)}{\sin \frac{1}{2}(XY + YZ)} \cdot \cot \frac{Y}{2}$$

$$\cos \frac{ZX}{2} = \frac{\cos \frac{1}{2}(XY + YZ)}{\cos \frac{1}{2}(Z + X)} \cdot \sin \frac{Y}{2}$$

XY =	88° 20′						
YZ =	60 50						
XY + YZ =	149 10						
XY − YZ =	27 30						
½(XY + YZ) =	74 35	sec =	10·575385	cosec =	10·015915	cos =	9·424615
½(XY − YZ) =	13 45	cos =	9·987372	sin =	9·376003		
Y/2 =	37 45	cot =	10·111100	cot =	10·111100	sin =	9·786906
½(Z + X) =	78° 2′ 8″	tan =	10·673857			sec =	10·683391
½(Z − X) =	17 39 48	...	...	tan =	9·503018		
Z =	95 41 56				38° 16′ 20″	cos =	9·894912
X =	60 22 20				2		
					76 32 40		

Ans. X = 60° 22′ 20″ ; Z = 95° 41′ 56″ ; XZ = 76° 32′ 40″.

ART. 22.—*To express the cosine of an angle of a spherical triangle in terms of the sines and cosines of the sides.*

Let O be the centre of the sphere, and PQR a spherical triangle. Let tangent at P to arc PR meet OR produced in T; and let tangent at P to arc PQ meet OQ produced in S. Join ST. Thus angle TPS is angle P of spherical triangle, and TOS measures the side RQ; SOP the side PQ; and TOP the side PR.

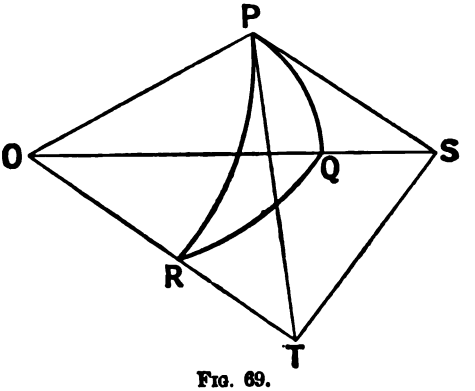


FIG. 69.

In triangle SPT—

$$ST^2 = SP^2 + PT^2 - 2SP \cdot PT \cos P \quad . \quad . \quad . \quad . \quad I.$$

Again, in triangle TOS—

$$ST^2 = SO^2 + OT^2 - 2SO.OT\cos SOT$$
$$= SP^2+PO^2+PT^2+PO^2 - 2SO.OT.\cos RQ$$

since OPS and OPT are right angles—

$$= 2PO^2 + SP^2 + PT^2 - 2SO.OT\cos RQ \quad \text{II.}$$

Subtract I. from II.

$$\therefore 0 = 2PO^2 + 2SP.PT\cos P - 2SO.OT\cos RQ$$

$$\text{Hence } \cos RQ = \frac{PO}{OT}\cdot\frac{PO}{SO} + \frac{PT}{OT}\cdot\frac{SP}{SO}\cos P$$
$$= \cos TOP.\cos POS + \sin TOP.\sin POS.\cos P$$
$$= \cos PR.\cos PQ + \sin PR.\sin PQ.\cos P.$$

This is the fundamental formula, and, where ABC is the triangle and sides denoted by a, b, c, is expressed—

$$\cos a = \cos b.\cos c + \sin b.\sin c.\cos A.$$

The formula used in Case I. is derived from this in the following manner:—

$$\cos a = \cos b.\cos c + \sin b.\sin c.\cos A$$
$$\therefore \cos A = \frac{\cos a - \cos b.\cos c}{\sin b.\sin c}$$
$$\text{Hence } 1 + \cos A = 1 + \frac{\cos a - \cos b.\cos c}{\sin b.\sin c}$$
$$\text{or } 2\cos^2\frac{A}{2} = \frac{\sin b.\sin c + \cos a - \cos b.\cos c}{\sin b.\sin c}$$
$$= \frac{\cos a - \cos(b+c)}{\sin b.\sin c}$$
$$= \frac{2\sin\frac{1}{2}(a+b+c).\sin\frac{1}{2}(b+c-a)}{\sin b.\sin c}$$
$$\text{Let } s = \tfrac{1}{2}(a+b+c) \quad \therefore s - a = \tfrac{1}{2}(b+c-a)$$
$$\therefore \cos^2\frac{A}{2} = \frac{\sin s.\sin(s-a)}{\sin b.\sin c}$$
$$\text{or } \cos\frac{A}{2} = \sqrt{\frac{\sin s.\sin(s-a)}{\sin b.\sin c}}$$

Exercises.—Oblique-angled Spherical Triangles.

1. If BC = 82° 26′, AC = 92° 2′, AB = 104° 28′. Solve.
2. If BC = 140°, AC = 70°, AB = 80°. Solve.
3. If BC = 113° 2′, AC = 82° 40′, and C = 138° 50′. Solve.
4. If AC = 60°, AB = 84° 22′, and A = 95°. Find BC.
5. If BC = 115° 20′, AB = 84° 42′, and B = 65° 30′. Find A and C.
6. If BC = 90°, AC = 75° 29′, AB = 84° 13′. Find A.
7. Sun's R.A. and Dec. being $1^h\ 30^m\ 10^s$, and 9° 26′ 30″ N. Find his long.
8. The R.A. of a star is $16^h\ 49^m\ 48^s$, and its Dec. is 7° 23′ 10″ N.; find its long. and lat., the obliquity of the ecliptic being taken to be 23° 27′ 30″.

9. In a right-angled spherical triangle C = 90°, show tan a = tan c . cos B = tan A . sin b; also if B = b, show triangle must be isosceles.

10. Two ports A and B, both in northern hemisphere, are in long. 90° W. and 10° W. Two ships leave A and B respectively, proceeding each upon the great circle which touches the parallel of the port at the point of departure.

Ship from A steams to the eastward at uniform rate of 12 knots, that from B to the westward at 17 knots; they meet each other in 5 days. Find lat. and long. of point of meeting.

CHAPTER VI.

THE SAILINGS.

ART. 23. **Parallel Sailing.**—In parallel sailing, the ship's course is supposed to continue on the same parallel of latitude. There are three things concerned, viz. the Latitude, the Departure or Meridian Distance, and the Difference of Longitude.

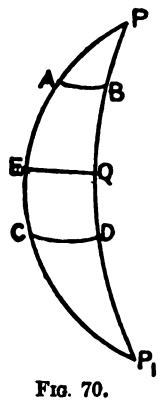


FIG. 70.

Definitions.—(1) *Latitude:* the distance of the ship north or south from the Equator.

(2) *Departure* (or Meridian Distance): the distance in geographical miles between two places on the same parallel of latitude.

(3) *Difference of Longitude:* the arc of the Equator corresponding to the Meridian Distance.

Illustration.—AB and CD are Departures at different distances from the Equator; EQ is the D. long., or corresponding Arc of the Equator.

Remark.—It is seen that the Meridian Distance decreases as the Latitude increases, and vanishes altogether at the Poles.

The formulæ which express the relation between the three terms are—

1. D. long. = dep. × sec lat.
2. Dep. = diff. long. × cos lat.
3. Sec lat. $= \frac{\text{diff. long.}}{\text{dep.}}$

These are the three *cases* in Parallel Sailing, and show how the D. long. or Dep. or Lat. may be found when the other two terms are given.

Let P be the pole, PE and PQ meridians of longitude, EQ

an arc of the Equator, DL an arc of a parallel of latitude, and C the centre of the Earth.

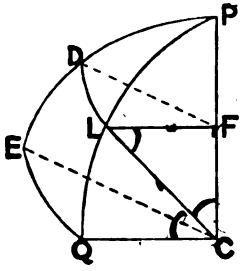


Fig. 71.

Then QL or ED will represent the lat. of the parallel DL, EQ is the diff. long., and DL the departure.

Join CL, and draw FL perpendicular to PC.

It is evident that FL is a radius of the small circle of which DL is a part, and CQ a radius of the great circle of which EQ is a part; that is, a radius of the Earth. And since the arcs are proportional to the radii of the two circles—

Arc DL : are EQ :: rad. FL : rad. CQ
therefore DL : EQ :: FL : CL

because CL is also a radius of the Earth;

$$\text{or } \frac{DL}{EQ} = \frac{FL}{CL} = \sin FCL, \text{ or } \cos QCL$$

because CFL is a plane right-angled triangle, and QCL is the complement of FCL.

Now, the angle QCL is measured by the arc QL, which is the latitude of the parallel DL—

$$\text{therefore } \frac{\text{dep.}}{\text{D. long.}} = \cos \text{lat.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

from which are derived the formulæ—

$$\text{Dep.} = \text{D. long.} \times \cos \text{lat.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{D. long.} = \frac{\text{dep.}}{\cos \text{lat.}} \text{ or dep.} \times \sec \text{lat.} \quad . \quad . \quad . \quad (3)$$

Now, the formula $\frac{\text{dep.}}{\text{D. long.}} = \cos \text{lat.}$ expresses the relation between the base, hypotenuse, and angle at the base of a right-angled plane triangle. Such a triangle may therefore be used to derive the formulæ of parallel sailing, by calling the base departure, the hypotenuse diff. of longitude, and the angle at the base latitude.

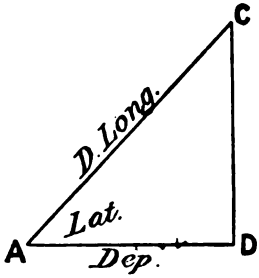


Fig. 72.

$$\text{Then } \frac{\text{dep.}}{\text{D. long.}} = \cos \text{lat.} \quad . \quad . \quad . \quad (1)$$

$$\text{and } \frac{\text{D. long.}}{\text{dep.}} = \sec \text{lat.} \quad . \quad . \quad . \quad (2)$$

From (1) dep. = D. long. × cos lat. (3)
from (2) D. long. = dep. × sec lat. . (4)

which are the same as those derived from the sphere.

ART. 24. **Middle-latitude Sailing.**—This is a combination of plane and parallel sailing, the dep. being common to both, but the latitude used is the "middle lat.," or mean of the two latitudes, for in this case the ship does not sail on a parallel of latitude.

It is evident that the formulæ for right-angled triangles will solve every case, but this method is only made use of in modern navigation for the purpose of finding the D. long. in the "day's work," the formula employed being—

D. long. = dep. × sec of middle lat.

The *mean* is not the true "middle lat.," being too small, but is sufficiently near for short distances.

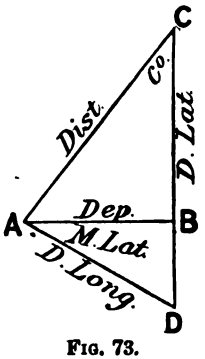


FIG. 73.

Examples.—

1. In lat. 46° 45′ the Departure made good was 110·5 miles: required the Difference of Longitude.

Construction.—In Fig. 74, scale $\frac{1}{10}$ inch = 10 miles. Draw horizontal line AB = 1·1 inches for departure, make angle BAC = 46° 45′, and draw BC perpendicular to AB: then AC represents D. long.

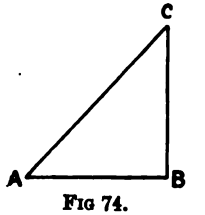


FIG 74.

Measure AC = 1·6 inches or 160 miles.

Dep. 110·5	...	log 2·043362
Lat. 46° 45′	...	sec 10·164193
D. long. = 161·3 ...		log 2·207555

2. In lat. 53° 14′ the D. long. being 3° 16′, required the Meridian Distance or Departure.

In Fig. 75 draw horizontal line AB. Make angle BAC = 53° 14′, and AC = 0·98 inch; scale $\frac{1}{10}$ inch = 20 miles. From C draw CB perpendicular to AB: then AB represents Dep.

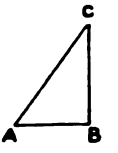


FIG. 75.

Measure AB = 0·6 inch or 120 miles.

D. long. 196′	... ...	log 2·292256
Lat. 53° 14′	... ...	cos 9·777106
Dep. = 117·3	... ...	log 2·069362

3. On what parallel of latitude would a Meridian Distance of 245 miles give a difference of longitude of 5° 54′?

In Fig. 76, scale $\frac{1}{10}$ inch = 20 miles, draw AB = 1·2 long, at B draw a perpendicular to AB, and with A as centre, radius = 1·77 inch, draw an arc, cutting this perpendicular in C: then BAC represents lat.

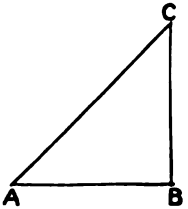


FIG. 76.

Measure BAC = 46°.

D. long. 354	... ...	log 2·549003
Dep. 245	... ...	log 2·389166
Lat. 46° 12′	... ...	sec 10·159837

Remark.—In the first two cases 10 is rejected from the sum of indexes, and in the third case 10 is added to the upper index, because in the Table of Log Sines, etc., the proper index is always increased by 10.

EXERCISES.

1. In lat. 16° 30′ N. the dep. made good was 167·6 miles: what will the difference of longitude be by parallel sailing?

2. The difference of longitude between two places in lat. 63° 19′ 30″ is 4° 20′: required the distance as measured along the parallel.

3. In lat. 11° 56′ N. the dep. made good was 365 miles: required the difference of longitude in minutes.

4. In what latitude is 200 miles of dep. equal to 600 minutes of longitude?

5. In lat. 35° 40′ S. the dep. made good was 9·76: required the difference of longitude measured on the parallel.

6. What is the difference of latitude between two parallels of latitude on different sides of the Equator, when between the same two meridians the dep. on the northern parallel is 250 miles, on the southern 350 miles, and on the Equator 470 miles?

7. On the parallel of 48° 12′ ran true west 1005 miles: if the longitude left was 9° 30′ W., what was the longitude in?

8. From two ports in lat. 40° 20′ N., distant 300 minutes of longitude, two ships sail N. for 300 miles: how many miles apart will they be, and in what latitude?

9. In lat. 33° 15′ N. ran east 10·95 miles: how many minutes of longitude will that represent?

10. If two ships in lat. 51° S. should sail directly north until the departure changes from 400 to 500 minutes, what latitude will they be in?

11. In lat. 72° 59′ the dep. was 370·5 miles: convert that into minutes of longitude.

12. In what latitude does 300 miles of dep. correspond to 600 minutes of longitude?

13. The latitude by Mer. alt. was 36° 35′ N.; longitude by afternoon sights, 137° 38′ E.; run since noon due east, 39 miles: find noon longitude.

14. A vessel sailed due east along the parallel of 45° 24′ S. for 29 miles from long. 43° 40′ E.: what is her longitude in?

15. What distance must a vessel sail on the parallel 48° 25′ N. to change her longitude 2° 25′?

16. A vessel sailing due east finds her difference of longitude to be 1° 19′, and distance made good 50′: what is her latitude?

17. How far on the parallel of 38° S. must a vessel sail from long. 47° 29′ E. to reach long. 49° 27′ E.?

18. For every 1° diff. long. I have to sail 45 miles due west: along what parallel am I sailing?

ART. 25. **Plane and Traverse Sailings.**—For short distances, the surface of the Earth may be regarded as a plane. In plane and traverse sailings no notice is taken of the curvature, and all cases may be solved by the formulæ for right-angled triangles.

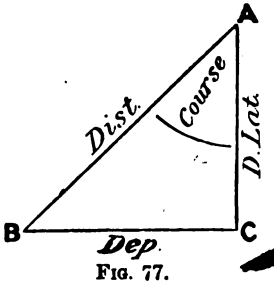


FIG. 77.

Thus in sailing from A to B, the hypotenuse represents the distance, the perpendicular the departure, and the base the D. lat., whilst the angle which AB makes with AC (lying true N. and S.) is the course.

In the Traverse Table the values of D. lat. and Dep. in miles are given for every degree of course

up to 89° and all distances up to 600 miles, being computed from the formulæ—

$$\frac{\text{D. lat.}}{\text{dist.}} = \cos \text{course}$$

$$\frac{\text{dep.}}{\text{dist.}} = \sin \text{course}$$

whence D. lat. = dist. × cos course . . . . (1)
and dep. = dist. × sin course . . . . (2)

When D. lat. and dep. are given, the course and distance made good may be found from the formulæ—

$$\tan \text{course} = \frac{\text{dep.}}{\text{D. lat.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{distance} = \text{D. lat.} \times \sec \text{course} \quad . \quad . \quad . \quad (4)$$

ART. 25. **The Day's Work and Traverse Sailing.**—The purpose of the Day's Work is to find the position of the ship at noon by "Dead reckoning," that is, from the courses and distances sailed since the preceding noon. The direct course and distance made good are also determined.

The courses given are by compass as per ship's Log Book, but the *true* courses must be found for computing a "Traverse Table."

Directions for finding the True Courses.—(*a*) Allow the leeway to the Right (R.) when the wind blows on the left or Port Side of the ship, and to the Left (L.) when on the Right or Starboard Side. (*b*) Allow E. deviation and variation (or total correction) to the Right (R.), and W. to the Left (L.).

NOTE.—The Leeway may be determined by towing the log-ship astern and observing the angle the line makes with the keel, but it is often merely estimated.

The Deviation is found by reference to the Deviation Card, and the Variation from a chart.

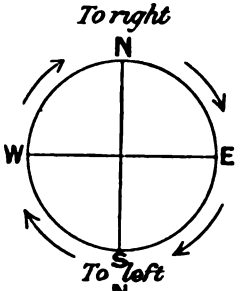
When the Departure is taken from a known point of land or light, the opposite course to the bearing of the point or light from the ship is called the "Departure Course," and the distance off is the distance belonging to that course.

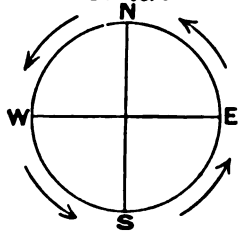
The "Set" and "Drift" of the current are found by chart, and are treated as a course and distance made by the ship, because the effect is the same on the Day's Work.

NOTE.—Should the "Set" and "Drift" of the current be required, it is the course and distance calculated by Mercator *from* the D.R. position found by Traverse *to* the observation position given in the question.

They may also be found by turning D. long. into Dep.: then with the D. lat. and Dep. find Co. and Dist. in Traverse Table, for Set and Drift required.

Illustrations and Minor Calculations.





Opposite course to bearing.

(1) S. 45° R.
Corr. 11 R.
S. 56 R.

(2) N. 65° L.
Leeway 20 L.
N. 85 L.
Corr. 3 R.
N. 82 L.

(3) N. 77° R.
17 R.
N. 94 R.
12 K.
N. 106 R.
180
S. 74 E.

(4) N. 57° L.
16 L.
N. 73 L.
2 L.
N. 75 L.

(5) S. 90° L.
19 R.
S. 71 L.
21 R.
S. 50 L.

(6) N. 52° L.
14 L.
N. 66 L.
Corr. 0
N. 66 L.

Example 1.

Hour.	Course steered.	Knts.	10ths.	Wind.	Leeway.	Deviation.	Remarks.
1	N. 65° W.	5	6	N. by E.	20°	12° W.	A point of land in lat. 46° 3′ S., long. 146° 15′ E., bore N.E. from ship, distant 12 miles, ship's head being W., and Deviation 4° W. Variation 15° E.
2		5	5				
3		5	8				
4		6	1				
5	N. 77° E.	6	2	N. by E.	17°	3° W.	
6		5	9				
7		5	7				
8		5	2				
9	N. 57° W.	5	2	N. by E. ½ E.	16°	17° W.	
10		5	3				
11		5	0				
12		5	5				
1	E.	5	7	N.N.E.	19°	6° E.	A current set N. magnetic 24 miles during the 24 hours from the time of departure to the end of the day.
2		6	3				
3		6	8				
4		7	2				
5	N. 52° W.	7	2	N.N.E.	14°	15° W	
6		7	3				
7		7	6				
8		7	9				
9	S. 80° E.	8	0	N.E. by N. ½ N.	21°	14° E.	
10		8	1				
11		8	2				
12		8	7				

Correct the courses, and find the course and distance made good; also the Latitude and Longitude in by inspection.

Traverse Table.

True courses.	Distance.	D. Lat.		Dep.	
		N.	S.	E.	W.
S. 56° W.	12		6·7		9·9
N. 82° W.	23	3·2			22·8
S. 74° E.	23		6·3	22·1	
N. 75° W.	21	5·4			20·3
S. 50° E.	26		16·7	19·9	
N. 66° W.	30	12·2			27·4
S. 30° E.	33		28·6	16·5	
N. 15° E.	24	23·2		6·2	
		44·0	58·3	64·7	80·4
			44·0		64·7
		D. lat. 14·3		Dep. 15·7	

The D. lat. and dep. are found nearest together under the course 48

and dist. 21 miles, which are the course and distance made good (course S. 48° W.).

Minor Calculations.

(7) S. 80 L. 21 R. — S. 59 L. 29 R. — S. 30 L.

(8) Current. N. 0° 15 R. — N. 15 R.

Lat. left	46° 3′ S.	Long. left	146° 15′ E.
D. lat.	14′ S.	D. long.	23′ W.
Lat. in	46° 17′ S.	Long. in	145° 52′ E.
	2)92° 20′		
Mid. lat.	46° 10′	...	sec 10·159541
Dep.	15·7	...	log 1·195900
Diff. long.	22·67	...	log 1·355441

By inspection: The M. lat. 46° as a course with dep. 15·7 in Lat. column gives 23′ long. in Dist. column.

In Fig. 77A, scale 1 inch = 10 miles, let ABC represent portion of meridian.

AB = 1·43 inch is D. lat.
BD = 1·57 inch is Dep.
where ABD is 90°.
Lay off BDC = 46° Mid. lat.
Then BAD is course = 48°.
AD is Dist. = 2·1 inch.
= 21 miles.
DC is D. long = 2·3 inch.
= 23 miles.

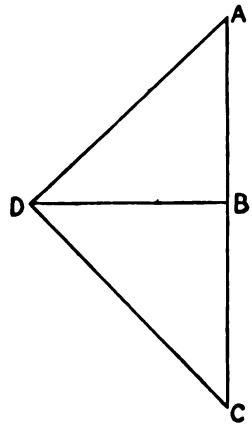


FIG. 77A.

Example 2.

Hour.	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Deviation.	Remarks.
					Pts.		
1	S.W.	8	2	S.E. by S.	1½	6° W.	A point of land in lat. 49° 57′ N., long. 5° 12′ W., bore by compass N.W. ½ N., distant 20 miles, ship's head being S.W., and Deviation as per log. Variation 19° W.
2		8	4				
3		9	2				
4		9	2				
5	S.W. by W.	8	9	S.S.E.	1¾	3° W.	
6		8	5				
7		7	6				
8		7	0				
9	S.W. by S.	6	7	S.E.	1½	2° E.	
10		6	5				
11		6	2				
12		5	6				
1	S.S E.	7	5	S.W.	2	7° E.	A current set E. by N. ½ N. (corr. mag.) 30 miles, during the 24 hours.
2		7	9				
3		8	5				
4		9	1				
5	S. by W. ½ W.	10	0	W.	1¾	4° E.	
6		10	1				
7		10	3				
8		10	6				
9	S. ¼ E.	11	3	W. by N.	½	2° E.	
10		11	6				
11		11	7				
12		12	4				

Required the course and distance made good, and the Latitude and Longitude in.

Minor Calculations.

(1) S. 39° L. Corr. 25 L. — S. 64 L.

(2) S. 62° R. Corr. 25 L. — S. 37 R.

(3) S. 76° R. 22 L. — S. 54 R.

(4) S. 51° L. 17 L. — S. 34 R.

(5) S. 45° L. 12 L. — S. 57 L.

(6) S. 3° L. 15 L. — S. 18 L.

(7) S. 8° L. 17 L. — S. 25 L.

(8) Current N. 73° R. Var. 19 L. — N. 54 R.

Traverse Table.

True courses.	Distance.	D. Lat. N.	D. Lat. S.	Dep. E.	Dep. W.
S. 64° E.	20		8·8	18·0	
S. 37° W.	35		28·0		21·1
S. 54° W.	32		18·8		25·9
S. 34° W.	25		20·7		14·0
S. 57° E.	33		18·0	27·7	
S. 18° E.	41		39·0	12·7	
S. 25° E.	47		42·6	19·9	
N. 54° E.	30	17·6		24·3	
		17·6	175·9	102·6	61·0
			17·6	61·0	
			158·3	41·6	

Course and dist. made good S. 15° E. 164 miles.

Lat. left 49° 57′ N. Long. left 5° 12′ W.
D. lat. 2° 38′ S. D. long. 1° 3′ E.

Lat. in 47° 19′ N. Long in 4° 9′ W.
2)97° 16′

Mid. lat. 48° 38′ ... ... sec 10·179880
Dep. 41·6 ... ... log 1·619093

D. long. 62·95 ... ... log 1·798973

Fig. 78.

In Fig. 78 scale 1 inch = 100 miles, ABC represents meridian.
AB = 1·58 inch is D. lat.
BD = 0·42 inch is Dep.
ABD = 90°.
Make BDC = 48½ = Mid. lat.
Then BAD is course = 15°.
AD is Dist. = 1·64 inch.
= 164 miles.
CD is D. long = 0·63 inch.
= 63 miles.

Note 1.—The D. long. can also be found by inspection thus: Take the mid. lat. as a course and find dep. in D. lat. column; the dist. corresponding is the D. long.

Note 2.—When course and leeway are in "points," it is most convenient to allow the leeway before changing points into degrees, then apply the correction.

Note 3.—To calculate course and distance made good.

D. lat. 158·3 S., dep. 41·6 E.

In triangle ABC, C = 90, AC = 158·3, BC = 41·6.

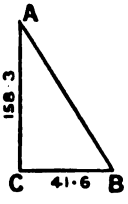


Fig. 79.

Find— A = course
AB = distance.

Now— $\tan A = \frac{BC}{AC} = \frac{41·6}{158·3}$ and $\frac{AB}{AC} = \sec A$

$\therefore AB = AC \times \sec A$

41·6 log = 11·619093
158·3 log = 2·199481 ... ... 2·199481

A = 14° 43′ 26″ tan = 9·419612 sec = 10·014501

163·7 log = 2·213982

Course S. 14° 43′ 26″ E., distance 163·7 miles.

EXERCISES.

1.

Hour.	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Deviation.	Remarks.
					Pts.		
1	N.N.W.	17	1	W. ¼ S.	1	7° E.	
2		16	2				
3		14	3				A point of land in lat. 0° 34′ S., long. 0° 17′ W., bearing by compass S. ¾ E., distant 49 miles, with the ship's head at N.N.W. Deviation as per log. Variation 30° W.
4		13	4				
5	N. by W.	13	2	N.W. by W. ¾ W.	2	36° E.	
6		15	5				
7		17	6				
8		16	7				
9	N.N.E.	15	8	East	3	19° W.	
10		10	9				
11		12	1				
12		16	2				
1	N.E. by E. ¼ E.	13	3	S.E. by E.	3½	5° W.	
2		17	4				
3		10	5				
4		15	8				A current set correct magnetic South 4 miles from the time departure was taken to the end of the day.
5	S.W. ¼ W.	14	9	S. by E. ½ E.	½	4° E.	
6		13	3				
7		15	9				
8		17	9				
9	S.S.W.	11	4	West	2	5° E.	
10		14	9				
11		13	8				
12		16	9				

Correct the courses for Deviation, Variation, and Leeway, and find the course and distance from the given point, and the Latitude and Longitude in.

2.

Hour.	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Deviation.	Remarks.
1	N. 85° W.	12	6	S.S.W.	—	14° E.	
2		12	7				
3		12	4				A point in lat. 55° 14′ N., long. 8° 20′ W., bearing by compass S. 23° E., ship's head N. 85° W. Deviation as per log., distant 7 miles.
4		12	—				
5		12	2				
6		12	8				
7		12	5				
8		12	4				
9		12	5				
10		12	6				
11		12	3				
12		12	—				

(Continued on next page.)

Hour	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Devia-tion.	Remarks.
1	N. 82° W.	11	9	S.W.	—	12° E.	Variation during first 8 hours, 28° W.; during second 8 hours, 30° W.; during third 8 hours, 32° W. A current set correct magnetic S.S.E. ½ E. 2¼ miles per hour for all day.
2		11	8				
3		11	6				
4		11	9				
5		12	1				
6		12	4				
7		12	5				
8		12	3				
9		12	2				
10		12	—				
11		11	9				
12		11	4				

Find the course and distance made good; Latitude and Longitude of the ship.

3.

Hour	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Devia-tion.	Remarks.
1	N. by E. ¼ E.	9	1	S.E. by E.	Pts. ¼	18° E.	A point of land in lat. 18° 59′ S., long. 179° 53′ W., bearing by compass E.S.E., distant 19 miles, with the ship's head at N. by E. ¼ E. Deviation as per log. Variation 9° E.
2		9	3				
3		9	9				
4		8	7				
5	S.E. by E.	6	4	S. by W.	1½	23° W.	
6		6	1				
7		6	3				
8		6	2				
9	E. by N. ¼ N.	6	0	N. ½ E.	1¾	3° W.	
10		5	9				
11		5	8				
12		5	3				
1	S.W. ¾ W.	5	3	S.S.E.	2¼	0	A current set correct magnetic S.S.E. ¾ E., 15 miles from the time departure was taken to the end of the day.
2		5	1				
3		5	2				
4		5	4				
5	S.S.W.	7	9	W. by N.	1	11° W.	
6		8	2				
7		8	4				
8		9	5				
9	N.N.E.	9	6	N.W ½ W.	1¼	16° E.	
10		10	0				
11		10	3				
12		10	1				

Correct the courses for Deviation, Variation, and Leeway, and find the course and distance from the given point, and the Latitude and Longitude in.

4.

Hour.	Course steered.	Knts.	10ths.	Wind.	Leeway.	Deviation.	Remarks.
					Pts.		
1	S. by W. ½ W.	16	3	W. ½ S.	1¼	7° E.	A point of land in lat. 58° 30′ S., long. 64° 54′ W., bearing by compass N.W. ¼ N., distant 60 miles, with the ship's head at S. by W. ½ W. Deviation as per log. Variation 21° E.
2		17	1				
3		10	2				
4		9	4				
5	W. by S. ¼ S.	11	9	N.W. by N.	¾	9° E.	
6		12	0				
7		13	0				
8		14	1				
9	S.W. ¼ W.	16	9	S.S.E	¼	8° W.	
10		14	8				
11		12	8				
12		13	5				
1	N.E. by N.	10	9	W.S.W.	0	11° W.	A current set correct magnetic W. ½ S. 39 miles, from the time departure was taken to the end of the day.
2		11	8				
3		12	8				
4		13	5				
5	N. by E.	14	8	N.W. by W.	½	12° W.	
6		15	8				
7		16	8				
8		17	6				
9	W. by N.	9	7	S.W. by S.	¼	41° W.	
10		1	7				
11		2	7				
12		6	9				

Correct the courses for Deviation, Variation, and Leeway, and find the course and distance from the given point, and the Latitude and Longitude in.

5.

Hour.	Course steered.	Knts.	10ths.	Wind.	Leeway.	Deviation.	Remarks.
					Pts.		
1	N.N.W. ½ W.	4	1	N.E.	1¾	2° W.	A point in lat. 29° 59′ N., long. 132° 54′ E., bearing by compass N.N.E. ½ E., ship's head N.W. by W. Deviation 6° W., distant 15 miles.
2		3	3				
3		3	1				
4		1	5				
5	E.S.E.	2	1	N.E.	2	7° E.	
6		3	1				
7		3	1				
8		4	7				
9	S. ¾ E.	6	3	E.S.E.	2¼	2° W.	
10		6	9				
11		5	4				
12		6	4				

(Continued on next page.)

Hour.	Course steered.	Knts.	10ths.	Wind.	Lee-way.	Devia-tion.	Remarks.
					Pts.		
1	N.E. ¼ N.	9	7	E.S.E.	1½	8° E.	
2		7	4				
3		0	2				
4		2	7				
5	W. ½ N.	3	5	S.S.W. ½ W.	1¼	9° W.	
6		4	3				Variation 1° E.
7		3	6				
8		3	6				
9	N. by E.	8	5	E. by N.	¼	6° E.	
10		1	1				
11		8	1				
12		9	3				

The position by observation was found to be lat. 30° 25′ N., long. 133° 7′ E. Find the set and drift of the current.

6.

Hour.	Course steered.	Knots.	10ths.	Winds.	Lee-way.	Devia-tion.	Remarks.
1	S. by W.	12	1	W. by S.	¾ pt.	9° W.	
2		13	2				
3		11	3				
4		12	4				
5	N. ¾ E.	12	9	East	¼ pt.	6° E.	
6		13	8				
7		14	7				
8		11	6				
9	East	11	2	S.S.E.	½ pt.	19° E.	A point in lat. 54° 22′ N., long. 178° 26′ E., bearing by compass N.W. ½ W., ship's head N. ¾ E. Deviation as per log, distant 8 miles. Variation 12° E.
10		12	2				
11		12	2				
12		13	4				
1	S.E.	9	3	S.S.W.	¾ pt.	11° E.	
2		11	4				
3		12	7				
4		10	6				
5	S.S.W. ¼ W.	12	6	N.W.	—	12° W.	
6		11	2				
7		14	9				
8		15	3				
9	E. ¾ N.	13	9	North	¼ pt.	18° E.	
10		12	9				
11		11	9				
12		13	3				

The ship by observation was found in lat. 51° 59′ N., long. 178° 5′ W. Find set and drift of current.

ART. 27. **Mercator's Sailing.**—The problem in Mercator's Sailing commonly used, and the only one in the Board of Trade Examination for Mates and Masters, is to find the *Course* and *Distance* from place to place, the latitudes and longitudes of both places being given.

The data used in the calculation are—

The difference of latitude.

The difference of longitude.

The meridional difference of latitude.

Definitions.—(1) D. lat.: The sum or difference of the two latitudes (according as they are of different names or of the same name) reduced to miles.

(2) D. long.: The sum or difference of the two longitudes (according as they are of different names or of the same name) reduced to minutes of arc.

N.B.—When the sum of longitudes exceeds 180°, subtract from 360°.

(3) Mer. D. lat.: The sum or difference of the meridional parts for the two latitudes.

Formulæ.—(1) To find the Course—

$$\text{Tangent of course} = \frac{\text{D. long.}}{\text{mer. D. lat.}}$$

(2) To find the Distance—

$$\text{Distance} = \text{secant of course} \times \text{D. lat.}$$

It may be graphically represented by two similar triangles, DEC and ABC, the angle at C being the course, which is common to both triangles. The sides of the smaller triangle represent the elements of plane sailing. The extended meridian CB represents the meridional D. lat. found from the meridional parts for the two latitudes; then the side AB will be the D. long. in its correct proportion to the departure. The object aimed at in Mercator's method is to find the rhumb course from point to point. For this purpose, the diff. lat. is extended by means of the meridional parts so as to bear the same proportion to the diff. long. as the true diff. lat. bears to the departure. In the right-angled triangle ABC—

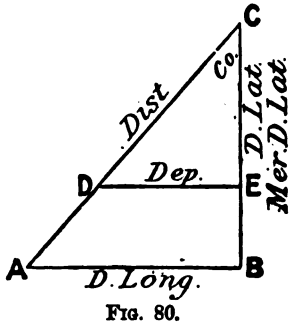


FIG. 80.

$$\text{Tan course} = \frac{\text{D. long.}}{\text{mer. D. lat.}} \quad \ldots\ldots \quad (1)$$

and in the triangle DEC—

$$\frac{\text{dist.}}{\text{D. lat.}} = \text{sec course}$$

$$\text{therefore dist.} = \text{D. lat.} \times \text{sec course} \quad \ldots \quad (2)$$

These are the only formulæ required for computing the true rhumb course and the distance on that course. Evidently this distance is longer than the distance on a great circle, but the method has the advantage of leading to the destination without change of course.

Examples.—

1. Find the Course and Distance from A to D on Mercator's principle.

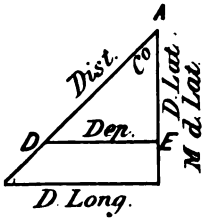


FIG. 81.

Lat. of A, 51° 36′ N.; long. of A, 8° 32′ W.
„ D, 38° 32′ N.; „ D, 28° 38′ W.

51° 36′ N.	Mer. parts	3626	8° 32′ W.		
38 32 N.	„	2509	28 38 W.		
13 4		1117	20 6		
60			60		
784			1206		

D. long. 1206 ... ... log 3·081347
Mer. D. lat. 1117 ... ... log 3·048053

Course S. 47° 11½′ W. ... ... log tan 10·033294
Course 47° 11½′ ... ... log sec 10·167780
D. lat. 784 ... ... log 2·894316

Distance 1154 miles ... ... log 3·062096

NOTE.—The Course is reckoned from S because D is south of A, and towards W. because D is west of A.

2. Find the Course and Distance from P (Hawraki) to B (Hawaii).

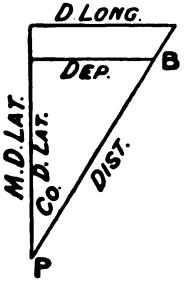


FIG. 82.

Lat. of P, 35° 59′ S.; long. of P, 175° 4′ E.
„ B, 19° 50′ N.; „ B, 156° 40′ W.

35° 59′ S.	Mer. parts	2317	175° 4′ E.		
19 50 N.	„	1215	156 40 W.		
55 49	Mer. D. lat.	3532	331 44		
60			360 0		
D. lat. 3349			28 16		
			60		
			D. long. 1696		

D. long. 1696 ... ... log 3·229426
Mer. D. lat. 3532 ... ... log 3·548021

Course N. 25° 39′ E. ... ... log tan 9·681405
Course 25° 39′ ... ... log sec 10·045056
D. lat. 3349 ... ... log 3·524915

Distance 3715 ... ... log 3·569971

NOTE.—The Course is reckoned from N. because B is north of P, and E. because it crosses the meridian of 180°.

3. Required the Course and Distance by Mercator from A to B.

Lat. of A, 6° 15′ N.; long. of A, 79° 30′ E.
,, B, 36° 10′ S.; ,, B, 113° 45′ E.

Lat. 6° 15′ N. Mer. parts 376 Long. 79° 30′ E.
,, 36° 10′ S. ,, 2330 ,, 113° 45′ E.

42 25 Mer. D. lat. 2706 34 15
60 60

D. lat. } 2545 D. long. 2055

D. long. 2055 ... ... log 3·312812
Mer. D. lat. 2706 ... ... log 3·432328

Course S. 37° 13′ E. log tan 9·880484
Course 37° 13′ ... log sec 10·098894
D. lat. 2545 ... ... log 3·405688

Distance 3196 miles ... log 3·504582

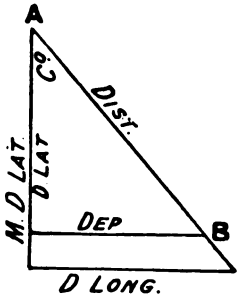


FIG. 83.

4. Required the Course and Distance by Mercator's principle from A, in lat. 57° 42′ S.; long. 70° 3′ W., to B. in lat. 46° 58′ S.; long. 168° 20′ E.; also the Course to steer by compass, supposing the variation to be 22° E., and the deviation of the compass 7° 30′ W.

In Fig. 84, ABC is portion of meridian. Scale 1 inch = 2000 miles.

AC = 0·53 inch is Mer. D. lat.
ACD = 90°.
CD = 3·65 inch is D. long.
AB = 0·32 inch is D. lat.
BE parallel to CD is Dep.

Then CAD = 82° is true Course.
AE = 2·24 inches.
= 4480 miles is Dist.

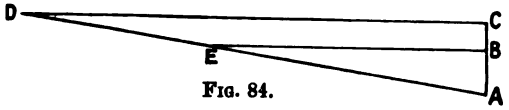


FIG. 84.

Lat. 57° 42′ S. Mer. parts 4260 Long. 70° 3′ W.
,, 46° 58′ S. ,, 3200 ,, 168° 20′ E.

10 44 Mer. D. lat. 1060 238 23
60 360 0

D. lat. } 644 121 37
60

D. long. 7297

D. long. 7297 ... ... log 3·863144
Mer. D. lat. 1060 ... ... log 3·025306

N. 81° 44′ 5″ W. ... ... log tan 10·837838
764

74 gives 5″

Course 81° 44′ 5″ ... ... log sec 10·842372
D. lat. 644 ... ... log 2·808886

Distance 4480 miles ... ... log 3·651258

NOTE.—When the Course is large, it may be advisable to work to seconds, as in this case, in order to ensure the accuracy of the distance within a mile.

To find the Course by compass :

True course	N. 81° 44′ W.
Variation	22° 0′ E.
Magnetic course	N. 103 44 W.
Deviation	7 30 W.
Compass course	N. 96 14 W.
Or	S. 83 46 W.

EXERCISES.

Required the Compass Course and Distance from A to B by calculation on Mercator's principle.

1. Lat. A, 50° 49′ N.; long. A, 0° 0′.
   ,, B, 48° 17′ N.; ,, B, 54° 47′ W.
   Variation 19° 45′ W.; deviation 20° W.
2. Lat. A, 11° 11′ S.; long. A, 0° 37′ W.
   ,, B, 0° 0′; ,, B, 0° 0′.
   Var. 23° 30′ W.; dev. 10° 17′ E.
3. Lat. A, 52° 57′ N.; long. A, 158° 33′ W.
   ,, B, 39° 30′ S.; ,, B, 178° 48′ E.
   Var. 25° E.; dev. 15° W.
4. Lat. A, 14° 9′ S.; long. A, 180° 0′.
   ,, B, 0° 0′; ,, B, 174° 57′ E.
   Var. 9° E.; dev. 5° 30′ E.
5. Lat. A, 35° 15′ S.; long. A, 115° 15′ E.
   ,, B, 51° 30′ S.; ,, B, 178° 52′ W.
   Var. 7½° W.; dev. 15½° W.
6. Lat. A, 51° 24′ N.; long. A, 9° 36′ W.
   ,, B, 38° 36′ S.; ,, B, 170° 24′ E.
   Var. 25½° W.; dev. 14½° E.
7. Lat. A, 0° 0′; long. A, 81° 27′ W.
   ,, B, 61° 10′ S.; ,, B, 170° 59′ E.
   Var. 2° 20′ E.; dev. 20° 40′ E.
8. Lat. A, 42° 56′ S.; long. A, 100° 36′ W.
   ,, B, 10° 36′ N.; ,, B, 180° 0′.
   Var. 17° 10′ W.; dev. 13° 50′ E.
9. Lat. A, 36° 19′ S.; long. A, 30° 46′ W.
   ,, B, 36° 12′ S.; ,, B, 58° 23′ W.
   Var. 31° 15′ W.; dev. 14° 26′ W.
10. Lat. A, 0° 0′; long. A, 31° 17′ W.
    ,, B, 63° 28′ N.; ,, B, 28° 16′ W.
    Var. 12° 20′ W.; dev. 15° 30′ E.
11. Lat. A, 32° 22′ N.; long. A, 33° 15′ E.
    ,, B, 37° 4′ N.; ,, B, 15° 40′ E.
    Var. 5° 35′ W.; dev. 12° 29′ W.
12. Lat. A, 0° 17′ S.; long. A, 59° 47′ W.
    ,, B, 1° 59′ N.; ,, B, 120° 13′ E.
    Var. 6° 40′ E.; dev. 13° 50′ W.

ART. 27.—**Current Sailing.**

1. Given the course and rate of sailing of a ship, and the "set" and "drift" of a current during the same interval of time, to find the course and distance made good.

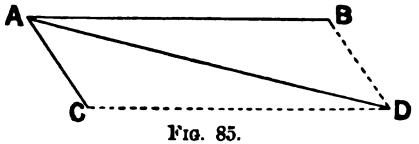


FIG. 85.

Let AB represent the course and the distance sailed, and AC the "set" and "drift" of the current. Draw CD parallel to AB, and BD parallel to AC, and join AD; then AD will represent the course and distance made good during the time elapsed.

Here the angle BAC is known, and therefore ABD is also known, because it is equal to 180° − BAC. Also the sides AB and BD of the triangle ABD are known (because BD = AC). Hence two sides and included angle of the oblique-angled triangle ABD are known, from which the other angles and the third side AD can be computed. From the given direction of AB and the angle BAD the course of AD may be found.

Example 1.—A ship steers E. by N. at 10 miles per hour in a current setting S. by W. 3 miles per hour. Find the course and distance made good.

The angle BAC = 10 points, therefore ACD = 6 points (or 67° 30′), because the two together are equal to two right angles. Applying the formula for two sides and including angle—

$$\tan \tfrac{1}{2}(A - D) = \frac{CD - AC}{CD + AC} \cdot \tan \tfrac{1}{2}(A + D)$$

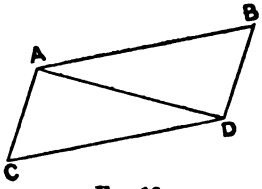


FIG. 86.

	180° 0′	½(A + D)	56° 15′	...	tan 10·175107
	67 30	CD − AC	7	...	log 0·845098
A + D	112 30				11·020205
		CD + AC	13	...	log 1·113943
½(A + D)	56 15				
CD	10	½(A − D)	38 52	...	tan 9·906262
AC	3	½(A + D)	56 15		
CD + AC	13				
CD − AC	7	∠ CDA	17 23		(being the lesser angle opposite to lesser side)

Course of AB N. 78 45 E.

N. 96 8 E.
180 0

Course made good S. 83 52 E.

To find distance made good (AD).

$$\frac{AD}{AC} = \frac{\sin ACD}{\sin CDA}$$

$$\text{therefore } AD = \frac{AC \times \sin ACD}{\sin CDA}$$

ACD	67° 30′	...	... sin	9·965615
AC	3	...	... log	0·477121
CDA	17° 23′	...	cosec	10·524673
AD	9·3	...	... log	0·967409

2. Given the course to a given point, the set and drift of the current, and the rate of sailing of the ship; to find the course to steer and the distance made good towards the point in a given time.

Let A be the place of the ship, and B the destination, and AC the set and drift of the current. From C set off CD equal to the distance the ship would sail in the given time, cutting AB in the point D. Then CD gives the course to steer, and AD is the distance made good towards B.

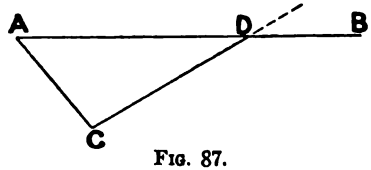


FIG. 87.

In the triangle ACD, the two sides AC and CD are given, and the angle at A opposite to the latter, from which the other angles and remaining side can be found by the rule of sines (Case I.).

Example.—Given the direct course from A to B, E. by S., and the set and drift of the current S.S.E. 4 miles per hour, the ship's rate of sailing being 7 miles per hour, required the course to steer in order to counteract the effect of the current, and the distance made good towards B in 2 hours?

Draw AC in a S.S.E. direction, and set off on it 8 miles (drift in 2 hours), and from C set off 14 miles (2 hours' sailing), cutting AB in the point D, and

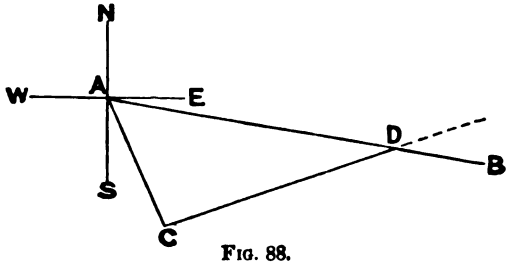


FIG. 88.

join CD. The line CD will give the course to steer, and AD is the distance made good.

Here AC = 8, CD = 14, and the angle CAD (opposite to CD) = 56° 15′.

$$\text{Then } \frac{\sin D}{\sin A} = \frac{AC}{CD}$$

$$\text{and } \sin D = \frac{AC \times \sin A}{CD}$$

A	56° 15′	...	...	sine	9·919846
AC	8	...	...	log	0·903090
					10·822936
CD	14	...	...	log	1·146128
			D 28° 22′	sine	9·676808

Course from A to B S. 78 45 E.

S. 107 7 E.
180 0

Course to steer N. 72 53 E.

To find AD.

A = 56° 15′
D = 28 22

84 37
180 0

C = 95 23

$$\frac{AD}{CD} = \frac{\sin C}{\sin A}$$

$$\therefore AD = \frac{CD \times \sin C}{\sin A}$$

C	95° 23′	...	...	sin	9·998080
CD	14	...	...	log	1·146128
A	56 15	...		cosec	10·080154
Dist. made good	16·76	...	...	log	1·224362

3. The ship's position by observation and by dead reckoning being known, the set and drift of the current may be found. (Solved by right-angled triangles.)

Example.—At noon on a certain day, according to dead reckoning from the previous noon, a ship was in lat. 45° 30′ N., long. 28° 15′ W.; but by observation she was found to be in lat. 46° 15′ N., long. 27° 8′ W. Required the set and drift of the current.

		M. pts.			
D.R. lat.	45° 30′ N.	3073	Long.	28° 15′ W.	
Obs. ,,	46 15 N.	3137	,,	27 8 W.	
	45 N.	64		67 E.	
D. long.	67 ...	log	1·826075		
M. D. lat.	64 ...	log	1·806180		

Set of current N. 46° 19′ E. tan 10·019895

46° 19′	...	sec	10·160728
D. lat. 45	...	log	1·653213
Drift in 24 hours	65·1	log	1·813941

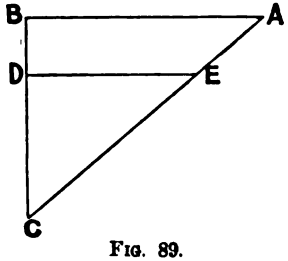


FIG. 89.

$$\text{Rate per hour} = \frac{65{\cdot}1}{24} = 2{\cdot}7 \text{ miles.}$$

NOTE.—This is a case of finding a course and distance by Mercator, and is solved by the formulæ for right-angled triangles.

Examples 1 and 3 can be solved approximately by means of the Traverse Table.

Example 1.—

Courses.	Dist.	N.	S.	E.	W.
E. by N.	10	2·0	—	9·8	—
S. by W.	3	—	2·9	—	0·6
		2·0	2·9	9·8	0 6
			2·0	0·6	
			0·9	9·2	

Course and distance made good, S. 84° E. 9·3 miles.

Example 3.—D. lat. 64 and dep. 67 give course 46° . . . . . (1)
Course 46°, with D. lat. 45, give dist. 65 . . . . (2)

Questions in current sailing may be varied in many ways, but all of them can be solved on the principles of the three foregoing examples.

ART. 28.—**The "Danger Angle."**—When sailing near the land where there are outlying dangers, a safe passage may be secured by calculating the angle subtended at a safe distance by the height of the top of a light-house or cliff above sea-level. Setting this angle on the sextant, the navigator so shapes his course that this vertical angle is not exceeded, allowance being also made for the dip of the land horizon (page 191 Norie or 35 Raper).

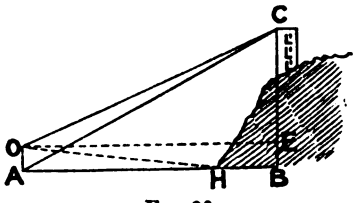


FIG. 90.

Referring to the figure, the height BC and the safe distance AB being known (by the chart)—

$$\text{Tan} \angle \text{BAC} = \frac{\text{CB}}{\text{AB}}$$

Let AO be the height of observer's eye above sea-level, OE a horizontal line, and H the base of the cliff or land horizon; then $\angle$ EOH is the dip, EB = OA, and CE = CB − EB.

Now, tan $\angle$ COE = $\frac{\text{CE}}{\text{OE}}$, and the dip being added to $\angle$ COE, the danger angle COH is found.

Example.—The height of the top of a lighthouse which stands inland 500 feet is 725 feet, and, owing to the existence of a sunken rock, it is not safe to pass nearer the land than 1¼ mile. Required the greatest vertical angle for safe navigation which the top of the lighthouse should make with the base of the cliff on which it stands, the height of eye being 20 ft.

Referring to the figure—

$$CE = 725 - 20 = 705 \text{ ft.}, \quad AH = 6600 \text{ ft.}$$
$$\text{and AB or } OE = 6600 + 500 = 7100 \text{ ft.}$$
$$\text{Now, } \tan COE = \frac{CE}{OE} = \frac{705}{7100}$$

705	...	...	log 2·848189
7100	...	...	log 3·851258
COE = 5° 40′	...	...	tan 8·996931
+ Dip 10			

∠ COH 5 50 the danger angle required.

To find the Width of a River or Channel.—Let C be a prominent object on the opposite bank, and A and B two stations whose distance apart is known, B being chosen so that the ∠ ABC is a right angle. The horizontal angle at A, subtended by B and C, can be measured by means of a sextant or prismatic compass.

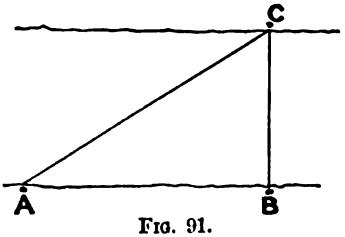


FIG. 91.

$$\text{Then } \frac{CB}{AB} = \tan A$$
$$\text{and } CB = AB \times \tan A$$

Example.—Given the distance AB = 600 yards, and the angle CAB = 42° 30′, required the width (CB) of the river.

A	42° 30′	...	tan 9·962052
AB	600	...	log 2·778151
BC =	549·8	...	log 2·740203

therefore the width of the river is 550 yards nearly.

MISCELLANEOUS PROBLEMS.

1. Two ports in lats. 38° S. and 50° S. are on the same meridian; a vessel leaves the former, and, after sailing due W. 300 miles, steers due S. until the second parallel is reached. How far is she then distant from the second port?

2. Two places, A and B, are in the same latitude, and in longs. 10° W. and 20° 20′ W. respectively; a ship, sailing 10 knots, took exactly 48 hours to pass from one to the other. Find the latitude.

3. A vessel sailed from lat. 35° 20′ N., long. 30° W., 250 miles due N., 250 miles due W., 250 miles due S., and 250 miles due E. Required her latitude and longitude in.

4. A ship which sails 8 knots per hour, within 6 points of the wind, has to reach a port 120 miles S.S.W. of her, wind S.W. How long will she take?

5. A ship, after steaming S.E. by E. 12 knots per hour for 23 hours, from a position lat. 25° 36′ S., long. 62° 35′ E., found herself in lat. 27° 25′ S., long. 66° 45′ E. Find set and drift of current.

6. A ship has to reach a port 30 miles N.N.W. of her; the wind is N. by E., and she can sail 7 knots within 6 points of the wind, making ¼ point leeway; she starts on the starboard tack, wishing to make her port in two tacks. After what interval must she go about?

7. An observer in a boat finds that the angle of elevation of the top of a lighthouse standing upon a cliff is 32°; after the boat has been rowed 100 yards directly towards the lighthouse, the elevation of the same point is 71°. Find the distance of the boat from the foot of the cliff at both observations; and, if at both points of observation the angle subtended by the lighthouse is found to have the same value, find the heights of cliff and lighthouse.

8. The upper part of a mast is 30 feet long, and subtends, at a point 150 feet in a horizontal line from the base, the same angle as that subtended at the same point by a man 6 feet high standing at the base. Find height of the mast.

9. A steamer is being steered S.W. ½ W., at the rate of 12 miles per hour, in a known current setting W.N.W. 2½ miles per hour. Required the course and distance made good per hour.

10. Given the course and distance to a port, N. 75° W. 196 miles; a steady current is setting S. 15° W. 1·9 mile per hour, and the vessel steams 10 knots. What course should be steered to counteract the current, and what progress is made towards the port in 12 hours?

11. A ship's position is, by dead reckoning, lat. 48° 21′ S., long. 43° 28′ W.; and by observation, lat. 47° 16′, long. 41° 3′ W. Assuming the discrepancy to be due to a current, what was its set and drift during the time elapsed since the previous observations?

12. The true course to destination is N. 33° W., and distance 75 miles; but after steaming 5 hours on that course at 14 miles per hour, it was found to bear N. by W. (true), and distant by cross-bearings 10 miles. What was the effect of the current?

13. How long should it take a steamer to reach her destination, bearing W.N.W. 85 miles distant, if steering the proper course to counteract the effect of a steady current setting S. 20° W. at the rate of 2¾ miles per hour, the ship's rate of steaming being 13 knots?

14. A departure is taken at 6 P.M., from a position determined by cross-bearings in lat. 51° 18′ N., long. 9° 42′ W.; her course is set W. ½ S., and after steaming at the rate of 14 knots per hour until noon the following day, the position is found by observation to be lat. 51° 5′ N., long. 16° 20′ W. Required the set and drift of the current.

15. From a place A, two ships start at same time steaming 10 knots and 8 knots respectively due E. After two hours the two ships bear S. 46° W. and S. 66° W. from a place B on the same parallel as and 30 miles E. of A. Find set and drift of current.

16. From a ship at anchor a torpedo boat known to be steaming at 15 knots was observed, at 9 A.M., to bear N. 20° W., at $9^h\ 10^m$ she bore N. 5° E., and at $9^h\ 30^m$ N. 35° E. Determine course steamed by T. B. and her distance from ship in her last position.

CHAPTER VII.

TIDES.

ART. 29.—Observation reveals the fact that the waters around our coast are not at the same height at all hours of the day. At times great tracts are uncovered which at other times have several feet of water over them. This is a consequence of what are known as the "Tides," and is due to the attraction by the moon and sun of the waters of the ocean, combined with the rotation of the earth on its axis. The changes in the distances and relative positions of these bodies account for the general variations in the character of the tides, but local influences modify their action, and make the determination of high and low water at any place a matter of extreme difficulty. The rise and fall occurs twice in about $24\frac{3}{4}$ hours, which is the mean value of the lunar day, and tends to show that the moon's influence plays the most important part, being more than twice that of the sun's.

Let the figure represent the earth surrounded by an envelope of water, shown by the dotted circle.

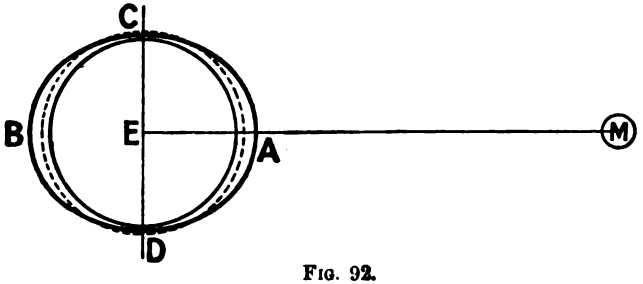


FIG. 92.

Under the action of the moon M, the waters assume an elliptical form, being piled up on the sides nearest to and furthest from the moon, and correspondingly reduced on the sides at right angles to this direction, thus causing the waters to be higher at A and B, in the direction of the line of centres EM; and lower

at C and D at right angles to this direction. As the earth rotates from west to east, this crest moves from east to west. The action of the sun is similar to the moon, and causes two solar tides in a day.

When the centres of the sun and moon are in a line, that is, at new moon, the bodies being in conjunction, or at full moon, when they are said to be in opposition, the solar and lunar effects correspond, and we have the highest or *Spring* tides.

When the bodies are 90° apart or in quadrature, the solar low water corresponds with the lunar high water, and the lowest or *Neap* tides occur.

As the moon moves eastward faster than the sun, the combined effect will be to produce a crest to the westward of the moon during the first and third quarters, and thus the time of high water will be hastened; also an opposite effect during the second and fourth quarters, when the time of high water will be retarded. This is called the *priming* and *lagging* of the tides.

The middle between the levels of high water and low water at Springs is called the *mean level* of the sea (M.T.L.).

The *datum line* is the level of low water of ordinary Springs, (M.L.W.S.) and the height of a tide is measured above this line unless it is stated otherwise in the Admiralty Tide Tables, where full information is given at the foot of the page for each port showing what the *zero* of predictions is.

The interval which elapses between the moon's transit each day and the time of H.W. next following is the *Lunitidal Interval;* the mean of the lunitidal intervals for all days between new and full moon is the *Corrected establishment of the Port;* the lunitidal interval on the days of new and full moon is the *Vulgar establishment,* and is registered in Roman figures on Admiralty charts as H.W.F. and C.

ART. 31.—In the Admiralty Tide Tables are given the times of High and Low Water and Heights above the zero of predictions selected for the various ports, around the British Isles and at other places chosen through the world, called "standard" ports. The Tables are in two parts, at the end of Part I. is an index of all standard ports, and of ports referred to a particular standard, and full information is tabulated for each day of the month throughout the year on the pages mentioned. At the end of Part II. is a general index, and on referring to the proper pages the lat., long., H.W.F. and C., and rise at Springs and Neaps are found for each place. Part II. is used when insufficient, or no information is given in Part I. for the required place.

The tides in Part I. are entered in order of occurrence, P.M. times and heights in *block* type.

The times given are STANDARD or mean time of certain meridians. Local and Greenwich times may be obtained by applying D. Long. in time. The "time differences" show how much earlier (if marked −) later (if marked +) high and low water occur at the required port than at the standard.

When no time difference is entered, or place not found in Part I., turn up index Part II. and on the page or pages will be found the Greenwich times of H.W.F. and C. at the required port and at the selected standard. The difference between these is the time difference, and is added (+) when full and change at place is greater than that at standard, subtracted (−) when less.

The port to be selected as standard is the nearest one geographically to the ship.

REMARK.—When the difference is +, and the sum amounts to more than 12^h, reject 12^h and change from A.M. to P.M. on the same day, or from P.M. to A.M. on the day after: it may be necessary to take the P.M. time of tide for the day before, in order to obtain the A.M. time of tide on the required day.

When the difference is −, and the time entered in the Tables is less than the difference, borrow 12^h, and change from P.M. to A.M. on the same day, or from A.M. on the day after to P.M. on the required day.

Examples.—

1. Find the M.T.G. and M.T.S. of H.W. on September 20th off Pembroke Dock.

M.T.G. H.W. Pemb. Dock	No A.M.	NOON.
Long. − 20^m		− 20^m
∴ M.T.S. H.W. Pemb. Dock		$11^h\ 40^m$ A.M., No P.M.

2. Find the M.T.G. and M.T.S. of H.W. on August 22nd off Llanelly.

M.T.G. H.W. Pemb. Dk.	$0^h\ 1^m$ A.M.	$0^h\ 36^m$ P.M.
Diff.	+ 1	+ 1
M.T.G. H.W. Llanelly	0 2 A.M.	0 37 P.M.
Long. W.	− 17	− 17
M.T.S. H.W. Llanelly	11 45 P.M.	0 20 P.M.
	∴ No A.M.	

3. Find the M.T.G. of H.W. on July 22nd off Limerick.

Dublin T. H.W. Galway	$10^h\ 1^m$ A.M.	$10^h\ 30^m$ P.M.
Diff.	+ 1 34	+ 1 34
Dublin T. H.W. Limerick	11 35 A.M.	0 4 A.M. on 23rd.
Long. W.	+ 25	+ 25
M.T.G. H.W. Limerick	0 0	0 29 A.M. on 23rd.
	∴ Midnt.	No P.M.

4. Find the M.T.G. of H.W. on Oct. 21st off St. Mary's.

M.T.G. H.W. Devonport	0h 31m A.M.	1h 9m P.M.
Diff.	− 1 17	− 1 17
M.T.G. H.W. St. Mary's	11 14 P.M.	11 52 A.M.
	on 20th.	No P.M.

5. Find the M.T.G. and M.T.S. of H.W. on Sept. 21st off Lizard Point.

H.W. F. & C. Devonport	6h 0m
,, Lizard Point	5 21
Diff.	− 0 39

M.T.G. H.W. Devonport	0h 30m P.M.	
Diff.	− 0 39	
M.T.G. H.W. Lizard Point	11 51 A.M.	No P.M.
Long. W.	− 0 21	
M.T.S. Lizard Point	11 30 A.M.	No P.M.

Exercises.

1. Find the M.T.G. of high water—
   at Bridlington on Nov. 29th.
   ,, Spurn Point on Dec 18th.
   ,, Dingle on Aug. 10th.
   ,, Hilbre Island on Oct. 13th.
   ,, Clydebank on Sept. 10th.

2. Find the M.T.S. of high water—
   at Rotterdam on Jan. 19th and 20th.
   ,, Ayr on Nov. 9th and 10th.
   ,, Crinan on Dec. 21st and 22nd
   ,, Port Carlisle on Sept. 28th and 29th.
   ,, Wicklow on Sept. 30th and Oct. 1st.

3. Find the M.T.G. and M.T.S. of high water—
   at Quaco on July 8th.
   ,, Strangford Quay on Oct. 29th.

4. Find the M.T.G. of high water—
   at Ardrishaig on Nov. 9th
   H.W. F. and C. at Greenock 12h 24m.
   ,, ,, Ardrishaig 12h 15m.
   Also at Southampton on Jan. 22nd.
   H.W. F. and C. at Portsmouth Dockyard 11h 45m.
   ,, ,, Southampton { 0h 39m / 11 2
   Times H.W. at Portsmouth 0h 44m A.M. 1h 1m P.M.

CHAPTER VIII.

REDUCTION OF SOUNDINGS.

ART. 32.—Near to land in foggy weather, the greatest security for the navigator is the frequent use of the "lead," and feeling his way by "ground navigation." A line of soundings and specimens of the bottom (brought up by the "arming" of the lead) will generally enable him to fix his position on the chart, or give him due warning of danger.

In Board of Trade inquiries into cases of stranding, the first importance is properly attached to the taking of soundings, and woe to the captain who may be found to have neglected this precaution.

The soundings marked on charts are from the Mean Low-water Level of spring tides, but the actual depth at a place varies with Springs and Neaps and the interval from high water. The difference between the sea-level at the "time of cast" and the Mean Low water of Springs is the *Reduction of Soundings*.

The Heights of Tides in the Admiralty Tables are measured from the same zero or datum line as used for the chart soundings, but some local tables, *e.g.* the Liverpool or Holden's Tables, give the heights from a datum line five feet below the Mean Sea-level.

In the diagram, let O (zero) be the mean level of low water spring tides, H the mean high water springs, and M the mean level of the sea or half-tide level; also let h_1, h_2, h_3, h_4 be the high-water marks of various tides, and l_1, l_2, l_3, l_4 the corresponding low-water marks, and B the sea-bottom.

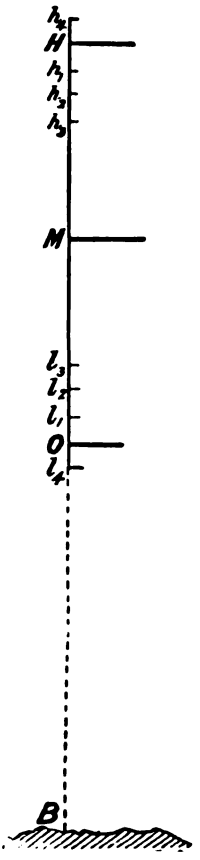


FIG. 93.

Now, if the lead were cast when the sea-level is at O, it would give the same sounding as the chart, but at other times there would be a difference depending on the height of the tide in the tables and the interval from high water. This difference is the Reduction of Soundings.

The Half Range of any tide is found by subtracting the half mean spring range (H.M.S.R.) from the height, *e.g.* $Mh_1 = Oh_1 - O.M.$

When the half-range, the time from high water and the time occupied in rising or falling are known, reference to the Tidal Diagrams No. I. to V. in the Admiralty Tables, *or* a diagram drawn by the student himself, gives the difference of level between M and the actual surface of the sea, which, being added to or taken from the H.M.S.R., gives the height above mean low water, or *zero*.

ART. 33.—In calculating a correction to a cast or reduction of soundings we proceed in the following order. *Firstly.*—Obtain the interval between the time of cast and the time of H.W.: from this derive the angle of swing. *Secondly.*—Find the Mean Tide Level (M.T.L.) and Height of Tide at ship: their difference gives the half tidal range, or half diurnal range (H.D.R.). *Thirdly.*—With the angle and H.D.R. take either from the traverse table in the D.Lat. column using angle as Co., and H.D.R. as dist., or from the diagram drawn, distance ship is from M.T.L. *Fourthly.*—Apply this distance to the M.T.L. for the distance the ship is from datum, which gives Reduction.

To find Interval and Angle.—The Interval is the elapsed time between the times of H.W. at place, and of cast: both times being reckoned from the same meridian. The time any tide takes to Rise or Fall is given at foot of column for those ports where H.W. only is predicted; at ports for which both H.W. and L.W. are predicted it is found by subtracting *preceding* time of L.W. from H.W. used, for *time to Rise:* and by subtracting time of H.W. used from time of *succeeding* L.W., for *time to Fall.*

Interval must always be *less* than *time* of *Rise* or *Fall* used.

Times in Tide Table are *not* necessarily *G.M.T.*

Since from H. to L.W. tide sweeps through 180°,

$$\text{Angle} = \frac{\text{Interval in minutes} \times 180°}{\text{Time of Rise or Fall in minutes}}$$

Examples.—Find the interval and angle:—

1. On August 29th at $9^h\ 14^m$ A.M. M.T.S. off Tenby.

M.T.G. Pembroke Dock	29th	$6^h\ 46^m$	A.M.
Time Diff.		$-0\ 35$	
M.T.G. H.W. Tenby	29th	6 11	A.M.
Tenby Long. W.		$-0\ 19$	
M.T.S. H.W. Tenby	29th	5 52	A.M.
,, cast	29th	9 14	A.M.
Interval		3 22 after H.W.	∴ Falling.

Time to fall $6^h\ 15^m$;

$$\text{Hence angle} = \frac{202 \times 180}{375} = 97°$$

2. On December 16th at $9^h\ 40^m$ A.M. M.T.G. off Lamlash.

M.T.G. H.W. Greenock,	Dec. 16th	$4^h\ 27^m$	P.M.
	Time Diff.	$-0\ 15$	
,, Lamlash,	Dec. 16th	4 12	P.M.
,, cast	Dec. 16th	9 40	A.M.
	Interval	6 32	before H.W. ∴ Rising.
	H.W. Greenock	$4^h\ 27^m$	P.M.
	preceding L.W. ,,	9 47	A.M.
	Time to rise	6 40	

$$\text{Angle} = \frac{392 \times 180}{400} = 175°$$

N.B.—Since only Time Diff. for H.W. is given, time of rise or fall at ship is taken to be the same as at standard port.

3. On July 23rd at $8^h\ 15^m$ P.M. M.T.S. off Welchpool, N.B.

Stand. time H.W. St. John, July 23rd			$6^h\ 38^m$	P.M.	Following L.W.	$1^h\ 6^m$	A.M.
	Time Diff.		$+0\ 2$		Time Diff.	0 10	
,, ,,	Welchpool	...	6 40	P.M.	Welchp'l L.W.	1 16	A.M.
	D. Long. W.		$-0\ 28$		,, H.W.	6 40	P.M.
M.T.S. ,,	Welchpool	...	6 12	P.M.	To fall	6 36	
	Cast	... ...	8 15	P.M.			
	Interval		2 3				

$$\text{Angle} = \frac{123 \times 180}{396} = 56° \text{ after H.W.} \quad \therefore \text{Falling.}$$

N.B. (1) Since Time Differences for both H. and L.W. are given, they must both be used to find time of fall at ship.

N.B. (2) Should *no* Time Difference be given, find its amount from H.W. F. and Cs. entered in Part II., and use 6 hours as time of rise or fall.

EXERCISES.

Find the Interval and Angle :—

1. On Dec. 24th at $11^h\ 30^m$ P.M. M.T.S. off Grimsby.
2. ,, Nov. 5th at $3^h\ 10^m$ P.M. M.T.S. off Spurn Point.
3. ,, Oct. 25th at $10^h\ 15^m$ P.M. M.T.G. off Bridlington.
4. ,, Sept. 26th at $1^h\ 10^m$ P.M. M.T.S. off Donaghadee.
5. ,, Feb. 19th at $4^h\ 15^m$ A.M. M.T.S. off Beachy Head.
6. ,, Jan. 22nd at $8^h\ 15^m$ P.M. M.T.S. off Ramsgate.

To find the Mean Tide Level and Half Daily Range.—(*a*) The height of tides above Datum is entered for all standard ports at H.W., and at most ports for L.W. also. When H.W. heights only are predicted, the M.T.L. is given at the foot of the column. When both H.W. and L.W. heights are predicted the

M.T.L. is half the sum of these heights, unless the L.W. height is * signifying below Datum, when half the difference must be taken. To find the H.D.R. subtract the M.T.L. from the height of tide.

Thus on Dec. 2nd, A.M., off Hull.

	ft.	in.
Ht. H.W. ...	17	11
M.T.L. ...	10	5
H.D.R. ...	7	6

On Oct. 7th, A.M., off Liverpool, rise and fall.

	ft.	in.		ft.	in.		ft.	in.		ft.	in.
Ht. H.W.	23	8	H.W.	23	8	H.W.	23	8	H.W.	23	8
preceding L.W.	6	7	M.T.L.	15	1½	following L.W.	7	3	M.T.L.	15	5½
2)30	3		H.D.R.	8	6½	2)30	11		H.D.R.	8	2½
M.T.L. rise	15	1½				M.T.L. fall	15	5½			

On Sept. 13th, P.M., off Liverpool, rise and fall.

Rise.						*Fall.*					
	ft.	in.		ft.	in.		ft.	in.		ft.	in.
Ht. H.W.	30	1	H.W.	30	1	H.W.	30	1	H.W.	30	1
preceding L.W.	*0	6	M.T.L.	14	9½	following L.W.	0	4	M.T.L.	15	2½
2)29	7		H.D.R.	15	3½	2)30	5		H.D.R.	14	10½
M.T.L.	14	9½				M.T.L.	15	2½			

(*b*) The height of tides for places referred to standard ports is found by applying the height differences according to sign to the heights at standard. These height differences are tabulated for Springs and Neaps, and, strictly speaking, are only correct for those days, which may be determined by reference to the Tables, when the days on which the highest H.W. occur are Springs, and those on which the lowest H.W. are entered are Neaps. These days may be on the days of full or new moon, or of quadrature, or one or two days after these phenomena. Astronomical data are entered at the beginning of the Tables. An interval of seven days may be allowed between Springs and Neaps, and so the correct height difference may be determined for any intermediate day by allowing $\frac{1}{7}$th of the difference between the Spring and Neap difference for each day elapsed since last Spring or Neap. To find the M.T.L. for places referred to a standard at which only H.W. is predicted, OR to a standard at which both H.W. and L.W. are predicted, but only the height differences for H.W. are given, apply $\frac{1}{2}$ SPRING Diff. to the M.T.L. at standard.

For places at which the Difference are given for both H.W. and L.W., find the corrected Height Difference and thence the heights of H. and L.W. at the place, from which the M.T.L. for the required port may be directly obtained as for a standard.

Examples.—Find the M.T.L. and H.D.R. off Spurn Point on Dec. 13th, P.M.

On looking down the Tables, the days of Springs are seen to be on the 11th and 27th, our date is two days from Springs.

	ft.	in.
Spring Diff.	−2	2
Neap ,,	−1	7
Diff. for 7 days	−0	7
,, 2 ,,	−0	2
Ht. Diff. for date ∴	2	0

	ft.	in.
M.T.L. Hull	10	5
½ Spring Diff.	−1	1
M.T.L. Spurn	9	4

7
2
7)14
2

	ft.	in.
Ht. Hull	19	1
,, Diff.	−2	0
,, Spurn	17	1
M.T.L. ,,	9	4
H.D.R. ,,	7	9

Off Dungeness on Feb. 9th, A.M., falling tide. Springs is on the 6th, hence three days from.

	ft.	in.
S. Diff.	+3	7
N. ,,	+4	8
Diff. for 7 days	+1	1
,, 3 ,,	+0	6
Ht. diff. for date	+4	1

13
3
7)39
6

	ft.	in.
H.W. Ht. Dover	18	1
L.W. ,,	0	4
	2)18	5
M.T.L. Dover	9	2½
½ S. Diff. Dungeness	+1	9½
M.T.L. ,,	11	0

	ft.	in.
	18	1
Diff.	+4	1
Ht. Dungeness	22	2
M.T.L. ,,	11	0
H.D.R. ,,	11	2

N.B.—Always use ½ SPRING Diff. to find M.T.L. at ship.

Off Ramsgate, January 6th, A.M., rising. Our date two days to springs.

	ft.	in.
Springs H.W. Diff.	−3	1
Neaps ,,	−1	6
Diff. for 7 days	−1	7
,, 2 ,,	−0	5
∴ Ht. Diff. H.W.	−2	8
Ht. H.W. Dover	18	9
Diff.	−2	8
Ht. H.W. Ramsgate	16	1
M.T.L. ,,	7	10
H.D.R. ,,	8	3

	ft.	in.
L.W. Diff.	+0	2
,,	−1	4
	−1	6
	−0	5
L.W.	−0	3
Ht. L.W.	*0	2
Diff.	−0	3
Ht. L.W.	*0	5
,, H.W.	16	1
	2)15	8
M.T.L. Ramsgate	7	10

EXERCISES.

Find the M.T.L. and H.D.R.—

1. On August 22nd, A.M., off Mellon.
2. „ Sept. 12th, P.M., off Valentia.
3. „ July 20th, A.M., off Foynes Island.
4. „ Feb. 14th, P.M., rise and fall off Rye Bay.
5. „ Jan. 8th, A.M., „ „ Hastings.
6. „ August 17th, A.M., „ „ Seal Cove.
7. „ Oct. 22nd, P.M., „ „ Warren Point.
8. „ Sept. 11th, A.M., rise off Annan Foot.

To illustrate the working of the whole problem several examples will now be worked in full.

Example 1.—Find the Reduction of Soundings off Spurn Point on November 18th at $8^h\ 38^m$, P.M., M.T.G.

M.T.G. of H.W. Hull	0^h	3^m A.M.
Time Diff.	-1	4
„ H.W. Spurn	10	59 P.M.
„ Cast	8	38 A.M.
Interval	2	21 before, ∴ rising. Takes 6 hrs. to rise.

$$\text{Hence angle} = \frac{141 \times 180}{360} = 70\tfrac{1}{2}^\circ$$

	ft.	in.		ft.	in.
Ht. H.W. Hull	16	0	M.T.L. Hull	10	5
Neap Diff.	-1	7	½ S. Diff.	-1	1
Ht. H.W. Spurn	14	5	M.T.L. Spurn	9	4
M.T.L. „	9	4			
H.D.R. „	5	1			

Now, see Fig. 94, draw a vertical line, and a horizontal line for the M.T.L. as OA; from O measure down 9′ 4″ to scale for zero, and measure up 5′ 1″ for H.W.; with centre O, radius = H.D.R. draw semicircle. At O, draw OB making 70½° with vertical cutting semicircle in a point through which draw a horizontal line for "level of tide at cast," meeting vertical in V.

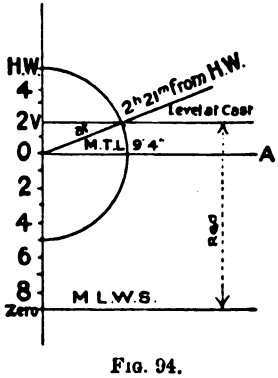


FIG. 94.

Measure OV = 1′ 10″, this is distance of ship at time of cast above M.T.L.

N.B.—Without the aid of a protractor, OV may be found in the D. lat. column of the Traverse Table, using angle 70½ as course, and H.D.R. 5′ 1″ = 61″ as distance, this gives 22″.

M.T.L.	9′	4″
OV +	1	10
Reduction to zero	11	2 to be subtracted from cast.

Since the zero at Hull is the level of mean low water springs, this is also the reduction to M.L.W.S.

Example 2.—Find the correction to cast on July 14th at $8^h\ 5^m$, P.M., M.T.S. off Dingle.

Dub. time H.W. Galway	$4^h\ 3^m$ P.M.	
Diff.	−39	
,, ,, Dingle	3 24 P.M.	
D. Long. W.	−16	
M.T.S. time H.W. Dingle	3 8 P.M.	
M.T.S. Cast	8 5 P.M.	
Interval	4 57 (falling)	

Takes $5^h\ 55^m$ to fall

$$\therefore \text{ angle} = \frac{297 \times 180}{355} = 151°.$$

	ft.	ins.
Ht. H.W. Galway	15	2
Diff.	−4	4
,, ,, Dingle	10	10
,, M.T.L. ,,	5	9
H.D.R. ,,	5	1

Now draw figure (Fig. 95).

	ft.	ins.
M.T.L.	5	9
OV	−4	4
Reduction to zero	1	5 to be subtracted.

	ft.	ins.
M.T.L. Galway	8	2
½ S. Diff.	−2	5
M.T.L. Dingle	5	9

3 days to Springs.

	ft.	ins.
Sp. Diff.	−4	9
Np. ,,	−3	10
In 7 days	−	11
,, 3 ,,	−	5
Ht. Diff.	−4	4

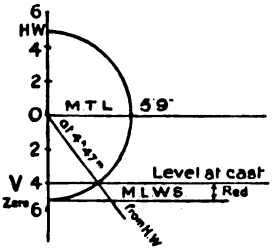


FIG. 95.

Since the zero is ·75 foot = 9 inches below the level of M.L.W.S. the Reduction to M.L.W.S. is 8″ to be subtracted from cast.

Example 3.—Find the Reduction on Sept. 27th at $4^h\ 10^m$, P.M., M.T.S. off Wicklow.

Stand. time H.W. Kingstown	$11^h\ 4^m$ P.M.
Diff.	− 43
,, ,, Wicklow	10 21 P.M.
D. Long. E.	+ 1
M.T.S. Wicklow	10 22 P.M.
M.T.S. Cast	4 10 P.M.
Interval	6 12 (rising)

To rise takes $6^h\ 25^m$.

$$\text{Angle} = \frac{372 \times 180}{385} = 174°.$$

	ft.	ins.
Ht. L.W. Kingstown	1	0
,, H.W. ,,	11	9
	2)12	9
M.T.L. ,,	6	4½
½ S. Diff.	−1	4½
M.T.L. Wicklow	5	0

	ft.	ins.
H.W. Kingstown	11	9
Diff.	− 2	9
,, Wicklow	9	0
M.T.L. ,,	5	0
H.D.R. ,,	4	0

Now draw Fig. 96.

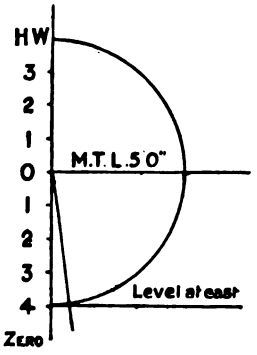


FIG. 96.

M.T.L.	5 0
O.V.	4 0
Reduction to zero	1 0 to be subtracted

Since zero is ·84 foot = 10 inches below the level of M.L.W.S.

Reduction to that level is 2″ to be subtracted from cast.

Example 4.—Find the Reduction on Sept. 12th at 5ʰ, A.M., M.T.S. off Donaghadee.

Stand. time	H.W. Kingstown	11ʰ 1ᵐ A.M.
	Diff.	− 2
,, ,,	Donaghadee	10 59 A.M.
	D. Long. E.	+ 3
M.T.S. ,, ,,		11 2 A.M.
	Cast	5 0
	Interval	6 2 rising.

To rise takes 6ʰ 29ᵐ

$$\therefore \text{angle} = \frac{362 \times 180}{389} = 168°$$

	ft.	ins.
Ht. L.W. Kingstown	0	6
Diff.	−0	10
Ht. L.W. Donaghadee	−0	4
,, H.W. ,,	12	10
	2)12	6
M.T.L. ,,	6	3

	ft.	ins.
H.W.	12	4
	+	6
H.W.	12	10
M.T.L.	6	3
H.D.R.	6	7

Now draw Fig. 97.

	ft.	ins.
M.T.L.	6	3
OV.	6	5
Red. to zero		2 to add to cast

Since zero is ·84 foot = 10 inches below the level of M.L.W.S.
Reduction to M.L.W.S. is 1 foot to be added to the cast.

This problem may also be utilised to find the depth of water to be found at any time at a given spot on a chart, by adding the Reduction found to the depth marked on the chart; if from this the ship's draught is subtracted,

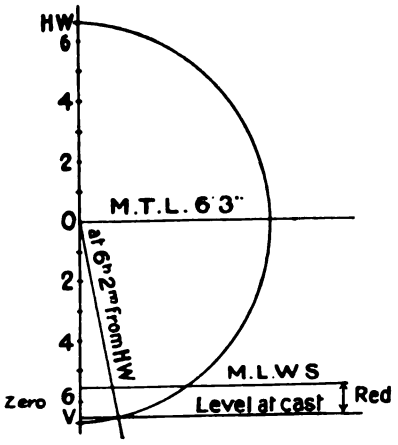


FIG. 97.

the depth of water under the ship's bottom may be found; or, again, the time may be found when a vessel of given draught will find sufficient water to pass an obstruction.

EXERCISES.

Find the correction to be applied to the cast, in order to reduce the soundings to datum and M.L.W.S. :—

1. On Oct. 14th, at 5h 56m P.M., M.T.G. off Hull.
2. „ July 24th, at 4h 30m A.M., M.T.S. off Galway.
3. „ Nov. 1st, at 1h 20m A.M., M.T G. off Grimsby.
4. „ Aug. 15th, at 1h 10m A.M., M.T.S. off Limerick.
5. „ Sept. 20th, at 6h 20m P.M., M.T.G. off Tenby.
6. „ Dec. 18th, at 10h P.M., M.T.G. off Spurn Point.
7. „ Aug. 6th, at 2h 10m A.M., M.T.S. off St. John (N.B.).
8. „ Nov. 3rd, at 7h 40m A.M., M.T.G. off Abu Shahr.
9. „ Sept. 28th, at 4h 45m A.M., M.T.G. off Fowey.
10. „ Oct. 19th, at 5h 20m P.M., M.T.S. off Exmouth.
11. „ Oct. 23rd, at 7h 10m P.M., M.T.S. off St. Mary's.
12. „ Oct. 7th, at 2h 40m P.M., M.T.G. off Beaumaris.
13. „ Sept. 13th, at 6h 20m A.M., M.T.S. off Workington.
14. „ Oct. 12th, at 0h 15m P.M., M.T.S. off Dartmouth.
15. „ Sept. 13th, at 5h 10m A.M., M.T.S. off Whitehaven.
16. „ Nov. 20th, at 5h 15m A.M., M.T.S. off Henjam Island.
17. „ July 15th at 10h 16m P.M., M.T.G. off St. Andrews (N.B.).
18. „ a Spring-tide day 1½ hours from H.W. off Al Basra.
19. „ a Neap-tide day 2 hours from L.W. off Kuweit.
20. The depth of water on a bar at the entrance to a harbour at mean low water springs is 10 ft. 6 in. If the time of high water is 10h 51m A.M., find the depth of water on the bar at 9h A.M., the height of tide for day being 20 ft. 3 in. Spring range 22ft.

21. On a certain day it is high water at Falmouth at $7^h\ 20^m$ P.M. Height of tide 15 ft. 6 in.; mean tide level 8 ft. What is the earliest time in the afternoon when there will be not less than 20 feet of water over a shoal marked 2 fathoms on the chart?

22. At St. Helier, Jersey, the M.T.L. is 15 ft. 9 in., and a certain tide rises 25 ft. 6 in. Find the height of the tide $1^h\ 30^m$ before high water.

23. On Deal Bank the depth marked on the chart is 18 ft. What depth of water would there be over it on Feb. 6th at $1^h\ 30^m$ A.M. M.T.G.? If a vessel's draught was 25 ft., what depth of water would she have under her?

24. A vessel whose draught is 23 ft. 6 in. wishes to berth at inner harbour where average depth on chart is 2 fathoms. Heights of tide at high and low water on the day are 14′ 10″ and 1′ 10″; time of H.W. $9^h\ 56^m$ A.M., previous time L.W. $3^h\ 43^m$ A.M. What is the earliest time on the day when there will be sufficient water?

CHAPTER IX.

TIME.

ART. 34. **Time and Arc.**—It is often necessary to change Time ($^{h\ m\ s}$) into Arc ($^{\circ}\ '\ ''$), or the contrary.

Twenty-four hours are the equivalent of 360°;

therefore	1^h	is equivalent to	15°
	1^m	„ „	15′
	1^s	„ „	15″
also	4^m	are „ „	1°
	4^s	„ „	1′

Two rules for converting Arc into Time are deduced from the above, viz.—

(1) Divide the $^{\circ}\ '\ ''$ by 15.
(2) Multiply „ „ by 4, and divide by 60.

NOTE.—The latter is generally preferred, because of the easier division.

Example.—Convert 34° 16′ 30″ into Time.

3)34° 16′ 30″		34° 16′ 30″
		4
5)11 25 30	or	6,0)13,7 6 0
$2^h\ 17^m\ 6^s$		$2^h\ 17^m\ 6^s$

EXERCISES.

Find the equivalent values in *Time* of the following *Arcs*:—
2° 59′ 30″; 179° 0′ 45″; 15° 0′ 15″; 67° 52′ 55″; 0° 17′ 20″; 10° 19′ 40″; 155° 11′ 36″; 93° 0′ 45″; 117° 38′ 30″; 144° 54′; 176° 39′ 35″; 3° 23′; 0° 59′ 45″; 0° 0′ 15″; 0° 15′; 0° 48′; 13°; 168° 9′; 61° 48′ 45″.

By reversing the above rules, Time may be changed into Arc, viz.—

(1) Multiply by 15.
(2) Multiply by 60 and divide by 4.

Example.—Change $7^h\ 32^m\ 15^s$ into arc.

$7^h\ 32^m\ 15^s$		$7^h\ 32^m\ 15^s$
3		60
22 36 45	or	4)452 15 0
5		
113° 3′ 45″		113° 3′ 45″

EXERCISES.

Change into *Arc* the following *Time intervals*:—

$1^h\ 12^m\ 30^s$; $10^h\ 19^m\ 55^s$; $8^h\ 0^m\ 10^s$; $6^h\ 59^m\ 18^s$; $11^h\ 59^m\ 58^s$; $3^h\ 0^m\ 19^s$; $5^h\ 59^m\ 22^s$; $9^h\ 1^m\ 37^s$; $0^h\ 15^m\ 15^s$; $2^h\ 25^m$; $0^h\ 10^m\ 35^s$; $0^h\ 0^m\ 45^s$; $2^h\ 24^m\ 44^s$; $11^h\ 0^m\ 19^s$; $11^h\ 13^m\ 30^s$; $5^h\ 0^m\ 5^s$; $8^h\ 28^m\ 52^s$; $10^h\ 0^m\ 12^s$; $0^h\ 0^m\ 12^s$; $0^h\ 1^m\ 40^s$.

ART. 35. **Time and Longitude.**—The rules for Time and Arc may be employed to find—

(1) The difference between Greenwich Time and the Time at a place whose Longitude is given; and

(2) The Longitude corresponding to a given difference of Time.

But it must be borne in mind that, as the Earth rotates from west to east, places in E. longitude have sunlight earlier than Greenwich, and places in W. longitude later. Therefore the Time at a place in E. longitude is always *fast* of or *greater* than the Time at Greenwich, and at a place in W. longitude it is always *slow* of or *less* than Greenwich Time.

ART. 36.—**Difference of Time due to Longitude.**

Examples.—

1. What is the difference between the time at a place in 72° 50′ E. long. and the time at Greenwich?

3)72° 50′		72° 50′
5)24 16 40	or	4
$4^h\ 51^m\ 20^s$		6,0)29,1 20
		$4^h\ 51^m\ 20^s$

Ans. The time at place is $4^h\ 51^m\ 20^s$ fast of Greenwich time.

2. What is the difference between Greenwich time and the time at a place in 124° 10′ 45″ W. long.?

3)124° 10′ 45″		124° 10′ 45″
5)41 23 35	or	4
$8^h\ 16^m\ 43^s$		6,0)49,6 43 0
		$8^h\ 16^m\ 43^s$

Ans. The time at place is $8^h\ 16^m\ 43^s$ *behind*, or slow, of Greenwich time.

EXERCISES.

Find the difference between time at Greenwich and corresponding time in the following longitudes, and say which is the *greater* :—

10° 20′ W. ; 17° 45′ E. ; 110° 17′ E. ; 172° 36′ W. ; 100° 16′ 30″ E. ; 90° 4′ 45″ W. ; 0° 7′ 15″ E. ; 179° 56′ 45″ W. ; 1° 2′ 15″ W. ; 115° 0′ 30″ E. ; 0° 2′ 45″ E. ; 165° 0′ 15″ W.

ART. 37.—Longitude from Difference of Time.

Examples.—

1. The mean time at ship is January $5^d\ 2^h\ 14^m\ 40^s$, when mean time at Greenwich is January $5^d\ 9^h\ 45^m\ 20^s$: required the longitude.

M.T.S. January	5^d	2^h	14^m	40^s				
M.T.G. ,,	5^d	9	45	20				
Difference	7	30	40		or	7^h	30^m	40^s
			3				60	
	22	32	0			4)450	40	0
			5					
						112°	40′	0″
Long. W.	112°	40′	0″					

2. The mean time at ship is February $14^d\ 17^h\ 16^m\ 25^s$ when mean time at Greenwich is February $14^d\ 17^h\ 19^m\ 14^s$: required the longitude.

M.T.S. February	14^d	17^h	16^m	25^s				
M.T.G. ,,	14^d	17	19	14				
Difference	0	2	49		or	0^h	2^m	49^s
			3				60	
	0	8	27			4)2	49	0
			5					
						0°	42′	15′
Long. W.	0°	42′	15″					

3. Given M.T.G. March $30^d\ 2^h\ 19^m\ 55^s$, and M.T.S. March $29^d\ 21^h\ 16^m\ 58^s$: required the longitude.

M.T.G. March	30^d	2^h	19^m	55^s
M.T.S. ,,	29^d	21	16	58
Difference		5	2	57
		60		
	4)302	57	0	
Long. W.	75°	44′	15″	

4. Given M.T.G. December $31^d\ 19^h\ 11^m\ 25^s$, and M.T.S. January $1^d\ 3^h\ 17^m\ 50^s$: required the longitude.

M.T.G. December	31d	19h	11m 25s
M.T.S. January	1d	3	17 50
Difference		8	6 25
		60	
		4)486	25 0
Long. E.		121°	36′ 15″

5. Given M.T.S. July 2d 5h 15m 45s, and M.T.G. July 1d 22h 20m 30s: required the longitude.

M.T.S. July	2d	5h	15m 45s
M.T.G. „	1d	22	20 30
Difference		6	55 15
		60	
		4)415	15 0
Long. E.		103°	48′ 45″

NOTE.—The student may change Time into Longitude by any method he prefers.

EXERCISES.

Find the Longitudes from the following Simultaneous Times at ship and Greenwich :—

No.	M.T.S.				M.T.G.			
			h.	m.	s.		h.	m.	s.
1	1906, January	10d	7	40	25	1906, January 10d	1	14	42
2	„ February	14d	3	22	33	„ February 14d	10	29	54
3	„ March	31d	19	8	11	„ April 1d	2	28	31
4	„ April	20d	3	13	48	„ April 19d	19	3	54
5	„ May	13d	22	46	59	„ May 14d	4	2	14
6	„ June	1d	5	3	13	„ May 31d	21	11	19
7	„ July	23d	11	12	43	„ July 23d	11	10	24
8	„ August	1d	0	36	18	„ July 31d	23	59	48
9	„ September	30d	23	58	45	„ October 1d	0	1	5
10	„ October	29d	11	13	10	„ October 29d	22	49	17
11	„ November	22d	23	31	49	„ November 23d	1	0	28
12	„ December	1d	9	10	12	„ November 30d	22	2	12
13	1905, December	31d	18	50	40	„ January 1d	0	11	50
14	„ December	31d	13	20	25	„ January 1d	1	16	33

ART. 38.—**To find the Time at Greenwich** when the Time at a place or at ship and the Longitude are known.

Rule.—Add the difference of time for West Longitude, and subtract for East.

Examples.—

1. When it is apparent noon at ship on May 10th in long. 96° W., what is the time at Greenwich?

A.T.S. May	10d	0h	0m 0s
		6	24 0
A.T.G. May	10d	6	24 0

96°
4

6,0)38,4

6h 24m

2. It is apparent noon at a place in 175° 40′ E. on July 30th: required the time at Greenwich.

A.T. at place, July	30^d 0^h 0^m 0^s	175° 40′
	11 42 40	4
A.T.G. July	29^d 12 17 20	6,0)70,2 40
		11^h 42^m 40^s

3. The sun is on the Meridian of ship's place in long. 100° 35′ E. on May 15th: required the time at Greenwich.

A.T.S. May	15^d 0^h 0^m 0^s	100° 35′
	6 42 20	4
A.T.G. May	14^d 17 17 40	6,0)40,2 20
Or	15th 5 17 40 A.M.	6^h 42^m 20^s

4. When it is 6^h 50^m P.M. apparent time on February 10th at ship in long. 75° 50′ W., what is the time at Greenwich?

A.T.S. February	10^d 6^h 50^m 0^s	75° 50′
	5 3 20	4
A.T.G. February	10^d 11 53 20	6,0)30,3 20
		5^h 3^m 20^s

5. At a place in long. 115° 56′ E. it is 7^h 30^m A.M. mean time on June 15th: required the Greenwich time.

M.T. at place, June	14^d 19^h 30^m 0^s	115° 56′
	7 43 44	4
M.T.G. June	14^d 11 46 16	6,0)46,3 44
		7^h 43^m 44^s

NOTE.—A.M. Civil time is changed into Astronomical time by adding the interval of 12 hours between noon and midnight, which makes the Astronomical day one less than the Civil day; but when it is P.M. the Astronomical day is the same as the Civil.

Astronomical time is always written thus: February 10^d 6^h 50^m 0^s, or June 14^d 19^h 30^m 0^s being reckoned from noon onwards to 24 hours.

Civil time is written thus: February 10th, 6^h 50^m 0^s P.M., or June 15th, 7^h 30^m 0^s A.M.

EXERCISES.

To find the Astronomical Time at Greenwich.

1906.	Civil time at ship. h. m. s.	Longitude of ship. ° ′ ″
1. May 16th	3 12 30 P.M.	59 16 30 W.
2. May 1st	5 8 10 A.M.	136 22 50 W.
3. June 13th	8 27 35 P.M.	174 14 45 E.
4. June 30th	11 2 49 A.M.	105 31 45 W.
5. June 1st	4 1 50 P.M.	96 17 15 E.
6. February 17th ...	7 43 28 P.M.	74 44 30 W.
7. February 28th ...	10 19 18 A.M.	112 14 45 W.
8. August 21st... ...	6 16 40 A.M.	43 13 30 W.
9. August 1st	2 54 14 P.M.	165 55 0 E.
10. August 10th ...	1 19 55 P.M.	15 50 15 E.
11. October 22nd ...	9 38 12 A.M.	155 0 45 W.
12. October 29th ...	10 56 11 P.M.	82 17 15 W

ART. 39.—**To find the Time at Ship** when the Time at Greenwich and the Longitude of Ship are known.

Rule.—Subtract the difference of time for West Longitude, and add for East, which is simply the reverse of the preceding rule. The reason has been previously explained (Art. 35).

Examples.—

1. Required the Time at ship in long. 65° E., when the A.T.G. is August 5th, $10^h\ 30^m$ A.M.

A.T.G. August	$4^d\ 22^h\ 30^m$	65°
	4 20	4
A.T.S. August	$5^d\ 2\ 50$	6,0)26,0
		$4^h\ 20^m$

2. What is the A.T.S. in long. 120° 15′ W. when the A.T.G. is $2^h\ 35^m$ P.M. on January 2nd?

A.T.G. January	$2^d\ 2^h\ 35^m$	120° 15′
	8 1	4
A.T.S. January	$1^d\ 18\ 34$	6,0)48,1·0
Or January 2nd	6 34 A.M.	$8^h\ 1^m$

3. Required the Mean Time at ship in long. 64° W. when the M.T.G. is August $15^d\ 5^h\ 17^m\ 35^s$.

M.T.G. August	$15^d\ 5^h\ 17^m\ 35^s$	Long. 64° W.
	4 16 0	4
M.T.S. August	$15^d\ 1\ 1\ 35$	6,0)25,6
		$4^h\ 16^m$

EXERCISES.

1. What is the mean time at ship in long. 123° 27′ 30″ E. when the M.T.G. is October $14^d\ 9^h\ 15^m\ 24^s$?
2. What is the M.T.S. in long. 35° W. when the M.T.G. is May 6th $2^h\ 3^m\ 46^s$ P.M.?
3. Find the M.T.S. in long. 143° 27′ 30″ W. when the M.T.G. is September 1st $4^h\ 37^m\ 23^s$ A.M.
4. When the M.T.G. is June 2nd $11^h\ 19^m\ 14^s$ P.M., what is the mean time at ship in long. 95° 46′ E.?
5. Required the M.T.S. in long. 97° 16′ 45″ E. when the M.T.G. is October 19th $5^h\ 8^m\ 40^s$ P.M.
6. Find the M.T.S. when the M.T.G. is May 1st $5^h\ 49^m\ 54^s$ A.M. in long. 164° 48′ E.

ART. 40.—**Rating Chronometers.**—The ship's chronometer is relied on at sea to give the Mean Time at Greenwich (M.T.G.), its error on leaving port and its rate of gaining or losing per day being known.

An "Error" and "Rate" are usually supplied when the chronometer leaves the shop, but as the Rate is liable to be affected by changes of temperature and other causes during a voyage, it

is advisable to ascertain the correct Error when opportunity offers at ports the ship may visit.

"Rating a chronometer" means finding the rate at which it gains or loses per day by dividing the sum or difference of the errors on different dates by the number of days elapsed.

Examples.—

1. A chronometer was found to be 4ᵐ 30ˢ fast of mean noon at Greenwich on January 15th, 1906, and on March 30th, 1906, it was 7ᵐ 35ˢ fast of mean noon at Greenwich. Required the rate.

m. s.
Fast 4 30 on January 15.
Fast 7 35 on March 30.

3 5 gained in 74 days.
60

74)185(2·5 gain per day.
148

370
370

2. A chronometer was 14ᵐ 25ˢ fast of mean noon at Greenwich on September 1st, 1906, and at mean noon at Greenwich on December 4th, 1906, it was 13ᵐ 14·5 fast. Required the rate.

m. s.
Fast 14 25 on September 1.
Fast 13 14·5 on December 4.

1 10·5 lost in 94 days.
60

94)70·5(·75ˢ losing daily.
658

470
470

3. A chronometer was 1ᵐ 42ˢ slow of M.T.G. at mean noon on January 1st, 1906, and on March 31st at mean noon Greenwich time it was 6ᵐ 9ˢ slow. Required the rate.

m. s.
Slow 1 42 on January 1.
Slow 6 9 on March 31.

4 27 lost in 89 days.
60

89)267(3ˢ losing daily.
267

4. On leaving Liverpool, January 19th, 1904, the ship's chronometer was 4ᵐ 2ˢ·5 slow of mean noon at Greenwich, and at Sydney on May 28th it was found to be 3ᵐ 10ˢ·5 slow of mean noon at Greenwich. Required the rate.

m. s.
Slow 4 2·5 on January 19.
Slow 3 10·5 on May 28.

52·0 gained in 130 days.
130)52·0(·4" gaining daily.
52·0

5. At New York on February 27th, 1904, a ship's chronometer was 18" slow at mean noon Greenwich time, and at Southampton on March 6th, at Greenwich mean noon it was 14" fast. Required the rate.

s.
Slow 18 on February 27.
Fast 14 on March 6.

32 gained in 8 days.

8)32

4" gaining daily.

6. A chronometer was fast of mean noon at Greenwich 2m 17" on January 14th, 1906, and on June 1st it was 3m 55"·5 slow of mean noon at Greenwich. Required the rate.

m. s.
Fast 2 17 on January 14.
Slow 3 55·5 on June 1.

6 12·5 lost in 138 days.
60

138)372·5(2"·7 losing daily.
276

965
966

ART. 41. **The time by chronometer being given, and two errors or one error and daily rate, to determine the correct G.M.T.**

Find the rate as explained above, multiply this by the number of days and decimals elapsed between the date of second error, and date of chronometer time. This gives the accumulated rate. Now take the time by chronometer expressed astronomically, being careful to make it correspond as far as possible with the Greenwich Time deduced from the Ship Time and approximate longitude, apply the second error and accumulated rate to obtain the correct G.M.T.

Examples.—

1. February 24th, about 8h 20m A.M. at ship in long. by account 74° 15′ W., the chronometer showed 1h 12m 20"·5, and had been found 13m 35" slow of mean noon at Greenwich on October 30th, and on December 24th it was 10m 55"·5 slow. Find G.M.T.

74° 15′
4

6,0)29,7 0

4ʰ 57ᵐ

Ship time Feb. 23ᵈ 20ʰ 20ᵐ
Long. W. + 4 57

Greenwich time 24ᵈ 1 17

Slow Oct. 30th 13ᵐ 35ˢ
,, Dec. 24th 10 55·5

Gain in 55 days 2 39·5

55)159·5(2ˢ·9
110
495
495

Time by chron. Feb. 24ᵈ 1ʰ 12ᵐ 20·5ˢ
Second error slow 10 55·5

24ᵈ 1 23 16
Accum. rate gained − 3 0

M.G.T. Feb. 24ᵈ 1 20 16

Elapsed time 62·05
rate 2·9

55845
12410

6,0)17,9·945

2ᵐ 59ˢ·9

2. October 18th, at 2ʰ 30ᵐ A.M., M.T. ship nearly in long. by D.R. 50° E.; the chronometer showed 11ʰ 3ᵐ 22ˢ, which had been found 4ᵐ 13ˢ·7 fast on July 24th, and losing daily 3ˢ·4. Find G.M.T.

4

6,0)20,0

3ʰ 20ᵐ

Ship time Oct. 17ᵈ 14ʰ 30ᵐ
Long E. − 3 20

Oct. 17ᵈ 11 10

Time by chron. Oct. 17ᵈ 11ʰ 3ᵐ 22ˢ
Error fast − 4 13·7

Oct. 17ᵈ 10 59 8·3
Accum. rate lost + 4 50·7

M.T.G. Oct. 17ᵈ 11 3 59

Elapsed time 85·5
rate 3·4

3420
2565

6,0)29,0·70

4ᵐ 50ˢ·7

3. February 3rd, approximate ship time 1ʰ 8ᵐ A.M., in long. about 31° W., when a chronometer, known to be 43ᵐ 19ˢ fast on January 13th, and gaining daily 2ˢ 5, showed 3ʰ 55ᵐ 30ˢ·5. Find correct G.M.T.

31°
4

6,0)1,24

2ʰ 4ᵐ

Ship time Feb. 2ᵈ 13ʰ 8ᵐ
Long. W. + 2 4

Greenwich time 2ᵈ 15 12

Elapsed time 20·6 days
rate 2·5

1030
412

51·50

Time by chron. Feb. 2ᵈ 15ʰ 55ᵐ 30ˢ·5
Error fast − 43 19

Feb. 2ᵈ 15 12 11·5
Accum. rate gained − 51·5

G.M.T. Feb. 2ᵈ 15 11 20

Exercises.

Find the chronometer "Rate" from the following errors on Greenwich Mean Noon:—

No.	1st date.	Error.	2nd date.	Error.
		h. m. s.		h. m. s.
1	August 28th	0 7 2·1 fast	December 9th	0 2 24·0 fast
2	September 30th	1 21 19·0 slow	January 28th	1 10 43·0 slow
3	April 12th	0 25 19·0 slow	July 1st	0 11 55·0 slow
4	April 1st	0 10 24·0 fast	August 15th	0 11 32·0 fast
5	February 27th	0 0 19·5 fast	July 31st	0 0 57·5 slow
6	December 27th	correct	February 20th	0 4 57·0 fast
7	June 1st	0 1 8·8 slow	September 30th	0 0 40·1 fast
8	June 16th	0 14 26·9 fast	September 30th	0 8 18·4 slow

9. November 27th, at about 10^h A.M., at ship in long. 123° 15′ E., when a chronometer showed $1^h\ 53^m\ 34^s{\cdot}6$, which was slow for G.M.N. $3^m\ 13^s$ on September 10th, and fast $2^m\ 30^s$ on October 20th. Find correct G.M.T.

10. October 31st, about $2^h\ 15^m$ P.M., at ship in long. 124° W., when a chronometer showed $10^h\ 7^m\ 33^s$, which was slow $10^m\ 46^s{\cdot}8$ on September 29th, and gaining $1^s{\cdot}01$ daily. Find the G.M.T.

11. March 20th, about 4^h P.M., at ship in long. 63° 45′ W., when a chronometer showed $7^h\ 12^m\ 1^s$, which was slow $1^h\ 2^m\ 4^s$ on January 2nd, and on February 1st was slow $1^h\ 3^m\ 54^s$. Find the G.M.T.

12. December 10th, about $2^h\ 15^m$ P.M., at ship in long. 161° W., when a chronometer showed $2^h\ 1^m\ 23^s$, which was fast $59^m\ 16^s$ on August 12th, and on October 31st was fast $1^h\ 2^m\ 16^s$ on G.M.N. Find G.M.T.

13. July 4th, at $7^h\ 17^m$ A.M., at ship in long. 75° W., when a chronometer showed 12 o'clock, it was fast of G.M. noon 38^s on April 15th, and gained $\cdot 75^s$ daily since. Find G.M.T.

14. November 27th, at 7^h A.M., at ship in long. 4° 45′ W., when a chronometer showed $7^h\ 17^m\ 58^s{\cdot}5$, which had been found $6^m\ 57^s$ slow for G.M. noon on August 21st, and slow $6^m\ 37^s{\cdot}5$ on September 20th. Find G.M.T.

15. December 14th, about $4^h\ 5^m$ P.M., M.T.S. in long. 11° 6′ W., when a chronometer showed $5^h\ 20^m\ 44^s$, which was fast of G.M. noon $2^m\ 38^s{\cdot}1$ on November 10th, and on November 27th was correct. Find G.M.T.

16. May 1st, about $6^h\ 30^m$ P.M., at ship in long. 91° E., when a chronometer showed 12^h, and was slow $24^m\ 54^s$ on January 11th for G.M. noon, and slow $24^m\ 34^s$ on March 2nd. Find G.M.T.

17. April 16th, about 4^h P.M., at ship in long. 131° 25′ E., when a chronometer showed $7^h\ 18^m\ 56^s$, which was slow of G.M. noon on February 9th $9^m\ 21^s{\cdot}5$, and was slow $3^m\ 19^s{\cdot}5$ on March 17th. Find G.M.T.

CHAPTER X.

ELEMENTS FROM THE "NAUTICAL ALMANAC."

ART. 42.—A great part of the labour involved in the calculation of problems in Nautical Astronomy is the *Reduction of the Elements* from the "Nautical Almanac;" therefore it is most important that the student should master this part of the work at the outset. The advantage of a thorough acquaintance with the "Nautical Almanac" is incalculable.

The necessary extracts for working all the problems in this Book will be found in the Appendix. In the case of the stars the "mean position" only is given, and this is the one used in the calculations, as the introduction of the "apparent places" would considerably enlarge the Appendix without giving a corresponding advantage.

The Sun's Elements.—These are given for *Noon at Greenwich*, therefore it is always necessary to know the *Greenwich Astronomical Time.*

When *Apparent Time* is given, the elements are taken from p. I. of the month; but when *Mean Time* is given, they are taken from p. II.

The variations or changes in one hour are on p. I.

All minutes and seconds in the Greenwich time are reduced to decimals of an hour.

(1) **Declination.**

Examples.—

1. Find the sun's declination for August 16th $8^h\ 36^m$ P.M. apparent time at Greenwich.

Declination at noon	13° 57′ 28″·0 N.	H.V. 47·03 page I., app. time
Cor.	− 6 44 5	8·6
Dcc. at $8^h\ 36^m$	13 50 43·5 N.	28218
		37624
		6,0)40,4·458
		6′ 44″·5

The correction is subtracted because the declination is decreasing.

2. Find sun's declination for September 29th $2^h\ 45^m$ A.M. M.T.G. ; astro. M.T.G. $28^d\ 14^h\ 45^m$.

Dec. for 29^d noon	$2°\ 9'\ 24''{\cdot}1$ S.	H.V. 58·41 page I., mean time.
Cor.	$-\ 9\ 00{\cdot}3$	9·25
Dec. for $28^d\ 14^h\ 45^m$	$2\ 0\ 23{\cdot}8$ S.	29205
		11682
		52569
		6,0)54,0·2925
		$9'\ 0''{\cdot}3$

Here we work from nearest noon, and obtain multiplier for H.V. by subtracting the $14^h\ 45^m$ in G.M.T. from $24^h = 9^h\ 15^m$.

The correction is subtracted since the declination decreases from 29th to 28th.

3. Find sun's declination for September 23rd $11^h\ 48^m$ P.M. M.T.G.

Dec. for 23^d noon	$0°\ 10'\ 58''{\cdot}1$ N.	H.V. 58·45
Cor.	$-\ 11\ 29{\cdot}7$	11·8
Dec. for $23^d\ 11^h\ 48^m$	$0\ 0\ 31{\cdot}6$ S.	46760
		64295
		6,0)68,9·710
		$11'\ 29''{\cdot}7$

Since the declination changes in name from N. to S., it must decrease. The declination has a maximum value twice during the year, about June 21st and December 24th, the value being about $23°\ 27'\ 15''$; it has no value about March 20th and September 23rd, the sun being on the equinoctial on those dates.

From March 20th to June 21st the sun's declination is N. and increasing.
,, June 21st to Sept. 23rd ,, ,, ,, ,, decreasing.
,, Sept. 23rd to Dec. 24th ,, ,, S. ,, increasing.
,, Dec. 24th to March 20th ,, ,, ,, ,, decreasing.

(2) Equation of Time.

Examples.—

1. Find the Eq. of Time for September 24th $8^h\ 21^m$ A.M. A.T.G.

Eq. of Time for Sept. 24^d noon	$7^m\ 43^s{\cdot}36$	H.V. ·866
Cor.	$-\ 3{\cdot}16$	3·65 for nearest noon.
,, ,, Sept. $23^d\ 20^h\ 21^m$	$7\ 40{\cdot}20$	4330
		5196
		2598
		3·16090

By looking on the top of the column, we may see how to apply this amount to A.T.G. in order to obtain M.T.G. A thick line divides the heading. There is a corresponding thick line in the Eq. of Time column between 1^d and 2^d: this shows that all amounts above the thick line must be applied as stated above the line in the heading, and *vice versâ* ; hence the amount found, $7^m\ 40^s{\cdot}2$, must be subtracted from the given A.T.G.

Thus A.T.G. Sept. $23^d\ 20^h\ 21^m\ 0^s$	
Eq. of Time	$-\ 7\ 40{\cdot}2$
M.T.G. Sept. $23^d\ 20\ 13\ 19{\cdot}8$	

2. Find Eq. of Time for September 1st $8^h\ 42^m$ P.M. M.T.G.; and find A.T.G.

Eq. of Time Sept. 1st noon		0^m	$9^s{\cdot}13$	H.V.	·779
	Cor.		6·78		8·7
,, ,,	$1^d\ 8^h\ 42^m$	0	2·35		5453
					6232
					6·7773

M.T.G. Sept. $1^d\ 8^h\ 42^m\ 0^s$
Eq. $-\ 2{\cdot}35$

A.T.G. ,, $1^d\ 8\ 41\ 57{\cdot}65$

Correction is subtracted from Eq. because it is changing its name, and the total amount is subtracted from M.T.G. because it is above the black line in column. Had the correction been more than the original amount of Eq., the corrected amount would have turned out below the line and additive.

The Eq. of Time has 4 maxima during the year, on Feb. 11th, May 15th, July 27th, Nov. 4th.

The Eq. of Time has 4 minima during the year, on April 15th, June 15th, Sept. 1st, Dec. 25th.

On the days on which the Eq. of Time vanishes, the mean and apparent suns are on the same hour circle.

(3) Right Ascension.

Examples.—

1. Find the R.A. of the apparent sun on August 19th at $8^h\ 27^m$ P.M. M.T.G.

R.A. August 19th noon		9^h	$51^m\ 21^s{\cdot}31$	H.V.	9·304
	Cor.		$+\ 1\ 18{\cdot}62$		8·45
,, ,,	$19^d\ 8^h\ 27^m$	9	52 39·93		46520
					37216
					74432
					78·61880

2. Find the R.A. of the apparent sun on August 25th at $3^h\ 15^m$ A.M. A.T.G.

A.T.G. August $24^d\ 15^h\ 15^m$

R.A. August 25th noon		10^h	$13^m\ 33^s{\cdot}02$	H.V.	9·187
	Cor.		$-\ 1\ 20{\cdot}39$		8·75
,, ,,	$24^d\ 15^h\ 15^m$	10	12 12·63		45935
					64309
					73496
					80·38625

In correcting the above elements, the H.V. as entered in the "Nautical Almanac" has been used without alteration; the H.V. differs, however, day by day, and therefore requires correcting itself; this is done by applying the amount that the H.V. changes in half the interval to be corrected for to the H.V. used: the element is then said to be corrected for *2nd differences.*

Example.—Find the R.A., Dec., and Eq. of Time for August 1st, 8h 30m P.M. A.T.G., correcting for 2nd Differences.

R.A. for August 1st noon		8h 42m 55·43	II.V. 1st	9·724	9·724
Cor.		+ 1 22·612	,, 2nd	9·698	Cor. — ·005
R.A. ,, ,,	8h 30m	8 44 18·042		·026	9·719
			½ Int.	4·25	8·5
				2550	48595
				850	77752
				24)·11050	82·6115
				·005	
Dec. for August 1st noon		18° 11′ 45″·0 N.	II.V. 1st	37″·26	37″·26
Cor.		— 5 17·8	,, 2nd	38·00	Cor. + ·13
Dec. ,, ,,	8h 30m	18 6 27·2 N.		·74	37·39
			½ Int.	4·25	8·5
				1700	18695
				2975	29912
				24)3·1450	317·815
				·13	
Eq. of Time August 1st noon		6m 10·22	H.V. 1st	·132	·132
Cor.		— 1·165	,, 2nd	·158	Cor. + ·005
,, ,,	8h 6m	6 9·055		·026	137
			½ Int.	4·25	8·5
				2550	685
				850	1096
				24)·11050	1·1645
				·005	

NOTE.—The *Polar Distance* of an object is its distance from the elevated pole, and is the complement of the declination. It is found by adding 90° to the dec. when dec. and lat. are of opposite names, and by subtracting the dec. from 90° when dec. and lat. are of same name.

EXERCISES.

Correct the declination, equation of time, and right ascension for the following Greenwich times.

1.	Apparent time	Aug. 3d	5h 48m.	4.	Mean time	Sept. 14d 10h 24m.
2.	,,	Aug. 7d	9 54.	5.	,,	Sept. 23d 6 12.
3.	,,	Aug 14d	14 36.	6.	,,	Sept. 23d 11 30.
				7.	,,	Sept. 23d 15 9.
				8.	,,	Sept. 23d 22 45.
				9.	,,	Sept. 29d 18 15

10. Find the A.T. ship when the Greenwich mean time is August 20^d 16^h 5^m 30^s in long. 64° E.
11. Find the M.T. ship when the A.T.G. is September 30^d 8^h 42^m in long. 164° 30′ W.
12. If the A.T.S. is September 25th 3^h 19^m 25^s A.M., in long. 84° 20′ E., find the M.T.G.

ART. 43. **Sidereal Time.**—This is tabulated for noon of each day on p. II., and is the Sidereal Time at mean noon; another name for it is Right Ascension of the Mean Sun. Since the Mean Sun is supposed to move uniformly in the equinoctial, the H.V. is constant. A table for correcting the Sidereal Time is found in the "Nautical Almanac;" or in Norie, table xxxviii. The equivalents taken from the "Nautical Almanac" are given in the Appendix.

Example.—Find the correct Sid. Time for August 25^d 13^h 15^m 25^s, M.T.G.

S.T. for August 25^d noon		10^h	11^m	$21^s \cdot 50$
Accel. for 13^h			2	8·13
,,	15^m			2·46
,,	25^s			·07
R.A.M. Sun		10	13	32·16

EXERCISES.

Correct the Sid. Times for following G.M.T.'s.

1.	August	19^d	18^h	10^m	32^s.
2.	,,	28	3	4	47.
3.	September	5	14	54	14.
4.	,,	29	2	49	24.

ART. 44. **Moon's Elements**—(1) **Semi-diameter.** (2) **Horizontal Parallax.** These are entered on p. III. of "Nautical Almanac" for "noon" and "midnight" of each day. The difference between any two consecutive entries is the variation in twelve hours, from which by proportion may be found the variation corresponding to the required interval. After being corrected for "time," the S.D. requires a further amount called "Augmentation" to be added to it, in consequence of the apparent increase in the disc of the moon, due to her approach to the observer as she increases her altitude. This is found in Table D, Norie. The H.P. also requires an amount called "Reduction" to be subtracted from it, because the H.P. entered in the almanac is the Equatorial H.P., and the observer is in some latitude where the earth's radius is shorter than the equatorial radius, and so the H.P. is reduced. This correction is found in Table E, Norie.

Example.—August 16^d at 5^h 14^m 24^s M.T.G. in lat. 35° S.; app. alt. moon, 47°. Find corrected S.D. and H.P.

S.D.		H.P.	
Noon	15′ 1″·38		55′ 2″·42
Midnight	15 5·37		55 17·04
Diff. in 12 hrs.	3·99		14·62
	5·24		5·24
	1596		5848
	798		2924
	1995		7310
	12)20·9076		12)76·6088
Change in 5^h 14^m·4	1·74		6·38
Noon	15′ 1″·38	Noon	55′ 2″·42
Corr.	+ 1·74	Corr.	+ 6·38
	15 3·12		55 8·80
Table D	+10·7	Table E	− 3·55
	15 13·82		55 5·25

Should the time exceed 12 hours, that is, fall between midnight and noon, the element is taken out for midnight and corrected towards the next day noon, using as a multiplier the hours in the Greenwich time *reduced* by 12.

EXERCISES.

Find the S.D. and H.P. of the Moon under the following conditions:—

1. August 7^d 7^h 24^m M.T.G. in lat. 40° N. Alt. 26°.
2. ,, 17^d 16 15 ,, ,, 50° S. ,, 36°.
3. ,, 24^d 20 36 ,, ,, 30° N. ,, 46°.
4. ,, 31^d 5 45 ,, ,, 20° S. ,, 56°.

(3) **Meridian Passage.**—This is tabulated in mean time for each day at upper and lower transit on p. IV. of the month, in Greenwich astronomical time. It is the *interval* between the transit of the Sun and Moon. As the Earth rotates from W. to E., and the Moon is constantly changing her angular distance eastward from the Sun (about 12° daily), it follows that this *interval* is *less* when the Moon crosses the meridian of a place in E. long., and *greater* for a place in W. long. than the time given in the "Nautical Almanac." The change for 24 hours is found by comparing with the *following* day for W. long. and the *preceding* day for E. long. The proportion of this change for the "Longitude in Time" is the correction required, and gives the time at place of the transit across the meridian of the place; by applying "Longitude in Time" to this, we obtain the M.T.G. of the transit across the ship's meridian.

NOTE.—(*a*) It is usual to ask for the Mer. pass. on a given *Civil Day*; therefore, if the corrected time of Mer. pass. exceeds 12 hours, it must be taken out for the preceding day, but for the same day if under 12 hours.

(*b*) The "Lower" Mer. pass. is used only when the Moon is observed *below* the pole.

(c) When two asterisks (* *) are seen in the upper Mer. pass. column, it is New Moon, and therefore no observation can be made.

Examples.—August 25th find the M.T.G. of Moon's Mer. pass. in long. 78° W.

Mer. pass. Aug.	25d 4h 39m·8	Increase to *following* day because of *West* long.	54m
Cor. for long.	+11·7	long. in time	5·2
M.T. at place	25d 4 51·5		108
Long.	+ 5 12		270
M.T.G.	25d 10 3·5	24	4)280·8 / 6) 70·2 / 11·7

78°	
4	
6,0)31,2	
5h 12m	

August 11th find M.T.G. of Moon's Mer. pass. in long. 156° E.
Since Mer. pass. on August 11th occurs at 17h 57m·0, we use—

Mer. pass. Aug.	10d 17h 13m·3	Decrease to preceding day because of *East* long.	43m·4
Cor. for long.	−18·8		10·4
M.T. at place	10d 16h 54·5		1736
Long.	−10 24		434
M.T.G.	10d 6 30·5	24	4)451·36 / 6)112·8 / 18·8

156°	
4	
6,0)62,4	
10h 24m	

EXERCISES.

Find the M.T.G. of the Moon's Mer. pass. under following conditions :—

1.	August	1st	in long.	172° 20′	W.
2.	,,	28th	,,	96 25	W.
3.	,,	2nd	,,	125 25	E.
4.	,,	27th	,,	89 45	E.
5.	,,	4th	,,	175 30	W.
6.	,,	15th	,,	142 48	E.
7.	,,	9th	,,	24 30	E.
8.	,,	7th	,,	55 35	E.

(4 and 5.) **The Moon's Right Ascension and Declination.**—The Moon's path in the heavens is inclined to the Sun's at an approximate angle of 5° 9′; and a great circle drawn through the direction of the Moon's motion at any time cuts the ecliptic in two opposite points called the *nodes*. At every revolution of the moon round the earth the nodes retrograde in the ecliptic, in a manner similar to the precession of the equinoxes; and are carried back 19° 20′ in one year, therefore taking 18½ years to complete the circle. Her greatest latitude is therefore 5° 9′; but her greatest declination varies from 23° 27′ + 5° 9′ to 23° 27′ − 5° 9′. This causes very large changes in her right ascension and declination from day to day, as in addition her angular motion

in her orbit is nearly 13° per day; therefore it has been found necessary to tabulate these data for every hour of the day. This is done in the "Nautical Almanac," pp. V. to XII. of each month, and the variations are given for 10^m.

In reducing these elements, it is convenient to find the var. in 1^m by moving the decimal point one place to the left, and then multiplying by the number of minutes and decimals of a minute in the M.T.G.

Example.—Correct the Moon's R.A. and Dec. for M.T.G. August 4th $12^h\ 24^m{\cdot}8$.

R.A. for $4^d\ 12^h$	$21^h\ 20^m\ 56^s{\cdot}87$	Var. in 1^m 2·363
Cor. for $24^m{\cdot}8$	+ 58·60	24·8
Corrected R.A.	21 21 55·47	18904
		9452
		4726
		58·6024
Dec. for $4^d\ 12^h$	16° 4′ 41″·9 S.	Var. in 1^m 7″·586
Cor. for $24^m{\cdot}8$	− 3′ 8·1	24·8
Corrected Dec.	16 1 33·8 S.	60688
		30344
		15172
		6,0)18,8·1328
		3′ 8″·1

Using 2nd differences.

R.A. for $4^d\ 12^h$	$21^h\ 20^m\ 56^s{\cdot}87$	12^h Var. in 10^m = 23·627	Var. in 1^m 2·3616
Cor.	+ 58·57	13^h ,, = 23·574	24·8
Corrected R.A.	21 21 55·44	Change in 60^m − ·053	188928
		½ Int. 12·4	94464
		212	47232
		636	58·56768
		60)·6572	
		Ch. ·011	
		∴ Req. var. = 23·616	
Dec. for $4^d\ 12^h$	16° 4′ 41·9″ S.	12^h Var. in 10^m 75·86	Var. in 1^m 7·604
Cor.	− 3 8·6	13 ,, ,, 76·75	24·8
Cor. Dec.	16 1 33·3 S.	Ch. in 60^m +·89	60832
		½ Int. 12·4	30416
		356	15208
		1068	6,0)18,8·5792
		6,0)1,1·036	3′ 8·6′
		Ch. ·18	
		∴ Req. var. = 76·04	

EXERCISES.

Reduce the Moon's R.A. and Dec. to the following M. Times at Greenwich :—

1. August $5^d\ 18^h\ 19^m\ 27^s$.
2. ,, 8 11 48 45.
3. ,, 15 6 37 24.
4. ,, 22 19 51 48.

ART. 45. **Planets' Elements.**—The planetary elements used are—

(1) **Right Ascension.**
(2) **Declination.**

These are entered for Mean noon at Greenwich on each day; no variation is given; and the corrections are found by using the difference between two consecutive days as diff. for 24 hours, and thence by proportion determining the change for hours and decimals in M.T.G.

Example.—Find the R.A. and Dec. of Jupiter for August $9^d\ 10^h\ 15^m$ M.T.G.

R.A. August 9^d noon	$6^h\ 8^m\ 17^s{\cdot}24$	Dec. August 9^d	23° 7′ 37″·9 N.
,, ,, 10^d ,,	6 9 07·77	,, ,, 10^d	23° 7′ 31·3
Diff. in 24 hrs.	+ 50·53		− 6·6
Int.	10·25		10·25
	25265		330
	10106		132
	5053		66
	24)517·9325		24)67·650
	Ch. 21·58		Ch. 2·8

R.A. August 9^d	$6^h\ 8^m\ 17^s{\cdot}24$	Dec. August 9^d	23° 7′ 37″·9 N.
Cor.	+ 21·58	Cor.	− 2·8
Corrected R.A.	6 8 38·82	Corrected Dec.	23 7 35·1 N.

(3) **Semi-diameter.**
(4) **Horizontal Parallax.**—These elements do not require correction, and are entered for mean noon at Greenwich. The S.D. is occasionally used in "Lunar" problems.

(5) **Meridian Passage.**—This column in the "Nautical Almanac" is consulted to obtain the time for observing the Mer. Alt. of a planet for the purpose of finding the latitude. It also determines whether a planet is *east* or *west* of the meridian at a given time.

The correction for longitude, when necessary, is similar to that of the Moon's Mer. Pass.

Example.—Find the G.M.T. of Mer. Pass. of Jupiter on August 9th in long. 75° E.

Mer. Pass. August	8^d	21^h $0^m{\cdot}4$	Ch. to preceding day + 3·1
Cor. for long.		+ ·6	long. 5
		21 1·0	24)15·5
Long. in time		5 0	·6
M.T.G.	8^d	16 1·0	

EXERCISES.

1. Correct the R.A. and Dec. of Jupiter for M.T.G. August 10^d 14^h 54^m, and find his Mer. Pass. in long. 120° W.

2. Correct the R.A. and Dec. of Saturn for August 11^d 8^h 24^m M.T.G., and find his Mer. Pass. in long. 150° E.

ART. 46. **Stars' Elements.**—The only elements used in Navigation are Right Ascension and Declination; these do not require correction.

In the "Nautical Almanac" is a table giving the Mean Places of Stars where they are arranged in order of Right Ascension. The magnitudes are also stated by numbers, the smaller number denoting the brighter star. At the bottom of each page the proper name of the star is given; this may also be found in Table XIV., Norie. After the Mean Places, there is a very much extended table giving the Apparent Places of Stars for every 10th day of the month.

Directions.—Find the star in the table of "Mean Places," and note the R.A. This R.A. is then used to find the star in the extended table of "Apparent Places." The R.A. and Dec. are then taken out for the nearest date.

Examples.—In our table in the Appendix we find—

γ Pegasi	Mag.	2·9	R.A.	0^h	8^m	$23^s{\cdot}6$	Dec.	14°	39′	40″ N.
α Arietis	,,	2·2	,,	2	1	52·3	,,	23	1	6 N.
β Aurigæ	,,	2·1	,,	5	52	38	,,	44	56	19 N.
α Argûs	,,	−1·0	,,	6	21	51·9	,,	52	38	39 S.

Stars like "α Argûs" whose mag. is prefixed by a " − " sign are brighter than those of mag. 0; and these increase in brightness as the negative number increases. Thus Sirius mag. −1·4 is brighter than α Argûs.

CHAPTER XI.

SIDEREAL AND SOLAR TIME.

ART. 47.—As a consequence of the Earth's rotation from W. to E., all Celestial bodies have an apparent motion from E. to W., but not all at the same rate. A star completes its apparent revolution in exactly the same time as the Earth takes to turn once on its axis, but the Sun, Moon, and Planets all change their positions with respect to the Stars. The Sun moves to the eastward about 1° daily, and the Moon about 13° daily, whilst a planet may have a motion to the E. or W., and sometimes stands still amongst the Stars.

The *Apparent Motions* of the *Stars* and the *Sun* are those used for measuring *Time*.

A Sidereal Day is the interval between two successive transits of a star over the Meridian of a place, and all Sidereal Days are equal.

A Solar Day is the interval between two successive transits of the Sun's centre, but Solar Days are *not* all equal, as they increase or decrease in length according to the time of the year. The Sun cannot therefore be a *uniform time-keeper*. Yet, as Day and Night depend on the Sun, it is the Natural Controller of time for ordinary purposes, and its position with respect to the Meridian marks *Apparent Time*, as indicated by a sun-dial.

To secure a *Uniform Solar Time* and *Equal Solar Days*, a *Mean Sun* is adopted, which gives *Equal Days* of 24 *Equal Hours* all the year round. Its position—sometimes to the E., sometimes to the W., and sometimes (four times in the year) coinciding in Right Ascension with the true Sun—marks *Mean Time*.

But on account of the progress Eastward (about 1° daily) of this *Mean Sun*, it is evident that it will take a longer time to come round to the Meridian again than would a fixed Star—nearly 4^m, in fact.

Hence the *Mean Solar Day* is nearly 4^m longer than a Sidereal Day, but as both kinds of days are divided into 24 hours, it follows that *Solar* hours, minutes, and seconds are longer than the corresponding divisions of *Sidereal* Time. The exact relative values are given in two Tables in the "Nautical Almanac," on pp. 588 and 590, and in the Appendix.

Any star would do to measure a Sidereal Day, but astronomers have made the Sidereal Day begin at the moment when the point where the Ecliptic cuts the Equator—corresponding to the Sun's position on March 21st, and named the "First point of Aries"—is on the Meridian of a place.

Right Ascension is also measured from this point (see Definition).

N.B.—An hour of any kind, whether of Right Ascension, Sidereal Time, or Solar Time, corresponds to an arc of 15°, because $24 \times 15° = 360°$.

ART. 48.—**To find Sidereal Time.**—When the *Hour Angle* and the *Right Ascension* of any Celestial Object are known, the *Sidereal Time* may be found as follows:—

(*a*) **Hour Angle West.**—*Add* the Hour Angle to the Right Ascension (rejecting 24 if the sum exceeds 24 hours).

(*b*) **Hour Angle East.**—*Subtract* the H.A. from the R.A. (adding 24 hours if necessary).

Illustration.—PMP_1 is the Celestial Meridian of a place, and the other projected semicircles from P to P_1 are "Hour Circles" (15° apart). A is the "First point of Aries," and the Arc of the Equinoctial M.A. is the Sidereal Time (in this case, 4 hours). But the *same Arc* measured eastwards from A to M is the R.A. of the Meridian; therefore *Sidereal Time* and *Right Ascension of Meridian* are always *equal*.

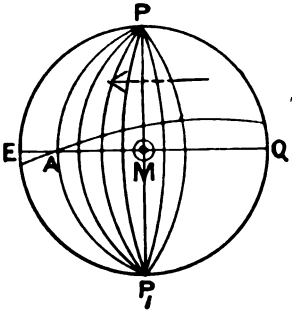


FIG. 98.

NOTE.—The student must regard the Meridian and Hour Circles as fixed, and the "First Point of Aries," as well as all celestial objects, in motion from E. to W. in the direction of the arrow.

Using the initials H.A. for Hour Angle, S.T. for Sidereal Time, R.A. for Right Ascension, and R.A.M. for Right Ascension of Meridian, the above rules may be stated concisely thus—

(*a*) H.A. West; S.T. or R.A.M. = R.A. + H.A.
(*b*) H.A. East; S.T. or R.A.M. = R.A. − H.A.

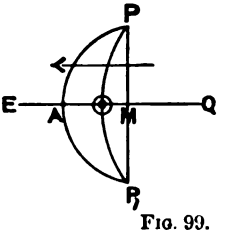


FIG. 99.

Examples.—

1. Given the Sun's H.A. $3^h\ 10^m\ 30^s$ W. of the Meridian, and Sun's R.A. $14^h\ 20^m\ 40^s$: required the Sidereal Time.

Here S.T. = R.A. + H.A.

Sun's R.A.	14^h	20^m	40^s
Sun's H.A.	3	10	30
S.T.	17	31	10

2. Given the Moon's H.A. $4^h\ 15^m\ 50^s$ E. of Meridian, and Moon's R.A. $10^h\ 19^m\ 35^s$: required the S.T.

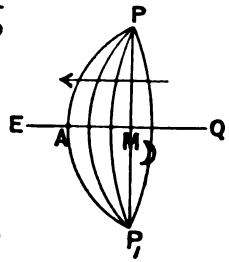


FIG. 100.

Here S.T. = R.A. − H.A.

Moon's R.A.	10^h	19^m	35^s
Moon's H.A.	4	15	50
S.T.	6	3	45

3. Given a star's H.A. $6^h\ 50^m\ 25^s$ E., and its R.A. $2^h\ 15^m\ 45^s$: required the S.T.

Star's R.A.	2^h	15^m	45^s (add 24^h)
Star's H.A.	6	50	25
S.T.	19	25	20 (always counted westward)

4. Given a Planet's H.A. $4^h\ 12^m\ 20^s$ W., and its R.A. $22^h\ 44^m\ 14^s$: required the S.T.

Planet's R.A.	22^h	44^m	14^s
Planet's H.A.	4	12	20
S.T.	2	56	34

NOTE.—The above calculation occurs in the "Lunar" and other problems, and should be clearly understood.

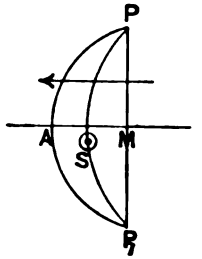


FIG. 101.

ART. 49. **To find Sidereal Time from Solar Time.**—Since *Apparent Time* is the *Westerly Hour* Angle of the Sun, and Mean Time is the *Westerly* Hour Angle of the *Mean Sun*, the formula for finding Sidereal Time from the Solar Time will be—

S.T. = R.A.M.S. + M.T.S.
or ,, = R.A.T.S. + A.T.S.

Examples.—

1. Given *Apparent* Time at ship in long. 78° W., August 14th, at $8^h\ 20^m$ A.M.: required the Sidereal Time.

A.T.S. Aug. 13^d	20^h	20^m	0^s	
long. W. +	5	12	0	
A.T.G. Aug. 14^d	1	32	0	
Sun's R.A. at noon	9^h	32^m	$39^s{\cdot}09$	
Cor.			14·40	
Reduced R.A.	9	32	53·49	
A.T.S.	20	20	0	
Sid. Time	5	52	53·49	

78°
4
6,0)31,2
$5^h\ 12^m$
H.V.
9·41
1·53
2823
4705
941
14·3973

2. Given *Mean* Time at ship in long. 84° 35′ W., September 23rd, 4ʰ 36ᵐ A.M.: required the Sidereal Time.

M.T.S. Sept.	22ᵈ	16ʰ	36ᵐ	0ˢ		
long. W.	+	5	38	20		
M.T.G. Sept.	22ᵈ	22	14	20		

84° 35′ / 4 / 6,0)33,8 20 / 5ʰ 38ᵐ 20ˢ

R.A. Mean Sun at noon (or Sid. Time)	12ʰ	1ᵐ	44ˢ·97	
Accel. for 22ʰ		3	36·84	see Appendix
Accel. for 14ᵐ			2·30	see Appendix
Accel. for 20ˢ			·05	see Appendix
Red. M.S.R.A.	12	5	24·16	
M.T.S.	16	36	0	
R.A.M. or Sid. Time	4	41	24·16	

3. Given M.T.G. September 28ᵈ 20ʰ 16ᵐ 50ˢ, long. 75° 20′ E.: find S.T.

M.T.G. Sept.	28ᵈ	20ʰ	16ᵐ	50ˢ		
long. E.	+	5	1	20		
M.T.S. Sept.	29ᵈ	1	18	10		

75° 20′ / 4 / 6,0)30,1 20 / 5ʰ 1ᵐ 20ˢ

S.T. at noon or R.A.M.S.	12ʰ	25ᵐ	24ˢ·28	
Accel. for 20ʰ		3	17·13	see Appendix
Accel. for 16ᵐ			2·63	see Appendix
Accel. for 50ˢ			·14	see Appendix
Red. M.S.R.A.	12	28	44·18	
M.T.S.	1	18	10	
R.A.M. or Sid. Time	13	46	54·18	

NOTE.—The Mean Sun's R.A. at noon is taken from the Sidereal Time column on p. II. of "Nautical Almanac," because the mean Sun being on the Meridian at Mean Noon, its R.A. at that moment is the same as the R.A.M. and the S.T. at Mean Noon.

EXERCISES.

Compute the S.T. from the following data:—

			h	m	s		long.
1. A.T.S.	...	August 2nd	3	15	0	A.M.	162° 30′ W.
2. M.T.S.	...	,, 4th	7	10	0	P.M.	105 45 E.
3. M.T.G.	...	,, 8ᵈ	23	14	42		36 30 E.
4. M.T.G.	...	Sept. 1ᵈ	5	15	25		124 37 W.
5. M.T.S.	...	,, 14th	8	30	0	A.M.	42 25 W.
6. A.T.S.	...	,, 24th	5	40	0	P.M.	155 12 E.

ART. 50.—**To find the Hour Angle of a Celestial Object.**—When

the *Sidereal Time* and the *Right Ascension* of a Celestial object are known, its *Hour Angle* can be found by subtracting one from the other, so as to obtain the least difference between them (borrowing 24 hours when necessary). The object is *West* of the Meridian when R.A. is *less* than S.T., and *East* when *greater*.

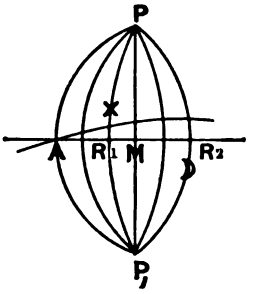


FIG. 102.

Illustration.—A is the first point of Aries. The arc MA is the S.T. of observation, AR_1 is the R.A. of a star west of the Meridian; its H.A. (the arc MR_1) is evidently = MA − AR_1, that is, S.T. − star's R.A.

Again, AR_2 is the R.A. of the Moon, and its H.A. (the arc MR_2) = AR_2 − MA, that is, Moon's R.A. − S.T., and it is East of the Meridian.

Examples.—

1. Given the S.T. $7^h\ 42^m\ 35^s$, and the R.A. of a star $6^h\ 40^m\ 13^s$: required its H.A.

S.T.	7^h	42^m	35^s
Star's R.A.	6	40	13
Star's H.A.	1	2	22 W. of Mer.

2. Given S.T. $19^h\ 10^m\ 40^s$, and R.A. of an object $1^h\ 2^m\ 50^s$: required its H.A.

S.T.	19^h	10^m	40^s
R.A.	1	2	50
H.A.	5	52	10 E. of Mer.

NOTE.—In this example the Sidereal Time is subtracted from the R.A. (which is therefore the greater), because it gives a difference less than 12 hours, and no object can be farther from the Meridian than 12 hours.

ART. 51.—All the Celestial bodies are so related to each other in position that when the H.A. of *one* of them is known, the H.A. of *any other* can be found by means of their known Right Ascensions; because it was shown in Art. 46 that S.T. is equal to the sum or difference of the R.A. and H.A. of a given object, and now it has been shown that the H.A. is the difference between the S.T. and the R.A. of any given object.

NOTE.—The H.A. can also be computed from the Altitude, as will be seen hereafter.

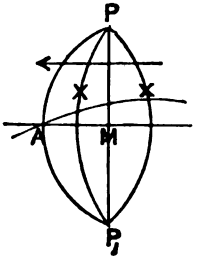


FIG. 103.

Examples.—

1. Given R.A. of Star Aldebaran $4^h\ 30^m\ 7^s$, and its H.A. $2^h\ 10^m\ 25^s$ west of Meridian: what is the H.A. of the Star Regulus whose R.A. is $10^h\ 3^m\ 0^s$?

R.A. Star Aldebaran	4^h	$30^m\ 7^s$
H.A. ,,	2	10 25 W.
S.T. (or R.A.M.)	6	40 32
R.A. Star Regulus	10	3 0
H.A. ,,	3	22 28 E. of Mer.

2. Given R.A. of Star Achernar $1^h\ 33^m\ 57^s$, and H.A. $4^h\ 50^m\ 47^s$ east of Meridian: required the H.A. of the Moon when its R.A. is $18^h\ 10^m\ 25^s$.

R.A. Star Achernar	1^h	$33^m\ 57^s$
H.A. ,,	4	50 47 E.
S.T. (or R.A.M.)	20	43 10
R.A. of moon	18	10 25
H.A. ,,	2	32 45 W. of Mer.

3. Given M.T.G. Aug. $16^d\ 8^h\ 14^m\ 20^s$ in long. 80° E.: find the Meridian Distance or Hour Angle of the Moon.

			Var. in 1^m
M.T.G. Aug.	$16^d\ 8^h\ 14^m\ 20^s$	80°	2·19
long. E.	5 20 0	4	14·3
M.T.S. Aug.	16^d 13 34 20	6,0)32,0	657
			876
		$5^h\ 20^m$	219
			31·317

S.T. at noon	9^h	35^m	$52^s·51$
Accel. for 8^h		1	18·85
Accel. for 14^m			2·30
R.A.M.S.	9	37	13·66
M.T.S.	13	34	20
R.A.M.	23	11	33·66
R.A. ☽	7	4	39·14
H.A. ☽	16	6	54·52 W. of Mer.
or	7	53	5·48 E. of Mer.

R.A. $16^d\ 8^h = 7^h\ 4^m\ 7^s·82$
Cor. 31·32
Red. R.A. 7 4 39·14

ART. 52.—**To find the Solar Time corresponding to given Sid. Time.**

Examples.—

1. Find the M.T.S. in long. 65° W. on Aug. 19th, when the S.T. is $7^h\ 55^m\ 20^s$.

S.T. or R.A.M.	7ʰ	55ᵐ	20ˢ
R.A.M.S. noon	9	47	42
Approx. M.T.S.	18ᵈ 22	7	38
long.	4	20	0
Approx. M.T.G.	19ᵈ 2	27	38
R.A.M.S. noon	9ʰ	47ᵐ	42ˢ·17
Accel. for 2ʰ			19·71
Accel. for 27ᵐ			4·44
Accel. for 38ˢ			·10
Red. R.A.	9	48	6·42
R.A.M.	7	55	20
M.T.S.	18ᵈ 22	7	13·58

or 19th 10ʰ 7ᵐ 13ˢ·58 A.M.

2. Find the M.T.S. in long. 82° E. on Sept. 20th, when the S.T. is 19ʰ 14ᵐ 35ˢ.

S.T. or R.A.M.	19ʰ	14ᵐ	35ˢ
R.A.M.S. Noon	11	53	52
Approx. M.T.S.	20ᵈ 7	20	43
long.	5	28	0
Approx. M.T.G.	20ᵈ 1	52	43
R.A.M.S. noon	11ʰ	53ᵐ	51ˢ·86
Accel. for 1ʰ			9·86
Accel. for 52ᵐ			8·54
Accel. for 43ˢ			·12
Red R.A.	11	54	10·38
R.A.M.	19	14	35
M.T.S.	20ᵈ 7	20	24·62

or 20th 7ʰ 20ᵐ 24ˢ·62 P.M.

EXERCISES.

1. Find the hour angle of Altair at 8ʰ 20ᵐ P.M. M.T.S. in long. 126° 20′ E. on August 12th.
2. Find the hour angle of Capella at 7ʰ 15ᵐ A.M. M.T.S. in long. 84° 35′ W. on September 20th.
3. Find the hour angle of the Moon on August 22nd at 5ʰ 26ᵐ P.M. M.T.S. in long. 34° W.
4. Find the hour angle of Jupiter on August 8ᵈ 17ʰ 15ᵐ M.T.G. in long. 56° 25′ E.
5. Find the M.T.S. in long. 74° E. on August 18th, when the S.T. is 6ʰ 14ᵐ 24ˢ.
6. Find the M.T.S. in long. 95° W. on September 14th, when the S.T. is 18ʰ 7ᵐ 36ˢ.

ART. 53.—**To compute the Time when a given Star is on the Observer's Meridian.**—When a star is on the meridian, the right

ascension of the star is the same as the right ascension of the meridian, or Sidereal Time, hence the problem is the same as finding the M.T.S. or A.T.S. corresponding to a given Sid. Time.

Examples.—

1. Find the M.T.S. when the star Procyon is on the observer's meridian in long. 35° W. on August 6th.

R.A. of Procyon or R.A.M. =	$7^h\ 34^m\ 23^s$	35°
R.A.M.S.	8 56 27	4
Approx. M.T.S. 5^d	22 37 56	6,0)14,0
Long. W.	2 20 0	$2^h\ 20^m$
Approx. M.T.G. 6^d	0 57 56	
R.A.M.S. noon	$8^h\ 56^m\ 27^s$	
Accel. for 58^m	9	
Red. R.A.	8 56 36	
R.A.*	7 34 23	
M.T.S. 5^d	22 37 47	

or 6th $10^h\ 37^m\ 47^s$ A.M.

Care must be taken in dating the approx. M.T.S. If the hours, as in the example, exceed 12, the date is the day before the date in the question; if the hours are less than 12, the date is the same as the question. The passages are required for the civil day, and hence the $5^d\ 22^h$ (astro. time) corresponds to the 6th 10^h A.M. (civil time).

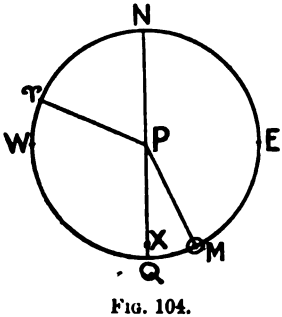


Fig. 104.

In Fig. 104, NEQW represents plane of equinoctial. NPQ is meridian, and the object X is always found between P and Q, as this represents our upper meridian. Distance to measure QX is the declination north or south. Find position of Υ 1st point of Aries by measuring to the westward the R.A. of star, which since the star is on the meridian represents the R.A.M. and Sidereal Time; then from Υ measure to the eastward the R.A. of Mean Sun, this gives M. Draw hour circles through Υ and M.

ΥWQ is R.A. of star and of meridian.
ΥQM is R.A.M.S.
QWEM is westerly hour angle of M is M.T.S.
$= 24^h + \text{ΥWQ} - \text{ΥQM}$.

2. Find the M.T.S. when Regulus is on the meridian in 94° E. on August 4th.

R.A. Regulus or R.A.M. =	$10^h\ 3^m\ 22^s$	94°
R.A.M.S.	8 48 34	4
Approx. M.T.S. 4^d	1 14 48	6,0)37,6
Long. E. −	6 16 0	$6^h\ 16^m$
,, M.T.G. 3^d	18 58 48	

R.A.M.S. noon	8h	44m	37s
Accel. for 18h		2	57
Accel. for 59m			10
Red. R.A.	8	47	44
R.A.*	10	3	22
M.T.S.	4d	1	15	38

or 4th 1h 15m 38s P.M.

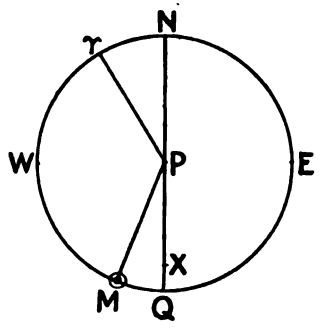


Fig. 105.

In Fig. 105, projected on plane of equinoctial, measure QW♈ = 10h, then ♈WM 8h 48m for M.

QM is M.T.S. when X is on meridian = ♈WQ − ♈WM.

3. Find the A.T.S. when Arcturus is on the meridian in 100° E. on September 2nd. Also by inspection.

R.A. Arcturus or R.A.M. =	14h	11m	22s
App. R.A. page I. or P.A.T.S.	10	42	44
Approx. A.T.S.	2d	3	28	38
Long. E. −		6	40	0
,, A.T.G.	1d	20	48	38
App. R.A. noon	10h	42m	44s
Cor.			29
Red. R.A.	10	42	15
R.A.*	14	11	22
A.T.S.	2d	3	29	7

or 2nd 3h 29m 7s P.M.

100°
4

6,0)40,0

6h 40m

H.V.
9·06
3·2

1812
2718

28·992

From Table XLIV., Norie, Mer. pass. Arcturus Sept. 1d 3h 26m
,, 6 3 9

Diff. in 5 days 17
,, 1 day 3
Hence by inspection 2nd 3 23
or 2nd 3h 23m P.M.

In Fig. 106

NEQW plane of equinoctical.
NPQ meridian.
X object. P pole. A true sun.
1st point of Aries. Draw hour circles as in Fig. 106.

♈WQ = R.A.* and R.A.M.
♈WM = R.A.T.S.
QM = Wly. H.A. of A.
= A.T.S.
= ♈WQ − ♈WM.

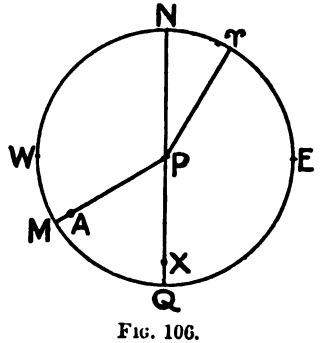


Fig. 106.

EXERCISES.

Find the ship time when the given star is on the observer's meridian from the following data:—

No.	Name of star.	Civil date.	Longitude.	Mean or App. Time.
1	α Andromedæ ...	August 6th ...	65° 0′ E.	App.
2	β Ceti	„ 8th ...	48 0 W.	Mean
3	α Arietis	„ 10th ...	137 0 E.	Mean
4	Achernar ...	„ 12th ...	172 0 W.	App.
5	Aldebaran ...	„ 14th ...	37 50 E.	Mean
6	Capella	„ 16th ...	14 0 W.	App.
7	Rigel	„ 18th ...	25 0 E.	Mean
8	Canopus	„ 20th ...	100 30 E.	App.
9	Betelgueuse ...	„ 22nd ...	150 25 W.	Mean
10	Regulus	„ 24th ...	29 45 E.	App.
11	Procyon	„ 31st ...	85 0 W.	Mean
12	Spica	Sept. 3rd ...	39 0 E.	App.
13	Antares	„ 10th ...	26 45 W.	Mean
14	Vega	„ 17th ...	13 45 E.	App.
15	Altair	„ 24th ...	175 45 W.	Mean

ART. 54.—**Bright Stars and Planets.**—Stars which may be considered as "bright," and suitable for Navigation purposes, are those of the first and second magnitudes. In the Table of "Mean Places of Stars" in the "Nautical Almanac," there are eighteen of the former and about twice as many of the latter, which gives the Navigator a choice of fifty-four objects suitable for observations, in addition to the Sun, Moon, and Planets.

It is not a work of much labour for the Student, by the help of a Star Atlas or a "Planisphere," to make himself familiar with most of the bright Stars and the Constellations to which they belong.

The *Planets* Venus, Mars, Jupiter, and Saturn are sometimes very conspicuous objects, and well adapted for observations, but they vary very much in brightness, according to their distance from the Earth. They may easily be recognized by their wanderings amongst the Stars, or by the times of their Meridian passage as given in the "Nautical Almanac."

With such an ample choice of objects, there is no excuse for the Navigator relying upon observations of the Sun only.

ART. 55.—**To find what Bright Stars are near the Meridian, or within a given Hour Angle at a Stated Time.**

Directions.—Find the S.T. corresponding to the given time, then look in the "Table of Mean Places" in the "Nautical Almanac" for the names of bright Stars whose Right Ascensions do not differ from the S.T. by more than the given Hour Angle.

N.B.—The star is E. of Mer. when its R.A. is *greater* than the S.T. (or R.A.M.), and W. when *less.*

It is N. of the observer when its Declination reads N. of the latitude, and S. when it reads S.

Remark.—As the object in view is to find what stars would be suitable for finding the latitude by Meridian and Ex-Meridian Altitudes, or for finding the Time and Azimuth, in *practice* it will be sufficient to take the Sun's R.A. at the *nearest* noon at Greenwich, since the error would not amount to 2 minutes in any case; but for *examination* purposes the R.A.M. must be exactly determined.

Examples.—

1. What bright stars will cross the meridian of a place long. 48° 30′ E., between the hours of 6 and 9 P.M. M.T.G. on September 30th.

G.M.T. Sept.	30^d	6^h	0^m	0^s	Sept. 30^d	9^h	0^m	0^s
Long. E.		3	14	0		3	14	0
M.T.S.	30^d	9	14	0		30^d 12	14	0
R.A.M.S.		12	33	17		12	33	17
					Accel. for 9^h		1	29
Accel. for 6^h				59				
R.A.M.		21	48	16	R.A.M.	24	48	46

Hence all stars whose R.A.'s fall between 21^h 48^m 16^s and 0^h 48^m 46^s will cross the meridian during these times. These stars are α Gruis, β Gruis, Fomalhaut, Alpheratz, and β Ceti.

2. Sept. 20^d 6^h 19^m M.T.G. in lat. 24° S., long. 125° 36′ E.; find what stars not less bright than 2·2 magnitude are within 2 hours E. and W. of the meridian, and above the horizon; state their meridian distances, and whether N. or S. of observer at transit.

M.T.G. Sept.	20^d	6^h	19^m	0^s	125° 36
Long. E.		8	22	24	4
M.T.S.	20^d	14	41	24	6,0)50,2 24
R.A.M.S. noon		11	53	52	
	6^h			59	8^h 22^m 24^s
Accel. for 19^m				3	
R.A.M.		2	36	18	

Hence limits of R.A. are from 0^h 36^m 18^s to 4^h 36^m 18^s.

Name of star.	Magnitude.	Meridian distance.	Bearing at transit.
β Ceti	2·2	1^h 57^m 26^s W.	N.
Achernar	0·5	1 2 5 W.	S.
γ' Andromedæ	2·2	0 38 11 W.	N.
α Arietis	2·2	0 34 26 W.	N.
α Persei	1·9	0 41 18 E.	N.
Aldebaran	1·1	1 54 14 E.	N.

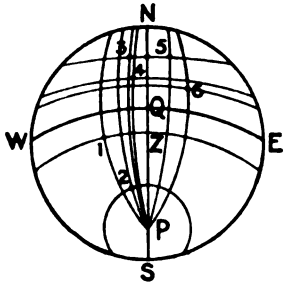


FIG. 107.

In Fig. 107, NESW is Ratl. horizon.
NZS is Meridian.
EQW is Equinoctial.
P pole ; Z zenith.

1, 2, 3, 4, 5, 6, the stars at the intersection of their respective hour circles and parallels of declination.

ZP1, ZP2, etc., are the hour angles or meridian distances of the stars.

EXERCISES.

From following data find names of stars which are visible, and state their hour angles and bearings at transit.

No.	Time.	Limit.	Mag.	Longitude.	Latitude.
1	August 1st $11^h\ 50^m$ P.M. M.T.S.	1^h E. and W.	2·6	48° 15′ W.	33° N.
2	„ 3rd 3^h A.M. A.T.S. ...	$1\frac{1}{2}$ E. and W.	2·0	56 45 E.	20 S.
3	„ 28th $3^h\ 40^m$ M.T.G. ...	1 E. and W.	1·0	72 30 E.	24 S.
4	Sept. $9^d\ 17^h\ 21^m\ 20^s$ A.T.G.	2 E. and W.	2·6	87 50 W.	5 N.
5	„ 19th 6^h A.M. M.T.S. ...	1 W. of Mer.	2·0	96 45 E.	40 N.
6	„ 29th $6^h\ 9^m$ A.M. A.T.S.	1 E. of Mer.	1·5	162 15 W.	24 S.

7. On May 27th at a place in long. 101° W., time shown by a sidereal clock was $13^h\ 27^m\ 38^s$, error of clock on local sidereal time being $2^h\ 17^m\ 47^s$. Find local mean time at the instant having given sidereal time at mean noon $4^h\ 20^m\ 15^s$.

8. Apparent R.A. Sun on page ii for mean noon is $20^h\ 57^m\ 47{\cdot}8^s$, and sidereal time on same page is $20^h\ 44^m\ 33{\cdot}2^s$. Find equation of time and show by diagram how it is to be applied to app. time.

9. Sun's app. R.A. on page i for app. noon was $14^h\ 35^m\ 28{\cdot}91^s$, and on page ii for mean noon was $14^h\ 35^m\ 31{\cdot}6^s$. Var. in R.A. for 1 hr. is $9{\cdot}883^s$. Find equation of time and show how it is to be applied to mean time.

CHAPTER XII.

CORRECTION OF ALTITUDES.

ART. 56.—The altitudes of all objects, whether observed at sea or on shore in an artificial horizon, must be corrected to reduce them to the true geocentric altitudes. The corrections employed are for index error, or correction; height of eye, or dip; semi-diameter of the object; refraction; and parallax. The method of applying them may be understood from the following remarks.

(*a*) *Index Error.*—This correction is due to an imperfection in the sextant used, by which the zero mark on the arc is placed out of position. As already explained, when the "0" on the vernier is in line with "0" on the arc, the index and horizon glasses should be parallel; if they (the zeros) are not in line an error is introduced; when the "0" on the vernier is to the left of the "0" on the arc (the glasses being parallel), it shows that the "0" on the arc is to the right of its true position, hence all angles will be in excess of their true value; the reading is "*on*" the arc and the error "*subtractive.*" The "0" on the arc being to the left of its true position makes all angles smaller than their true value; the reading is "*off*" the arc and the error "*additive.*"

The ind. err. is always applied according to the direction in the question.

(*b*) *Height of Eye.*—This correction allows for the depression or *dip* of the visible horizon below the sensible horizon; observed angles, being measured above the visible horizon, are too large, hence the correction is always *subtractive.* Its amount is found in Norie, Table v., or Raper, Table 30. Should a table not be handy, an approximate correction is found by taking the square root of the height of the eye in feet; this gives the "dip" in minutes of arc.

(*c*) *Semi-diameter.*—The altitude of an object's centre is the one required in the calculations, and in bodies which have a visible disc, like the sun and moon, it would be a matter of impossibility for an observer to say exactly where the centre is;

an observation of its limb or boundary is therefore taken. The limb nearer the horizon is called the lower limb, "*L.L.*"; an observation of this limb is less in altitude than that of the centre; the S.D. is therefore *added*. The limb further from the horizon is called the upper limb, "*U.L.*"; an observation of this limb gives too great an altitude, and the S.D. is *subtracted*.

The amount for the Sun is entered for noon of each day on page II., "Nautical Almanac."

The amount for the Moon is entered for noon and midnight of each day on page III., "Nautical Almanac."

The limb observed is always stated in the questions. When the corrections *a*, *b*, *c* are applied, the amount obtained is the *Apparent* altitude of the centre.

(*d*) *Refraction.*—Rays of light, in passing from a rarer to a denser medium, have their directions diverted towards the normal or perpendicular to the surface bounding the media. This is called the refraction or bending of the ray. Objects appear in the direction that the last portion of the rays enters the eye, and therefore, as the air is denser at the surface of the earth than higher up, the effect is to make all objects appear nearer the zenith than their true positions, hence the correction for *refraction* is always *subtractive*. Its amount is found in Norie, Table IV., or Raper, Table 31.

(*e*) *Parallax.*—Observations are of necessity made on the surface of the earth; these have to be reduced to the centre or rational horizon, which is below but parallel to the visible horizon; as all objects would appear higher in altitude at the centre than at the surface, the correction for *parallax* is always *additive*.

Its amount depends on the latitude of the observer, and the altitude and proximity of the object, and for the Sun is entered in Norie, Table VI., or Raper, Table 34.

In the case of the Moon the amount is entered in the "Nautical Almanac," page III., and must be corrected as before directed.

Refraction and parallax decrease as the altitude increases, and vanish when the object is in the zenith.

Norie, Table XVIII., gives the combined effect in the case of the Sun, and since Refraction is greater than Parallax, the amount found there is always subtractive.

Norie, Table XXX., and Raper, Table 39, give the amount of the Refraction and Parallax in altitude combined in the case of the Moon, and is additive, as the Parallax is more than the Refraction.

In practice the Refraction is taken out for the apparent altitude of the limb observed, that is, after the index error and dip have been applied.

When the Refraction and Parallax are applied to the apparent altitude, the *true* altitude is obtained.

Examples.—

1. September 23rd, the obs. alt. of the sun's L.L. was 46° 25′ 40″; height of eye, 24 feet; ind. cor. + 2′ 25″. Find the true alt.

Obs. alt. $\underline{\odot}$	46° 25′ 40″
Ind. cor.	+ 2 25
	46 28 5
Dip	− 4 48
	46 23 17
S.D.	+ 15 58
App. alt.	46 39 15
Ref.	− 54
	46 38 21
Par.	+ 6
True alt.	46 38 27

2. August 5th, the obs. alt. of the sun's U.L. was 14° 38′ 20″; height of eye, 32 feet; ind. cor. − 3′ 37″. Find the true alt.

Obs. alt. $\overline{\odot}$	14° 38′ 20″
Ind. cor.	− 3 37
	14 34 43
Dip	− 5 32
	14 29 11
S.D.	− 15 48
App. alt.	14 13 23
Ref.	− 3 38
	14 9 45
Par.	+ 9
True alt.	14 9 54

The stars appearing merely as points of light have no S.D., and, as they are so far distant, no appreciable parallax.

3. August 6th, the obs. alt. of the star α Trianguli Australis was 27° 26′ 10″ Find the true alt., ind. cor. − 5′ 27″, eye 41 feet.

Obs. alt.	27° 26′ 10″
Ind. cor.	− 5 27
	27 20 43
Dip	− 6 16
App. alt.	27 14 27
Ref.	− 1 50
True alt.	27 12 37

4. August 19th, in lat. 46° 25′ N., the obs. alt. of the Moon's L.L. was 42° 19′ 30″; ind. cor. + 3′ 11″; height of eye, 27 feet; M.T.G. 19^{d} 4^{h} 23^{m} 50^{s}. Find the true alt.

The Par. in alt. is obtained from formula—

Par. in alt. = H.P. × cos app. alt.
Or P.L. Par. in alt. = P.L. Hor. Par. + L. sec. app. alt.

S.D.		H.P.	
Noon 15′ 28″·8	4″·8	56′ 42″·9	17′·7
Cor. + 1·8	4·4	Cor. + 6·5	4·4
15 30·6	192	56 49·4	708
Tab. D. + 10·4	192	Tab. E. − 5·7	708
15 41	12)21·12	56 43·7 = 3403″·7	12)77·88
	1·8		6·5

Obs. alt. ☽	42° 19′ 30″		
Ind. cor.	+ 3 11		
	42 22 41		
Dip.	− 5 5		
	42 17 36		
S.D.	+ 15 41		
		3403·7 log = 3·531952	*Or* H P. P.L. = 5014
App. alt.	42 33 17	cos = 9·867250	sec = 1327
Ref.	− 1 2	2507 log = 3·399202	41′ 48″ P.L. = 6341
	42 32 15		
Par.	+ 41 47	6,0)250,7	
True alt.	43 14 2	41′ 47″	

Tab. XXX. 40′ 5″
Cor. for 43″ par. 32
,, 3′ alt. 8

Par. in alt. − ref. = cor. 40 45

5. August 9th, the obs. alt. of Jupiter's U.L. was 34° 8′ 50″; ind. cor. + 4′ 23″; height of eye, 25 feet.

Obs. alt.	34° 8′ 50″	
Ind. cor.	+ 4 23	
	34 13 13	
Dip	− 4 54	
	34 8 19	
S.D.	− 16	
App. alt.	34 8 3	H.P. = 1″·5; turn up Tab. XLVIII., which gives par. in alt. 1″.
Ref.	− 1 24	
	34 6 39	
Par.	+ 1	
True alt.	34 6 40	

ART. 57.—The *Artificial Horizon* is a plane horizontal mirror, or reflecting surface, used for taking altitudes when the sea horizon is obscured, or not visible. The liquid used is mercury, but treacle, oil, or tar will do as well, and they all depend on the same law in optics, viz. "The angles of incidence and reflection are equal." AB is a plane horizontal mirror, S a celestial object; its image, I, is seen by an eye placed at E as far perpendicularly below the surface as S is above it. Join EI, meeting AB in M, and join SM. The object S appears in the direction of the dotted line through E, but because of the immense distance S is off, this line and MS may be considered parallel. The angle FEI is measured with the sextant, while SMA is the altitude of S.

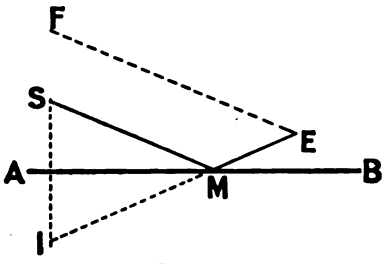


FIG. 108.

Now, observed angle FEI $= \pi -$ EMS
$=$ SMA + EMB
$=$ SMA + AMI
$=$ twice altitude of S.

Hence, when an altitude is taken with an artificial horizon the index correction is first applied, and then the result halved; moreover, as the horizon is a portion of, or parallel to, the sensible horizon, no correction for dip is necessary.

Example.—

August 12th, the obs. alt. of the sun's L.L. in artificial horizon was 103° 56′ 20″; ind. cor. − 4′ 18″; height of eye, 56 ft. Find the true alt.

Obs. alt. ⊙	103° 56′ 20″
Ind. cor.	− 4 18
	2)103 52 2
	51 56 1
S.D.	+ 15 49
	52 11 50
Ref.	− 44
	52 11 6
Par.	+ 5
True alt.	52 11 11

Exercises.

1. August 16th, the obs. alt. of the $\underline{\odot}$ was 50° 3′ 10″; ind. cor. − 4′ 19″; eye, 20 ft. Find true alt.

2. August 8th, the obs. alt. of $\overline{\odot}$ was 21° 13′ 20″; ind. cor. + 6′ 17″; eye, 15 ft. Find true alt.

3. September 1st, the obs. alt. of $\underline{\odot}$ was 19° 52′ 10″; ind. cor. − 3′ 25″; eye, 30 ft. Find true alt.

4. September 4th, the obs. alt. of $\underline{\odot}$ was 32° 24′ 30″; ind. cor. + 37″; eye, 17 ft. Find true alt.

5. August 6th, the obs. alt. of $\overline{\odot}$ was 47° 19′ 50″; ind. cor. + 4′ 29″; eye, 24 ft. Find true alt.

6. September 27th, the obs. alt. of $\underline{\odot}$ was 12° 17′ 20″; ind. cor. − 1′ 57″; eye, 50 ft. Find true alt.

7. August 13th, at $13^d\ 6^h\ 9^m\ 30^s$ M.T.G. the obs. alt. of Moon's L.L. was 55° 36′ 20″, in lat. 24° N.; ind. cor. + 2′ 41″; eye, 10 ft. Find true alt.

8. August 15th, $6^h\ 14^m$ A.M. M.T.S., in lat. 43° S., long. 45° E., the obs. alt. of Moon's U.L. was 15° 13′ 10″; ind. cor. − 3′ 17″; eye, 32 ft. Find true alt.

9. August 17th, the obs. alt. of star Rigel was 29° 17′ 20″; eye, 50 ft.; ind. cor. + 4′ 48″. Find true alt.

10. August 28th, the obs. alt. of star Altair was 47° 29′ 30″; eye, 35 ft.; ind. cor. − 3′ 26″. Find true alt.

11. August 12th, the obs. alt. of Planet Saturn in lat. 51° N. was 23° 14′; ind. cor. + 2′ 27″; eye, 36 ft. Find true alt.

12. August 8th, the obs. alt. of Planet Jupiter in lat. 42° S. was 53° 20′ 10″; ind. cor. − 37″; eye, 25 ft. Find true alt.

13. August 27th, the obs. alt. of Sun's L.L. in artificial horizon was 78° 14′ 20″; ind. cor. − 3′ 36″. Find true alt.

14. August 21st, the obs. alt. of Polaris in artificial horizon was 95° 37′ 40″; ind. cor. + 2′ 56″; eye, 27 ft. Find true alt.

15. August 20th, when the M.T.S. was $4^h\ 19^m\ 32^s$ P.M., in lat. 37° N., long. 147° 30′ W., the obs. alt. of Moon's L.L. in artificial horizon was 102° 21′ 30″; ind. cor. 0; eye, 100 ft. Find true alt.

CHAPTER XIII.

LATITUDE BY MERIDIAN ALTITUDE.

ART. 58.—Latitude is the angle at the earth's centre subtended by the arc of the meridian intercepted between the equator and the place. The arc of the declination circle intercepted between the observer's zenith and the equinoctial subtends the same angle; the latitude is therefore the declination of the observer's zenith, and may be determined easiest by meridian or ex-meridian observations.

The Altitude of the Pole is equal to the Latitude of the Observer.—Let ZSP_1N represent the celestial meridian of the observer O. Z is the zenith, QQ_1 the equinoctial. E is the centre of earth. Angle OEL or latitude is subtended by arc LO or arc QZ. P is the elevated pole, SN the horizon. PEN is altitude of pole.

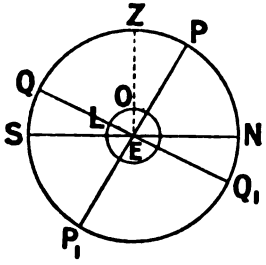


FIG. 109.

$$\text{Now, PEN} = 90° - \text{PEZ}$$
$$= \text{ZEQ}$$
$$\text{or altitude of pole} = \text{latitude.}$$

ART. 59.—*Proof of Latitude by Meridian Altitude.*—Let NESW represent the plane of the horizon produced to meet the celestial concave. NPZS the celestial meridian of the observer. P the elevated pole, Z his zenith, EQW the equinoctial. Let X_1, X_2, X_3, and X_4 be the positions of four celestial objects. Then QX_1, QX_2, QX_3 are the declinations of the first three, and PX_4 the polar distance of the fourth. Object 1 has south declination, the others north.

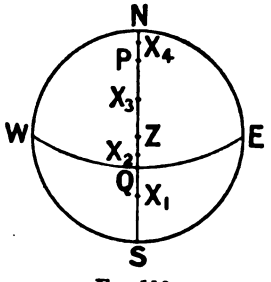


FIG. 110.

(1) $$\text{Lat.} = QZ = ZX_1 - QX_1$$
$$= \text{Mer. zen. dist.} - \text{dec.}$$

(2) $\text{Lat.} = QZ = ZX_2 + QX_2$
$= \text{Mer. zen. dist.} + \text{dec.}$

(3) $\text{Lat.} = QZ = QX_3 - ZX_3$
$= \text{Dec.} - \text{Mer. zen. dist.}$

From these the rules for finding lat. by mer. alt. may be deduced, viz. when dec. and zen. dist. are of *same* name the *sum* is the lat.; when dec. and zen. dist. are of *contrary* name the difference is the lat., which is of the name of the greater.

The zen. dist. is always of contrary name to the bearing.

(4) $\text{Lat.} = NP = PX_4 + NX_4$
$= \text{pol. dist.} + \text{alt.}$

Hence when an object is observed on the meridian between the pole and the horizon, *i.e. below* the pole, the lat. is the *sum* of pol. dist. and alt.

Therefore, to find lat. by mer. alt. the process is as follows:—

Object above the Pole.—

(1) Find Greenwich date, correct declination.

(2) Correct altitude, and subtract from 90° for zen. dist., which mark of contrary name to the bearing of the object.

(3) Dec. and zen. dist. of same name, add for lat.; dec. and zen. dist. of contrary name, subtract for lat., which is of same name as the greater.

Object below the Pole.—

(1) Find Greenwich date; in the case of the sun, the observation must be taken at *midnight*; in the case of the moon, use the *lower* mer. pass Correct declination, and subtract from 90° for pol. dist.

(2) Correct altitude.

(3) Add pol. dist. to alt. for lat., which must be of same name as dec.

Examples.—

1. August 17th, in long. 55° 45′ W. the obs. mer. alt. of the Sun's L.L. was 35° 10′ 30″ bearing north; ind. cor. − 4′ 50″; height of eye, 18 ft. Required the latitude.

55° 45′
4
6,0)22,3 0
3h 43m

A.T.S. August 17d	0h 0m 0s	Dec. 13° 38′ 32″·5 N.	H.V. 47″·59
Long. W.	3 43 0	Cor. 2 56	3·7
A.T.G. 17d	3 43 0	13 35 36·5 N.	33313
			14277
			6,0)17,6·083
			2′ 56″

Obs. alt. ⊙	35°	10′	30″	
Ind. cor.	−	4	50	
	35	5	40	
Dip.	−	4	9	
	35	1	31	
S.D.	+	15	50	
	35	17	21	
Ref.	−	1	20	
	35	16	1	
Par.			+ 7	
True alt.	35	16	8	N.
	90			
M.Z.D.	54	43	52	S.
Dec.	13	35	37	N.
Lat.	41	8	15	S.

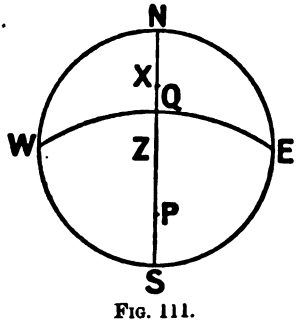


FIG. 111.

Construction.—Draw a circle NESW radius $\frac{9}{10}$ inch to represent rational horizon, centre point is the zenith Z. The vertical diameter is the ship's meridian on which the sun is situated (Fig. 111). Since the sun bears north, measure 35° to scale from the north point for position X of sun. His declination is 14° N., from X measure to scale 14° to the south (contrary name to declination) for the point Q where the equinoctial cuts the meridian. E and W are in the line through Z at right angles to the meridian. Through EQW draw an arc of circle for the equinoctial, the centre is somewhere on the meridian produced, and also on the perpendicular bisecting the line joining EQ, and so is at their intersection. Measure 90° from Q for the position of the elevated pole P. The centres for circles of declination passing through objects whose declination is of the same name as the latitude lie between the centre for the equinoctial and P, the greater the declination, the nearer P is the centre; when declination is of contrary name to latitude, the centre is beyond this point.

Explanation.—NESW represents Rational Horizon.
NZS ,, Meridian.
EQW ,, Equinoctial.
Z, zenith; P, pole; X, object.
QX is Dec.; NX is Mer. Alt.; ZX is zen. dist.

Then Lat. = ZQ
= ZX − QX.

2. August 26th, in long. 87° W., the obs. mer. alt. of the Sun's U.L. bearing south was 56° 30′ 30″; ind. cor., + 3′ 20″; eye, 16 ft. Required the lat.

87°
4

6,0)34,8

5h 48m

A.T.S. August 26d 0h 0m 0s
Long. 5 48 0

A.T.G. 26d 5 48 0

Dec. 10° 39′ 5″ N.
Cor. 5 1

10 34 4 N.

H.V. 51″·92
5·8

41536
25960

6,0)30,1·136

5′ 1″

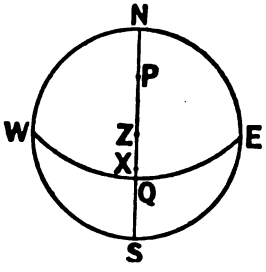


Fig. 112.

Obs. alt. ⊙ 56° 30′ 30″
Ind. cor. + 3 20

56 33 50
Dip. − 3 55

56 29 55
S.D. − 15 51

56 14 4
Ref. − 38

56 13 26
Par. + 5

True alt. 56 13 31 S.
90

M.Z.D. 33 46 29 N.
Dec. 10 34 4 N.

Lat. 44 20 33 N.

Construction (Fig. 112).—Draw NESW and NZS, as before. For X measure 56° from S, since sun bears south, and from X measure XQ to the south 10°. Draw EQW ; put in P as explained.

Explanation.—NESW represents Rational Horizon.
NZS ,, Meridian.
EQW ,, Equinoctial.
Z, zenith ; P, pole ; X, object.
QX is Dec. ; SX is Mer. Alt. ; ZX is zen. dist.

Then Lat. = ZQ
= ZX + QX

3. August 1st, in long. 49° E., the obs. mer. alt. of the sun's U.L. zenith south of sun was 76° 54′ 40″ ; eye, 25 ft. ; ind. cor. − 5′ 10″. Required the lat.

49°
4

6,0)19,6

3h 16m

A.T.S. August 1d 0h 0m 0s
Long. 3 16 0

A.T.G. July 31d 20 44 0

Dec. 18° 11′ 45″ N.
Cor. 2 3

18 13 48 N.

H.V. 37″·26
3·3

11178
11178

6,0)12,2·958

2′ 3″

Obs. alt. $\overline{\odot}$	76°	54′	40″	
Ind. cor.		− 5	10	
	76	49	30	
Dip.		− 4	54	
	76	44	36	
S.D.		− 15	47	
	76	28	49	
Ref.			− 14	
	76	28	35	
Par.			+ 2	
True alt.	76	28	37	
	90			
M.Z.D.	13	31	23	S.
Dec.	18	13	48	N.
Lat.	4	42	25	N.

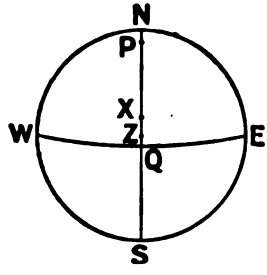


FIG. 113.

Construction (Fig. 113).—Draw NESW and NZS as before.

Since zenith is south of sun, measure 77° from N to X, as sun must bear north. Complete figure.

Explanation.—As given to Fig. 111.

Then Lat. = ZQ
= QX − ZX.

EXERCISES.

1. August 3rd, in long. 51° 45′ W., the obs. mer. alt. of the Sun's L.L. bearing north was 34° 15′ 30″; ind. cor. − 3′ 31″; eye, 16 ft. Find the lat.

2. August 5th, in long. 33° 30′ W., the obs. mer. alt. of the Sun's L.L., zenith north of sun was 50° 1′ 20″; ind. cor. + 4′ 14″; eye, 18 ft. Find the lat.

3. August 7th, in long. 60° E., the obs. mer. alt. of the Sun's L.L. bearing north was 19° 32′ 10″; ind. cor. + 2′ 37″; eye, 20 ft. Find the lat.

4. August 9th, in long. 97° 30′ E., the obs. mer. alt. of the Sun's U.L. zenith south of sun was 69° 47′ 40″; ind. cor. + 1′ 10″; eye, 17 ft. Find the lat.

5. August 11th, in long. 20° 15′ W., the obs. mer. alt. of the Sun's U.L. bearing north was 64° 4′ 50″; ind. cor. − 1′ 43″; eye, 19 ft. Find the lat.

6. August 13th, in long. 70° 30′ E., the obs. mer. alt. of the Sun's U.L. zenith south of sun was 50° 6′ 10″; ind. cor. − 1′ 56″; eye, 21 ft. Find the lat.

7. August 15th, in long. 10° E., the obs. mer. alt. of the Sun's L.L. bearing north was 78° 36′ 30″; ind. cor. − 2′ 59″; eye, 25 ft. Find the lat.

Under the following conditions find the latitude by mer. alt. of the Sun:—

	Date.	Longitude.	Mer. alt.	Limb.	Bearing.	Ind. cor.	Ht. of eye.
8	Aug. 17th	12° 0′ W.	80° 12′ 0″	U.	N.	−3′ 42″	27 feet.
9	„ 19th	180 0 W.	52 35 10	L.	S.	−6 25	29 „
10	Sept. 30th	57 19 E.	89 4 20	U.	S.	+5 38	30 „
11	„ 23rd	48 45 W.	48 57 50	L.	S.	+3 20	32 „
12	„ 23rd	97 30 W.	43 22 20	L.	N.	−4 17	34 „
13	„ 27th	13 25 E.	58 44 40	U.	S.	−2 34	22 „
14	„ 29th	9 14 W.	36 36 30	U.	N.	−2 41	45 „
15	„ 24th	165 0 E.	67 14 10	L.	S.	−3 58	36 „
16	„ 23rd	100 40 E.	52 19 30	U.	S.	+2 15	34 „
17	„ 24th	24 50 W.	22 28 20	L.	S.	−1 52	33 „
18	„ 24th	88 20 E.	30 31 0	U.	N.	+3 29	49 „

ART. 60.—Latitude by Meridian Altitude of a Star.

Example.—August 5th, the obs. mer. alt. of the star Canopus (α Argûs) bearing south was 54° 17′ 30″; ind. cor., −6′ 10″; eye, 24 ft Required the lat.

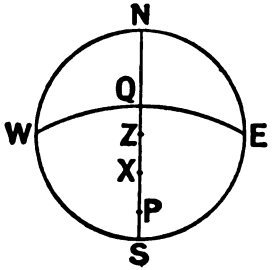


FIG. 114.

Given
Mer. alt. = SX
Dec. = QX
Req. lat. = ZQ
= QX − ZX
= QX − (90 − SX)

Obs. alt.	54°	17′	30″
Ind cor.		− 6	10
	54	11	20
Dip.		− 4	48
	54	6	32
Ref.		−	41
True alt.	54	5	51 S.
	90		
M.Z.D.	35	54	9 N.
Dec.	52	38	39 S.
Lat.	16	44	30 S.

EXERCISES.

1. The observed Mer. Alt. of Capella was 54° 29′ 15″ north of the observer; index error, − 3′ 10″; height of eye, 20 ft. Required the latitude.

2. The observed Mer. Alt. of Rigel was 84° 54′ 20″ north of the observer; index error, + 1′ 10″; height of eye, 21 ft. Required the latitude.

3. The observed Mer. Alt. of Sirius, zenith north of star, was 45° 30′ 30″; index error, + 1′ 30″; height of eye, 23 ft. Required the latitude.

4. The observed Mer. Alt. of Spica bearing south of observer was 36° 25′ 20″; index error, — 3′ 10″; height of eye, 24 ft. Required the latitude.

5. The observed Mer. Alt. of Fomalhaut bearing south of the observer was 10° 19′ 30″; height of eye, 25 ft. Required the latitude.

6. The Mer. Alt. of Aldebaran bearing south was 56° 23′; index correction, + 1′ 19″; height of eye, 28 ft. Required the latitude.

7. The Mer. Alt. of Spica in an Artificial Horizon was 86° 47′ 20″; index correction, — 2′ 18″; zenith north of the star. Find the latitude.

8. The Mer. Alt. of Markab bearing north was 30° 59′ 10″; index correction, + 4′ 2″; height of eye, 33 ft. Find the latitude.

9. The Mer. Alt. of α Arietis bearing south was 75° 18′ 40″; index correction, — 4′ 9″; height of eye, 19 ft. Required the latitude.

10. The Mer. Alt. of Algenib in an Artificial Horizon was 111° 57′ 10″; index correction, + 8′ 6″; zenith south of the star. Find the latitude.

11. The Mer. Alt. of Regulus in an Artificial Horizon was 114° 20′ 30″; index correction, — 5′ 16″; zenith north of the star. Find the latitude.

ART. 61. **Latitude by Meridian Altitude of the Moon.**—It is first necessary to find the M.T.G. when the Moon is on the observer's meridian, and then reduce the "Elements" as before directed.

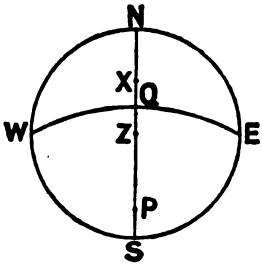
Example.—August 16th, in long. 126° 5′ E., the obs. mer. alt. of the Moon's L.L. bearing north was 42° 26′ 20″; ind. cor. — 3′ 30″; height of eye, 16 ft. Find the latitude.

The time of mer. pass. on the 16th is $21^h\ 55^m{\cdot}3$, which is really about 10 o'clock on the morning of the 17th, hence we use the passage for the 15th, and since we are in E. long. work from that day towards the 14th, as the transit occurs earlier.

Mer. pass.	$15^d\ 21^h\ 4^m{\cdot}9$	$49^m{\cdot}2$ (for 24^h)	126° 5′
Cor.	$-\ 17{\cdot}2$	$8{\cdot}4$	4
M.T.S.	$15^d\ 20\ 47{\cdot}7$	1968	6,0)50,4 20
Long.	$8\ 24{\cdot}3$	3936	$8^h\ 24^m{\cdot}3$
M.T.G.	$15^d\ 12\ 23{\cdot}4$	(4)413·28	
		(6)103·32	
		17·2	

	S.D.			H.P.		
Midngt.	14′ 57″·80	3·58		54′ 49″·29	13·1	Alt. 42°
Cor.	·12	·4		·44	·4	M.Z.D. 48 S.
	14 57·92	12)1·432		54 49·73	12)5·24	Dec. 20 N.
Aug. D.	+ 9·8	·12	Red. E.	— 2·4	·44	Approx. lat. 28 S.
	15 7·72			54 47·33		

Obs. alt. ☽	42° 26′ 20″	Dec.	20° 11′ 20″·2 N.	Var.	1·15
Ind. cor.	− 3 30	Cor.	+ 26·8		23·4
	42 22 50		20 11 47 N.		460
Dip.	− 3 55				345
					230
	42 18 55				
S.D.	+ 15 8				26·91C
App. alt.	42 34 3	Cos.	... 9·867167	Sec	... 1328
Ref.	− 1 2		3287 log = 3·516800		H.P. P.L. 5166
	42 33 1		2421 log = 3·383967		40′ 21″ P.L. 6494
Par.	+ 40 21				
True alt.	43 13 22 N.				
	90				
M.Z.D.	46 46 38 S.				
Dec.	20 11 47 N.				
Lat.	26 34 51 S.				



Given
Mer. alt. = NX
Dec. = QX
Req. lat. = ZQ
= ZX − QX
= 90° − NX − QX

FIG. 115.

EXERCISES.

1. August 21st, in long. 48° 30′ W., the obs. mer. alt. of the Moon's U.L. bearing north was 34° 17′ 20″; ind. cor. + 2′ 15″; eye, 16 ft. Find the lat.

2. August 22nd, in long. 97° 25′ W., the obs. mer. alt. of the Moon's L.L. bearing south was 53° 26′ 10″; ind. cor. − 3′ 20″; eye, 17 ft. Find the lat.

3. August 30th, in long. 62° 15′ E., the obs. mer. alt. of the Moon's U.L. zenith south of Moon was 61° 34′ 40″; ind. cor. + 5′ 46″; eye, 18 ft. Find the lat.

4. August 31st, in long. 57° 45′ E., the obs. mer. alt. of the Moon's L.L. zenith north of Moon was 24° 46′ 50″; ind. cor. − 4′ 27″; eye, 20 ft. Find the lat.

5. August 31st, in long. 120° W., the obs. mer. alt. of the Moon's L.L. zenith north of Moon was 68° 57′ 30″; ind. cor. + 3′ 33″; eye, 23 ft. Find the lat.

Under the following conditions find the latitude by mer. alt. of the Moon:—

	Date.	Longitude.	Mer. alt.	Limb.	Bearing.	Ind. cor.	Ht. of eye.
6	Aug. 4th	178° 30′ W.	38° 22′ 20″	L.	N.	−1′ 12″	34 feet.
7	„ 10th	21 30 E.	42 39 20	L.	S.	−2 19	36 „
8	„ 11th	94 15 E.	46 17 10	L.	S.	+3 5	43 „
9	„ 12th	127 30 E.	54 29 40	U.	N.	−2 8	46 „
10	„ 14th	139 45 W.	75 16 40	L.	N.	−4 57	49 „
11	„ 17th	158 24 E.	73 56 10	L.	N.	+2 35	54 „
12	„ 25th	88 35 E.	84 24 30	L.	S.	+1 24	56 „

ART. 62. Latitude by Meridian Altitude of a Planet.

Example.—August 9th, in long. 90° W., the obs. mer. alt. of the Planet Jupiter bearing north was 42° 10′ 20″; ind. cor. + 4′ 15″; eye, 27 ft. Find lat.

As before explained, the G.M.T. of the Planet's mer. pass. across the observer's meridian must be first found, in a similar way to the Moon's.

Mer. pass. August 8^d 21^h 0^m 4
Ret. − ·8
M.T.S. 8^d 20 59·6
Long. + 6
M.T.G. 9^d 2 59·6

Ret. in 24 hrs. 3^m·1
,, 6 ,, $\frac{3{\cdot}1 \times 6}{24}$ = ·8

Dec. 23° 7′ 37″·9 N.
Cor. − ·8
23 7 37 N.

Diff. for 24 hours 6″·6
3
24)19·8
·8

90°
4
6,0)36,0
6^h

Obs. alt. 42° 10′ 20″ N.
Ind. cor. + 4 15
42 14 35
Dip. − 5 5
App. alt. 42 9 30
Ref. − 1 3
42 8 27
Par. + 1
True alt. 42 8 28
90
M.Z.D. 47 51 32 S.
Dec. 23 7 37 N.
Lat. 24 43 55 S.

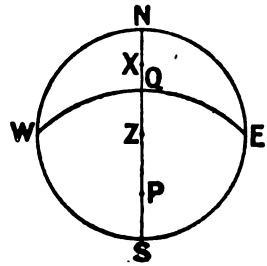


FIG. 116.

Given
Mer. alt. = NX
Dec. = QX
Req. lat. = ZQ
= ZX − QX
= 90 − NX − QX

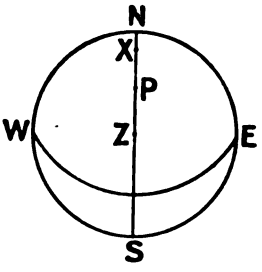
ART. 63. Latitude by Meridian Altitude of an Object below the Pole.

—An object whose Polar Distance is less than the elevation of the Pole above the horizon will never set, and will therefore be seen on the meridian *below* the Pole as well as *above* the Pole: when it has been proved that the lat. is found by adding together the Mer. Alt. and Pol. Dist.

Example.—August 10th, the obs. mer. alt. below the Pole in artificial horizon of the star α Ursæ Majoris was 33° 1′ 10″; ind. cor. + 3′ 26″. Find the latitude.

Obs. alt.	33°	1′	10″
Ind. cor.		+ 3	26
	2)33	4	36
	16	32	18
Ref.		− 3	10
True alt.	16	29	8
P.D.	27	44	29
Lat.	44	13	37 N.

Dec. 62° 15′ 31″ N.
90
P.D. 27 44 29



Given
Mer. alt. = NX
Pol.dist. = PX
Req. lat. = PN
= NX + PX

FIG. 117.

EXERCISES.

1. August 11th, in long. 48° 30′ W., the obs. mer. alt. of the Planet Saturn bearing north was 55° 16′ 40″; ind. cor. − 3′ 27″; eye, 30 ft. Find the lat.

2. September 19th, the obs. mer. alt. below the pole in artificial horizon of the star α' Crucis was 42° 8′ 50″; ind. cor. − 4′ 36″. Find the lat.

3. August 14th, the obs. mer. alt. below the pole in artificial horizon of the star α Cygni was 18° 11′ 20″; ind. cor. + 5′ 42″. Find the lat.

4. August 29th, in long. 135° 45′ E., the obs. mer. alt. of the Moon's L.L. under the south pole was 13° 27′ 50″, taken in artificial horizon; ind. cor. + 3′ 24″. Find the lat.

5. August 1st, in long. 94° 25′ W., the obs. mer. alt. of the Sun's U.L. under the north pole in artificial horizon was 12° 36′ 40″; ind. cor. − 2′ 50″. Find the lat.

6. September 9th, the obs. mer. alt. below the pole of β Centauri was 20° 47′ 10″; ind. cor. + 45″; height of eye, 37 ft. Find the lat.

ART. 64.—To find the Approximate Meridian Altitude of a Star, for setting the Sextant preparatory to observing the Meridian Altitude.

Referring to the Rule for finding the Latitude by a Meridian Altitude (Art. 59), it was shown that the Latitude is equal to the *sum* of the Meridian Zenith Distance and Declination when they are of the *same* name, and the *difference* when of *different* names.

Therefore it follows that the Meridian Zenith Distance is the *difference* of the Lat. and Dec. when they are of the *same* name, and the *sum* when of different names.

The Meridian Zenith distance subtracted from 90° gives the *Meridian Altitude.*

Examples.—

1. Given lat. 25° 30′ N., and a Star's Dec. 52° 20′ N., required the Mer. Alt. of the Star.

Lat. 25° 30′ N.
Dec. 52 20 N. } Same name—subtract.

Mer. Zen. Dist. 26 50 S.
90 0

Mer. Alt. 63 10 North of observer.

Draw NESW and NZS as before. Lat. being north measure ZQ 25° to south for Q, draw EQW, and set off QX 52° to north for X (Fig. 118).

Lat. = ZQ
Dec. = QX, zen. dist. = ZX.
Then Mer. Alt. = NX = 90 − ZX
= 90 − (QX − ZQ).

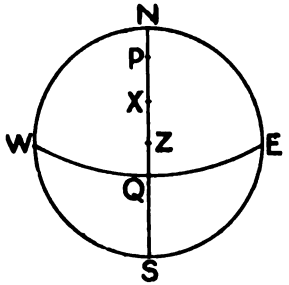


Fig. 118.

2. Suppose the latitude to be 24° 25′ S., and the Star's Dec. 32° 10′ N., required its Mer. Alt.

Lat. 24° 25′ S.
Dec. 32 10 N. } Different names—add.

Mer. Zen. Dist 56 35 S.
90 0

Mer. Alt. 33 25 North of observer.

In Fig. 119, lat. being south, measure ZQ = 24° to north for Q;
make QX = 32° to north for X.
Lat. = ZQ
Dec. = QX, zen. dist. = ZX.
Then Mer. Alt. = NX = 90 − ZX
= 90 − (QX + ZQ).

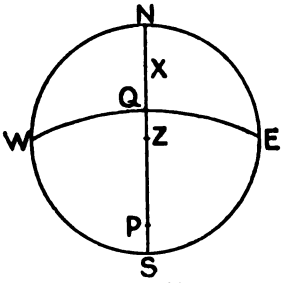


Fig. 119.

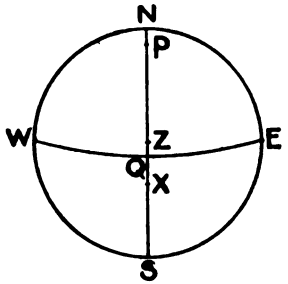


Fig. 120.

3. Given latitude of Ship 10° 15′ N., what is the Mer. Alt. of a Star whose Dec. is 20° 40′ S., and how does it bear from observer?

Lat. 10° 15′ N.
Dec. 20 40 S. } Different names—add.

Mer. Zen. Dist. 30 55 N.
90 0

Mer. Alt. 59 5 N. South of observer.

Here Fig. 120. Mer. Alt. = SX
= 90 − ZX
= 90 − (ZQ + QX).

N.B.—A Star is north or south of observer according as its Declination reads north or south of the latitude of observer.

If the corrections for Ref., Dip., etc., are applied to the true alt. to make an obs. alt., they must all be reckoned contrary to the manner indicated in the examples on the correction of altitudes.

Exercises.

Find the Meridian Altitudes of Stars from the following data, and state whether N. or S. of observer :—

No.	Latitude.	Name of star.	Declination.
	° ′		° ′
1	14 40 N.	α Draconis	64 52 N.
2	28 12 S.	α Bootis	19 42 N.
3	44 10 N.	α Scorpii	26 12 S.
4	14 25 S.	β Gruis	47 25 S.
5	52 19 S.	α Piscis Aust.	30 9 S.
6	0 0	Achernar	57 45 S.
7	0 0	β Geminorum	28 16 N.
8	35 28 N.	Sirius	16 35 S.
9	49 50 N.	Polaris	88 46 N.
10	3 56 S.	α Ursæ Majoris	61 3 N.

CHAPTER XIV.

LATITUDE BY EX-MERIDIAN ALTITUDE.

ART. 65.—Although the meridian altitude is the simplest method by which latitude may be determined, clouds or other causes may prevent an observation at the particular instant required; under these circumstances, the latitude is deduced from observations of the altitude taken when the object is *near* the meridian, that is, when its hour angle or meridian distance is small. There are two methods in use:—

(1) By calculating a correction which, when applied to the altitude out of the meridian, determines the object's altitude or zenith distance when on the meridian. This quantity is called a "Reduction," and the latitude by account is always used to determine its amount. The correctness of the result, therefore, depends on the accuracy with which the D.R. lat. is known.

(2) By direct spherics.

There are several variations of the first method, and the one we shall employ may be called the "log-rising" formula, which, according to Chauvenet, approximates more rapidly than the others.

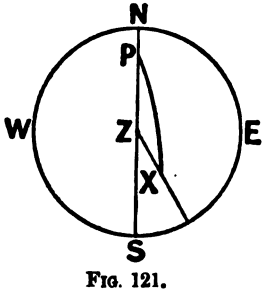


FIG. 121.

In figure, let X be object near the meridian; then in triangle PZX PX is Polar Distance, PZ colatitude, ZX zenith distance at time of observation, P the hour angle.

From the fundamental equation.

$$\text{Cos ZX} = \cos \text{PZ} \cdot \cos \text{PX} + \sin \text{PZ} \cdot \sin \text{PX} \cdot \cos \text{P}.$$

Let "a" denote the altitude, "l" the latitude, "h" the hour angle, and "d" the declination.

Then above becomes—

$$\begin{aligned}\text{Sin } a &= \sin l \cdot \sin d + \cos l \cdot \cos d \cdot \cos h \\ &= \sin l \cdot \sin d + \cos l \cdot \cos d - \cos l \cdot \cos d \left(2 \cdot \sin^2 \frac{h}{2}\right) \\ &= \cos (l \sim d) - \cos l \cdot \cos d \left(2 \cdot \sin^2 \frac{h}{2}\right)\end{aligned}$$

$$\therefore \cos (l \sim d) = \sin a + \cos l \cdot \cos d \left(2 \cdot \sin^2 \frac{h}{2}\right)$$

The quantity $\log\left(2 . \sin^2 \frac{h}{2}\right)$ is called the "log rising," and is tabulated in Norie, Table XXIX.

In practice the natural number corresponding to the second term on the right-hand side is found by adding together the L. cos. lat., L. cos dec., and log-rising hour angle; this number is added to the nat. sin of altitude; the nat. cos of sum is the zen. dist. when on the meridian; the lat. is then found as in meridian altitude.

ART. 66. **Latitude by Ex-meridian Altitude. Direct Method.**—Given the hour angle (h), the declination (δ), and the true altitude (a), to find the latitude.

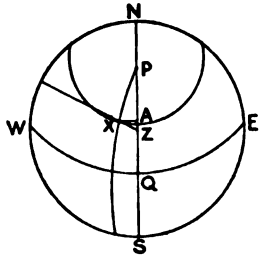


FIG. 122.

In the figure X is the object, PX the polar distance, ZX the zenith distance, the angle ZPX(h) the hour angle, and XA an arc of a great circle at right angles to the meridian, EQW the equator.

Then QZ is the latitude, and is equal to the sum or difference of the arcs ZA and QA.

In the right-angled spherical triangle PXA—

$$\begin{aligned} \cos h &= \cot p \,.\, \tan \mathrm{PA} \\ &= \tan \delta \,.\, \tan \mathrm{PA} \\ \therefore \tan \mathrm{PA} &= \cos h \,.\, \cot \delta \end{aligned}$$

But QA (arc 1) is the complement of PA, because PQ = 90°;

$$\therefore \text{cot arc 1} = \cos h \,.\, \cot \delta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, in the same triangle—

$$\begin{aligned} \cos p &= \cos \mathrm{PA} \,.\, \cos \mathrm{XA} \\ i.e.\ \sin \delta &= \sin \text{arc 1} \,.\, \cos \mathrm{XA} \\ \therefore \cos \mathrm{XA} &= \sin \delta \,.\, \text{cosec arc 1} \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

Also in the right-angled triangle XZA—

$$\begin{aligned} \cos \mathrm{XZ} &= \cos \mathrm{XA} \,.\, \cos \mathrm{ZA} \\ i.e.\ \sin a &= \sin \delta \,.\, \text{cosec arc 1} \,.\, \cos \mathrm{ZA} \end{aligned}$$

by substituting the value of cos XA in (2)—

$$\begin{aligned} \therefore \cos \mathrm{ZA} \text{ (arc 2)} &= \frac{\sin a}{\sin \delta \,.\, \text{cosec arc 1}} \\ \text{or cos arc 2} &= \sin a \,.\, \text{cosec } \delta \,.\, \sin \text{arc 1} \quad . \quad . \quad . \quad (3) \end{aligned}$$

Arc 1 is named the same as the dec., and arc 2 the same as the zenith distance, and the sum or difference, according as they

are of the same or different names, gives the latitude as in the meridian altitude problem.

For computation the formulæ may be arranged as follows:—

h — — —	...	cos		
δ — — —	...	cot	...	cosec
arc 1 — — —	...	cot	...	sine
a — — —	...	a — — —	...	sine
arc 2 — — —				cos
lat. — — —				

N.B.—When X is on the equinoctial (*i.e.* dec. = 0), then the perpendicular XA coincides with the equinoctial EQW and arc 1 disappears.

The lat. QZ is then found from the right-angled triangle ZXQ, in which QX measures the H.A., and ZX = co-alt. or zenith distance.

$$\text{Then } \cos ZX = \cos QX \times \cos QZ$$
$$\therefore \cos QZ = \frac{\cos ZX}{\cos QX}$$
$$= \cos ZX \times \sec QX$$
$$\textit{i.e. } \cos \text{lat.} = \sin \text{alt.} \times \sec \text{H.A.}$$

ART. 67.—**Latitude by Ex-meridian Altitude of the Sun, Star, etc.**—The hour angle of the Sun is the time from apparent noon.

The hour angle of any other celestial object is the difference between the R.A.M. and R.A. of object.

To allow for *Run* since determining error, it must be noticed that when the course has been to the eastward the ship is going to a place where the time is later; when to the westward, where time is earlier. Hence the diff. long. in time must be *added* for *easterly*, and *subtracted* for *westerly* run.

The process is as follows:—

(1) Find Greenwich time and hour angle.

(2) Correct declination and altitude.

(3) *a.* **Log-rising Method.**—Add L. cos lat., L. cos dec., and log-rising hour angle (Norie, Table XXIX.), increasing the index there found by "1"; reject 10's from sum and find natural number in Table XXIV.

Add this nat. no. to nat. sin true alt. in Table XXVI., and find in this same table nat. cos of sum; this is the zen. dist., which mark of contrary name to bearing. The sum or difference of zen. dist. and dec. according as they are of same or different names gives the latitude.

b. **Direct Method.**—Add L. cos hour angle, and L. cot dec., take back L. cot of result, which call arc 1, and name it the same as the dec.

Then add L. cosec dec., L. sin arc 1, and L. sin alt., reject 10's, and take back L. cos of result for arc 2, which name contrary to the bearing.

The sum of arcs 1 and 2 when they are of the same name, or difference when they are of contrary names, gives the lat., which is of same name as the greater.

ART. 68.—Latitude by Ex-meridian Altitude of Sun.

Example.—August 7th A.M. at ship, in lat. by account 33° 30′ N., long. 10° 57′ W., the obs. alt. of the Sun's U.L. was 72° 52′ 50″; ind. cor. + 3′ 4″; eye, 16 ft.; time by watch, 10ʰ 57ᵐ 58ˢ, which was slow of A.T.S. 44ᵐ 10ˢ; the d. long. made to the W. since the error was determined being 17¾ miles. Find the latitude by ex-meridian; and supposing the course and distance to be S. by W. 4 miles since observation, find the latitude at noon.

			Dec.	H V.
Time by watch, Aug.	6ᵈ 22ʰ 57ᵐ 58ˢ	long. 10° 57′	16° 37′ 8″·9 N.	41·50
Slow	+ 44 10	4 Cor.	− 16·6	·4
	6ᵈ 23 42 8	43ᵐ 48ˢ	16 36 52·3	16·600
Run. W.	− 1 11			
A.T.S.	6ᵈ 23 40 57		Obs. alt.	72° 52′ 50″
Long. W.	+ 43 48		Ind. cor.	+ 3 4
A.T.G.	7ᵈ 0 24 45			72 55 54
			Dip.	− 3 55
				72 51 59
			S.D.	− 15 48
				72 36 11
			R. & P.	− 14
			True alt.	72 35 57

First Method.

E.H.A. 19ᵐ 3ˢ log ris. = 3·538380
Lat. 33° 30′ 0″ cos = 9·921107
Dec. 16 36 52 cos = 9·981479
2760 log = 3·440966
T.A. 72° 35′ 57″ nat. sin 954236
956996 N. cos. = 16° 51′ 51″ N.
Dec. = 16 36 52 N.
Obs. lat. = 33 28 43 N.

Second Method.

E.H.A. 19ᵐ 3ˢ	cos = 9·998498		
Dec. 16° 36′ 52″	cot = 10·525219	cosec = 10·543740	
Arc 1 16 40 8 N.	cot = 10·523717	sin = 9·457640	
	T.A. 72° 35′ 57″	sin = 9·979656	
Arc 2 16 48 33 N.	... ...	cos = 9·981036	

Obs. lat. 33 28 41 N.
Run 3 54 S.
Noon lat. 33 24 47 N.

(Run S. by W. 4′ gives d. lat. 3′·9 to be applied to the S., as the obs. is taken before noon.)

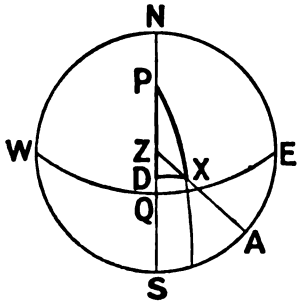


FIG. 123.

Third Method. Directly from figure.

X is position of object, AX its altitude, PX the pol. dist, P the hour angle, XD a perpendicular from X on the meridian.

In right-angled spherical triangle PDX, D = 90°, P the hour angle, PX the pol. dist. Find PD and DX.

$$\sin(\text{co. P}) = \tan PD \,.\, \tan(\text{co. PX})$$
$$\therefore \tan PD = \cos P \,.\, \tan PX \quad . \quad . \quad . \quad \text{I.}$$

Again—

$$\sin DX = \cos(\text{co. P}) \,.\, \cos(\text{co. PX})$$
$$= \sin P \,.\, \sin PX \quad . \quad . \quad . \quad \text{II.}$$

P = 19ᵐ 3ˢ cos =	9·998498	sin =	8·919209
PX = 73° 23′ 8″ tan =	10·525219	sin =	9·981479
PD = 73 19 52 tan =	10·523717	DX = 4° 33′ 48″ sin =	8·900688

Then in right-angled triangle ZDX, given D = 90°, ZX zen. dist., and DX, find ZD

$$\text{Sin(co. ZX)} = \cos ZD \,.\, \cos DX$$
$$\therefore \cos ZD = \cos ZX \,.\, \sec DX$$

DX =	4° 33′ 48″	sec =	10·001379
ZX =	17 24 3	cos =	9·979656
ZD =	16 48 34	cos =	9·981035

$$\text{Lat.} = ZQ = 90 - PZ = 90 - (PD - DZ)$$
$$= 90 - (73° 19′ 52 - 16° 48′ 34″)$$
$$= 90 - 56\ 31\ 18$$
$$= 33° 28′ 42″$$

EXERCISES.

1. August 1st P.M. at ship, in lat. by account 37° 20′ S., long. 133° E., the obs. alt. of the Sun's L.L., bearing north, was 33° 55′ 20″; ind. cor. + 2′ 5″; eye, 30 ft.; time by watch, 3ʰ 33ᵐ 1ˢ, which was found 8ʰ 50ᵐ slow of A.T.S. the same morning; run since error was determined S. 38° E., 23 miles. Find the latitude by ex-meridian, and if the run since noon was 3 miles on the same course, find lat. at noon.

2. August 6th A.M. at ship, in lat. by account 26° 20′ N., long. 80° 45′ W., the obs. alt. of the Sun's U.L., bearing south, was 78° 9′ 20″; ind. cor. − 3′ 16″; eye, 19 ft.; time by watch, 4ʰ 50ᵐ 25ˢ, which had been found fast 5ʰ 22ᵐ 18ˢ of A.T.S. the same morning; since error was determined the diff. long. made westward was 24·5 miles. Find the lat. by ex-meridian.

3. August 9th P.M. at ship, in lat. by account 5° 35′ S., long. 14° W., the obs. alt. of the Sun's L.L. near the meridian, bearing north, was 67° 10′ 10″; ind. cor. − 2′ 18″; eye, 35 ft.; time by watch, 1ʰ 23ᵐ 7ˢ, which was 58ᵐ fast of A.T.S.; diff. of long. made to the eastward since determining the error was 37¼ miles. Find the latitude.

4. August 13th A.M. at ship, in lat. by account 53° 15′ N., long. 168° 47′ E., the obs. alt. of the Sun's L.L. near the meridian, bearing south, was 51° 26′ 50″; ind. cor. + 3′ 5″; eye, 26 ft.; time shown by chron., 12ʰ 18ᵐ 44ˢ, which was 13ᵐ 20ˢ slow of M.T.G. Find the latitude.

5. August 17th P.M. at ship, in lat. by account 44° 50′ S., long. 180° W.,

the obs. alt. of the Sun's U.L. near the meridian, bearing north, was 31° 57′ 50″; ind. cor. + 48″; eye, 23 ft.; time shown by watch, 1ʰ 2ᵐ 5ˢ, which was 50ᵐ 54ˢ fast of M.T.G. Find the latitude.

6. August 22nd A.M. at ship, in lat. by account 2° 35′ S., long. 87° 40′ E., the obs. alt. of the Sun's L.L. near the meridian, bearing north, was 72° 1′ 20″; ind. cor. − 3′ 11″; eye, 31 ft.; time shown by chron., 5ʰ 18ᵐ 37ˢ, which was 12ᵐ 40ˢ slow of G.M.T. Find the latitude at obs., and if the run to noon was S. 46° E., 8 miles, find the lat. at noon.

7. August 26th P.M. at ship, in lat. by account 39° 20′ N., long. meridian of Greenwich, the obs. alt. of the Sun's L.L. near the meridian, bearing south, was 60° 45′ 20″; ind. cor. + 4′ 18″; eye, 17 ft.; time shown by chron. which had been found 14ᵐ 15ˢ slow of G.M.T. was 12ʰ 8ᵐ 14ˢ; run since error was determined, S. 52° W., 29 miles. Find lat. at sights, and if the run was 4 miles on the same course since noon, find lat. at noon.

8. September 1st A.M. at ship, in lat. by account 43° 22′ S., long. 93° W., the obs. alt. of the Sun's U.L. near the meridian was 38° 16′ 50″; ind. cor. + 2′ 13″; eye, 40 ft.; time by watch, 5ʰ 52ᵐ 37ˢ, which was 6ʰ 11ᵐ 40ˢ fast of A.T.S.; diff. long. made to the westward since determining the error, 25 miles. Find the latitude, and if the run to noon was S. 65° W., 5 miles, find lat. at noon.

9. September 29th P.M. at ship, in lat. by account 46° 25′ N., long. 40° 30′ W., the obs. alt. of the Sun's L.L. near the meridian was 40° 45′ 20″; ind. cor. 0; eye, 25 ft.; time shown by chron. 3ʰ 30ᵐ 40ˢ, which was fast 28ᵐ 36ˢ of G.M.T.; run since determining the error, S. 42° E., 18 miles. Find the latitude, and if the run since noon was 7 miles on the same course, find the lat. at noon.

10. September 24th A.M. at ship, in lat. by account 15° 38° N., long. 93° 26′ E., when a chron. estimated 10ᵐ 15ˢ fast of G.M.T. showed 5ʰ 25ᵐ 11ˢ, the obs. alt. of the Sun's L.L. was 73° 5′ 30″; ind. cor. − 2′ 5″; eye, 18 ft. Find the latitude by ex-meridian.

11. September 23rd P.M. at ship, in lat. by D.R. 51° 15′ N., long. 124° 30′ W., when a watch showed 12ʰ 5ᵐ 14ˢ, and had been found 6ᵐ 12ˢ slow of A.T.S., diff. long. made to the westward since determining the error was 19·5 miles; the obs. alt. of the Sun's U.L. was 39° 5′ 40″; ind. cor. + 2′ 9″; eye, 22 ft. Find the latitude by ex-meridian. Run since noon S. by W. 4 miles. Find the lat. at noon.

12. September 23rd P.M. at ship, in D.R. lat. 40° 18′ S., long. 165° 54′ W., when a chron. showed 11ʰ 18ᵐ 33ˢ, error on A.T.G. was 4ᵐ 15ˢ slow; the obs. alt. of the Sun's L.L. near the meridian was 49° 17′ 10″; ind. cor. −17″; height of eye, 41 ft. Find the latitude at sights, and if the run since noon was N. 35° E., 6 miles, find lat. at noon.

ART. 69.—Latitude by Ex-meridian Altitude of a Star.

Example.—August 16th at ship, lat. 31° 30′ S., long. 175° 30′ 30″ W.; time by chron., 16ᵈ 0ʰ 0ᵐ 10ˢ, which was fast of M.T.G. 10ˢ; obs. alt. of α Cygni bearing N. and W. of the observer, 11° 37′ 50″; ind. cor. + 2′ 30″; height of eye, 26 ft. Find the latitude.

Time by chron.	16ᵈ	0ʰ	0ᵐ	10ˢ	175° 30′ 30″		R.A.M.S.	9ʰ 35ᵐ 52ˢ·5	
Fast				10		4	M.T.S.	12 17 58	
M.T.G.	16ᵈ	0	0	0	6,0)702	2 0	R.A M.	21 53 50·5	
Long. W.		11	42	2			R.A.*	20 38 13·6	
					11ʰ 42ᵐ 2ˢ				
M.T.S.	15ᵈ	12	17	58			W.H.A.*	1 15 36·9	

First Method.

W.H.A. $1^h\ 15^m\ 37^s$ log ris. = 4·731890
Lat. 31° 30′ S. cos = 9·930766
Dec. 44 56 39 N. cos = 9·849908

32551 log = 4·512564
T.A. 11° 30′ 48″ N.S. 199596

232147 nat. cos ZD 76° 34′ 35″ S.
Dec. 44 56 39 N.

Lat. 31 37 56 S.

Obs. alt. 11° 37′ 50″
I.C. + 2 30

11 40 20
Dip. − 5 0

11 35 20
Ref. − 4 32

True alt. 11 30 48

Second Method.

W.H.A. $1^h\ 15^m\ 37^s$ cos = 9·975919
Dec. 44° 56′ 39″ N. cot = 10·000846 cosec = 10·150939

Arc 1 46 31 55″ N. cot = 9·976765 sin = 9·860792

T.A. 11° 30′ 48 sin = 9·300150

Arc 2 78 10 1 S. ... ... cos = 9·311881

Lat. 31 38 6 S.

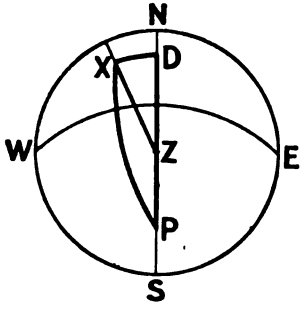


Fig. 124.

Third Method. From figure.

X is the star's position; XD a perpendicular on the meridian.

In triangle PDX, D = 90°; given Polar Dist. = PX; Hour angle = P. Find PD and DX.

Sin (co. P) = tan (co. PX) . tan PD

$$\therefore \tan PD = \frac{\cos P}{\cot PX} = \cos P \,.\, \tan PX \quad \text{I.}$$

Again—

$$\sin DX = \cos (\text{co. P}) \,.\, \cos (\text{co. PX})$$
$$= \sin P \,.\, \sin PX \quad \text{II.}$$

P = $1^h\ 15^m\ 37^s$ cos = 9·975919 sin = 9·510526
PX = 134° 56′ 39″ tan = 10·000846 sin = 9·849908

43° 28′ 5″ tan = 9·976765 13° 15′ 24″ sin = 9·360434
180

PD = 136 31 55 DX = 13° 15′ 24″

Now, in triangle ZDX, D = 90°; given zen. dist. = ZX and DX, find ZD.

Sin (co. ZX) = cos ZD . cos DX

$$\therefore \cos ZD = \frac{\cos ZX}{\cos DX} = \cos ZX \,.\, \sec DX$$

DX = 13° 15′ 24″ sec = 10·011730
ZX = 78 29 12 cos = 9·300153

ZD = 78° 10′ 1″ cos = 9·311883

Lat. = 90° − PZ = 90° − (PD − ZD)
= 90° − 58° 21′ 54″
= 31° 38′ 6″ S.

EXERCISES.

1. August 1st, about 10^h P.M. at ship, in lat. by account 38° 52′ S., long. 10° W., when a chron. showed $10^h\ 45^m$, and was fast $1^m\ 48^s$ of G.M.T., the obs. alt. of Vega near the meridian, bearing north, was 12° 30′ 40″; ind. cor. + 2′ 54″; eye, 20 ft. Find lat. by ex-meridian.

2. August 4th, about $11^h\ 15^m$ P.M. at ship, in lat. by account 51° 27′ N., long. 49° 30′ W., when a chron. correct for A.T.G. showed $2^h\ 36^m\ 53^s$, the obs. alt. of Altair near the meridian, bearing south, was 46° 49′ 20″; ind. cor. + 2′ 14″; eye, 45 ft. Find lat. by ex-meridian.

3. August 8th, about $10^h\ 50^m$ P.M. at ship, in lat. by D.R. 0° 2′ N., long. 169° 15′ E., when a chron. correct for M.T.G. showed $11^h\ 35^m\ 47^s$, the obs. alt. of α Pavonis near the meridian, bearing S. ½ E., was 32° 56′ 30″; ind. cor. − 37″; eye, 39 ft. Required the lat.

4. August 12th, at $1^h\ 5^m\ 32^s$ A.M. apparent time at ship, in lat. by D.R. 42° 30′ N., long. 174° 18′ W., the obs. alt. of Fomalhaut near the meridian was 17° 23′ 40″; ind. cor. + 1′ 11″; eye, 42 ft. Find the lat.

5. August 17th, about $2^h\ 45^s$ A.M. at ship, in lat. by account 17° 15′ N., long. 57° 12′ W., when a chron. correct for M.T.G. showed $6^h\ 32^m\ 38^s$, the obs. alt. of α Andromedæ near the meridian, bearing north, was 77° 59′ 10″; ind. cor. − 2′ 7″; eye, 50 ft. Required the lat.

6. August 26th, about $4^h\ 10^m$ A.M. at ship, in lat. by D.R. 8° 45′ S., long 69° 14′ E., when a watch which was $4^h\ 38^m\ 10^s$ slow of A.T.S. showed $11^h\ 29^m\ 41^s$; the diff. long. made to the westward since determining the error was 18·5 miles; the obs. alt. of Achernar near the meridian, bearing south, was 40° 11′ 30″; ind. cor. + 3′ 17″; height of eye, 38 ft. Find the lat.

7. September 4th, about $2^h\ 50^m$ A.M. at ship, in lat. by account 31° 50′ S., long. 100° 18′ E., when a chron., whose error on M.T.G. was $10^m\ 57^s$ slow, showed $7^h\ 58^m\ 14^s$, the obs. alt. of α Arietis near the meridian was 34° 58′ 40″; ind. cor. − 1′ 6″; height of eye, 41 ft. Required the lat. by ex-meridian.

8. September 9th, about $5^h\ 10^m$ A.M. at ship, in lat. by D.R. 53° 22′ N., long. 158° 35′ E.; when a watch, which had been found $10^h\ 32^m\ 10^s$ slow of A.T.S., showed $6^h\ 36^m\ 15^s$, the diff. long. made good to the eastward since determining the error was 32·5 miles; the obs. alt. of Aldebaran near the meridian was 52° 51′ 50″; ind. cor. + 2′ 11″; eye, 33 ft. Find the lat.

9. September 13th, about $5^h\ 15^m$ A.M. at ship, in lat. by account 23° 40′ S., long. 50° 19′ W., when a chron., whose error on M.T.G. was $13^m\ 15^s$ fast, showed $8^h\ 49^m\ 50^s$, the obs. alt. of Capella near the meridian was 20° 22′ 50″; ind. cor. − 2′ 9″; eye, 46 feet. Find the lat. by ex-meridian.

10. September 19th, about 5^h A.M. at ship, in lat. by D.R. 55° 35′ S., long. 60° 15′ W., when a chron. showed $9^h\ 10^m$, error $7^m\ 37^s$ fast of A.T.G., the obs. alt. of Rigel near the meridian was 42° 34′ 30″; ind. cor. + 2′ 11″; eye, 48 ft. Find the lat.

11. September 25th, about $8^h\ 30^m$ P.M. at ship, in lat. by D.R. 3° N., long. 68° 14′ E., the obs. alt. of α Cygni (Deneb) near the meridian was 48° 16′ 10″; ind. cor. − 2′ 3″; height of eye, 50 ft.; time by chron., $3^h\ 57^m\ 19^s$ correct M.T.G. Find the lat.

12. October 1st, about 9^h P.M. at ship, in lat. by account 20° 10′ N., long. 137° 46′ E., the obs. alt. of α Gruis, near the meridian, was 22° 11′ 30″; eye, 55 ft.; no ind. cor.; time by chron., 12^h, which was $8^m\ 45^s$ fast of A.T.G. Find the lat. by ex-meridian.

ART. 70. **Latitude by the Pole Star.**—On account of the proximity of the Pole Star to the North Pole, its azimuth is never large, and it is, therefore, a very suitable object for determining latitude by ex-meridian method at any time when

visible. Special formula has, however, been devised for this purpose, and tables calculated from this formula are entered in the "Nautical Almanac."

Thus, if l denote the latitude,
a „ „ altitude,
p „ „ polar distance,
h „ „ hour angle,
it may be shown that—

$$l = a \pm p \cdot \cos h + \tfrac{1}{2} \sin 1'' \cdot (p \sin h)^2 \cdot \tan a$$

$p \cdot \cos h$ is found in Table I., calculated for different R.A.M.'s from 0^h to 24^h; that is for different values of the hour angle and a constant value of the pol. dist.

The third term is found in Table II. and is always added; this is the second correction.

Table III. contains the third correction, always added, increased by 1′ in order to make it so; this 1′ is a constant to be subtracted from the true alt.; this correction in III. is an amount calculated for different dates to correct the mean values of R.A. and Dec. of Polaris used in constructing Tables I. and II.

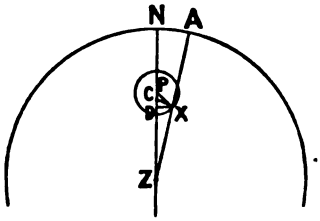


Fig. 125.

In figure, let X be position of Pole Star; draw XD perpendicular on meridian; along NZ cut off NC = AX its altitude.

PX = pol. dist., ZPX = hour angle.

Latitude = NP
= NC − CP
= NC − PD + CD

If the hour angle had exceeded 6 hours—

$$\text{latitude} = \text{NC} + \text{PD} + \text{CD}$$

PD is first correction, and considering the triangle PDX as a plane one—

$$\text{PD} = \text{PX} \cdot \cos \text{P}$$
$$\text{or 1st cor.} = p \cdot \cos h$$

and is added or subtracted according as the hour angle is greater or less than 6 hours.

$$\text{2nd cor.} = \text{CD}$$

In right-angled triangle ZDX—

$$\cos \text{ZX} = \cos \text{ZD} \cdot \cos \text{DX}$$
$$\text{or } \cos \text{ZC} = \cos \text{ZD} \cdot \cos \text{DX}$$
$$\therefore \cos (\text{ZD} + \text{CD}) = \cos \text{ZD} \cdot \cos \text{DX}$$
$$\cos \text{ZD} \cdot \cos \text{CD} - \sin \text{ZD} \cdot \sin \text{CD} = \cos \text{ZD} \cdot \cos \text{DX}$$

since CD is very small, $\cos \text{CD} = 1$

$$\text{Hence } \cos ZD - \sin ZD \,.\, \sin CD = \cos ZD \,.\, \cos DX$$
$$\sin ZD \,.\, \sin CD = \cos ZD \,(1 - \cos DX)$$
$$\text{or } \sin CD = \cot ZD \,.\, \left(2 \sin^2 \frac{DX}{2}\right)$$

As CD and DX are very small, we may write CD″ . sin 1″ for sin CD and $\frac{DX''}{2}$. sin 1″ for sin $\frac{DX}{2}$; also ZD is very nearly = ZX.

$$\text{Hence } CD'' \sin 1'' = \cot ZX \,.\, 2\left(\frac{DX}{2}\right)^2 \,.\, (\sin 1'')^2$$
$$i.e.\ CD = \tfrac{1}{2} \sin 1'' \,.\, (DX)^2 \,.\, \cot ZX$$
$$= \tfrac{1}{2} \sin 1'' \,.\, (PX \,.\, \sin P)^2 \,.\, \cot ZX.$$
$$\text{Hence approximately } l = a \pm p \,.\, \cos h + \tfrac{1}{2} \sin 1'' \,.\, (p \sin h)^2 \,.\, \tan a$$

ART. 71.—The process is as follows :—

(1) Find Greenwich time, and sidereal time of observation, which is Right Ascension of Meridian.

(2) Correct altitude and subtract 1′.

(3) Apply first correction from Table I., as there directed.

(4) Add second and third corrections from respective tables for latitude.

Example.—August 24th, at 12ʰ 12ᵐ 20ˢ mean time at ship, the obs. alt. of Polaris out of the meridian was 58° 35′ 10″, in longitude 28° 15′ W.; ind. cor. + 2′ 13″; height of eye, 27 ft. Find the latitude.

M.T.S. August 24ᵈ	12ʰ	12ᵐ	20ˢ
Long.	+1	53	0
M.T.G. August 24ᵈ	14	5	20

Obs. alt.	58°	35′	10″
Ind. cor.		+2	13
	58	37	23
Dip.		−5	5
	58	32	18
Ref.		−34	
T.A.	58	31	44
Const.		−1	0
Red. A.	58	30	44
1st cor.		−50	15
	57	40	29
2nd cor.		+37	
3rd cor.		+1	24
Lat.	57	42	30

	28°	15′	
		4	
6,0)11,3		0	
	1ʰ	53ᵐ	

R.A.M.S.	10ʰ	7ᵐ	24ˢ·9
Accel.		2	18·0
			·9
	10	9	43·8
M.T.S.	12	12	20
S.T. of obs. or R.A.M.	22	22	3·8
R.A. Polaris	1	26	12·9
E.H.A.	3	4	9·1

Cor. for	22ʰ	20ᵐ	= 0° 49′ 48″
,,	22	30	= 0 52 1
Diff. for	10		= 0 2 13
,,	1		= 0 0 13·3
,,	2		= 0 0 26·6
∴ 1st cor.			= 0 50 14·6

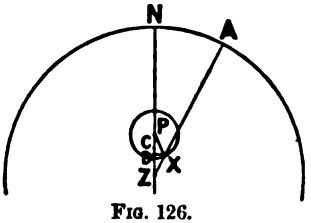


FIG. 126.

Given—

Alt. = AX = NC by construction
Hour angle = ZPX
Pol. dist. = PX
1st cor. = PD
2nd cor. = CD
Req. lat. = NP
= NC − PD + CD.

From formula—

Dec. = 88° 48′ 9″ N.
Pol. dist. = 1 11 51
p = 4311″
h = 3ʰ 4ᵐ 9ˢ
a = 58° 31′ 44″.

p = 4311″	log =	3·634578
h = 3ʰ 4ᵐ 9ˢ	cos =	9·841476
1st cor. 2993″	log =	3·476054
p = 4311″	log =	3·634578
h = 3ʰ 4ᵐ 9ˢ	sin =	9·857208
		3·491786
		2
	(p sin h)² =	6·983572
a = 58° 31′ 44″	tan =	10·213172
½ sin 1″	=	4·384545
2nd cor. 38″·13	log =	1·581289

True alt.	58° 31′ 44″
1st cor.	− 49 53
	57 41 51
2nd cor.	+ 38
Lat.	57 42 29

EXERCISES.

1. August 28th, about 5ʰ 50ᵐ A.M., in long. 48° 30′ W.; when a chron. correct for M.T.G. showed 9ʰ 2ᵐ 14ˢ; the obs. alt. of Polaris out of mer. was 36° 47′ 20″; ind. cor. − 2′ 17″; eye, 26 ft. Find the latitude.

2. August 5th, about 1ʰ 50ᵐ A.M., in long. 128° 50′ E.; when a chron., which was 10ᵐ 18ˢ fast of A.T.G., showed 5ʰ 15ᵐ 38ˢ; the obs. alt. of Polaris out of mer. was 46° 12′ 30″; ind. cor. + 3′ 46″; eye, 32 ft. Find the latitude.

3. August 1st, about 4ʰ 20ᵐ P.M., in long. 180° E.; when a chron. correct for M.T.G. showed 4ʰ 19ᵐ 50ˢ; the obs. alt. of the Pole star out of mer. was 29° 5′ 10″; ind. cor. − 4′ 15″; eye, 37 ft. Find the latitude.

4. August 2nd, at 8ʰ 10ᵐ 20ˢ P.M., apparent time at ship in long. 68° 34′ W.; the obs. alt. of α Urs. Min. out of mer. was 53° 36′ 40″; ind. cor. + 55″; eye, 29 ft. Required the latitude.

5. September 30ᵈ 8ʰ M.T.G.; when in long. 162° 36′ 15″ E.; the obs. alt. of Polaris out of mer. was 23° 14′ 50″; ind. cor. − 1′ 8″; eye, 42 ft. Find the latitude.

6. September 10th, about 8ʰ 15ᵐ P.M., in long. of Greenwich; when a chron. correct for A.T.G. showed 8ʰ 15ᵐ 38ˢ, the obs. alt. of the Pole star out of mer. was 32° 26′ 40″; ind. cor + 3′ 25″; eye, 34 ft. Find the latitude.

7. September 1st A.M., at ship in long. 95° 28′ E.; when a chron. correct for M.T.G. showed 7ʰ 3ᵐ 18ˢ; the obs. alt. of Polaris in artificial horizon out of mer. was 101° 50′ 20″; ind. cor. + 3′ 26″. Find the latitude.

8. September 30th, about 7ʰ 20ᵐ A.M., in long. 179° 45′ W.; when a chron. 12ʰ slow of A.T.S. showed 7ʰ 19ᵐ 32ˢ; the obs. alt. of α Urs. Min. in artificial horizon out of mer. was 96° 37′ 40″; ind. cor. − 2′ 48″. Find the latitude.

9. August 4th, at 9ʰ 18ᵐ 10ᵐ P.M., mean time at ship in long. 127° 16′ W.; the obs. alt. of Polaris in artificial horizon out of mer. was 61° 15′ 10″; ind. cor. + 2′ 42″; eye, 50 ft. Find the latitude.

CHAPTER XV.

COMPASS ERROR BY AMPLITUDES.

ART. 72.—Variation is the angle between the terrestrial and magnetic meridians. It shows how much the magnetic needle is disturbed from a true north and south direction by the action of the earth. It changes with the geographical position of the observer, and also with a lapse of time; the annual change is about 6′ annually. The amount of the variation for a particular year is entered on Admiralty charts.

Deviation is the angle the compass needle on a ship makes with the magnetic meridian. It shows how much the compass north is deflected from magnetic north by the action of iron in the ship or cargo. The amount varies with the direction of the ship's head, with the heel of the ship, and with her geographical position.

Compass Error is the angle the compass needle on a ship makes with the true meridian. It includes variation and deviation; and is the difference between the true and compass bearing of any object. The compass bearing of an object may be observed at any time, and one method of determining true bearings is by "Amplitude," which is the bearing of an object reckoned from east when rising, or from west when setting. Thus it is possible for the sun to be used for this purpose twice each day; and suitable stars can be selected continually at night.

ART. 73. **Proof of Amplitude.**—In the figure NESW is the horizon, X the position of an object on the horizon; the arc WX is its amplitude. In the right-angled triangle PNX, N = 90°; PN = latitude; PX = pol. dist. or co. dec.; NX = co. amp.

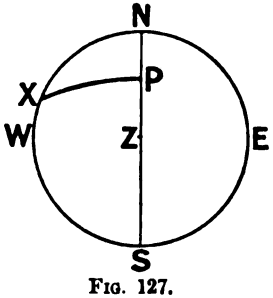


FIG. 127.

$$\text{Sin (co. PX)} = \cos PN \cdot \cos NX$$
$$\cos PX = \cos PN \cdot \sin WX$$
$$\text{since } \cos NX = \sin WX$$
$$\therefore \sin WX = \frac{\cos PX}{\cos PN}$$
$$= \cos PX \cdot \sec PN$$
$$\text{or sin amp.} = \sin \text{dec.} \cdot \sec \text{lat.}$$

The equinoctial cuts the horizon in the east and west points, hence when an object is on the equinoctial, that is, has declination = 0°, it rises due east and sets due west. Again, when an

observer is on the equator the formula becomes sin amp. = sin dec., since sec 0° = 1; hence in this case amp. = dec. It is also observed that when an object has north dec., it rises to the north of east, and sets to the north of west; if south dec., it rises south of east, and sets south of west; the name of the amplitude is therefore the same as the declination.

ART. 74. **To find the Time of rising or setting of a Celestial Object.**—In the figure the hour angle is ZPX, which is the supplement of NPX. Then from triangle NPX—

$$\sin(\text{co. P}) = \tan \text{PN} \cdot \tan(\text{co. PX})$$
$$\cos \text{P} = \tan \text{PN} \cdot \cot \text{PX}$$
$$\text{hence } \cos \text{H.A.} = -\tan \text{lat.} \cot \text{pol. dist.}$$

ART. 75.—The process for finding compass error by amplitude is as follows :—

(1) Find Greenwich time, and correct declination.

(2) To L. sin declination add L. sec latitude, reject 10 from index, and take back sin true amp., which reckon from E. if A.M. at ship, and W. if P.M. at ship; and same name as declination. Under true amp. place comp., bearing reckoned from E. or W. like the true amp.

(3) When the true and comp. amps. are of the *same* name N. *or* S., their *difference* is the *comp. err.*; when of *contrary* names, their *sum* is the comp. err. It is named E. or W. according as the true amp. reads to the right or left of the comp. amp.

(4) Under the comp. err. place the variation; if of *same* names *subtract;* if of *contrary* names *add*; this gives the deviation, which is to be named like the error, unless the error is subtracted from the variation, when it is of contrary name to error.

Examples.—

(1) September 28th, at 5h 53m A.M., mean time at ship in lat. 23° 25′ S., long. 12° 40′ W., the sun's compass bearing was E. ¾ N. Find the comp. err. and deviation for the position of the ship's head, if the variation from chart was 28° 30′ W.

M.T.S. Sept.	27d 17h 53m 0s	12° 40′
Long. W.	+ 50 40	4
M.T.G. Sept.	27d 18 43 40	6,0)5,0 40
		50m 40s
Dec. 28th noon	1° 46′ 1″·4 S.	H.V. 58″·45
Cor.	5 8	5·27
	1 40 53·4 S.	40915
		11690
		29225
		6,0)30,8·0315
		5′ 8″

Dec.	1° 40′ 53″	sin = 8·467483
Lat.	23 25 0	sec = 10·037328
True amp. E.	1° 49′ 57″ S.	sin = 8·504811
Comp. bearing E.	8 26 15 N.	
Comp. err.	10 16 12 E.	(Add because true and comp. amp. differ in name; named E, because true amp. reads to the right of the comp. amp.)
Var.	28 30 0 W.	
Dev.	38 46 12 E.	(Sum of comp. err. and variation because they are of different names; E. because comp. err. is E.)

In figure X is position of sun.
In triangle SPX, S = 90°.
Given—
SP = lat.
PX = pol. dist. or co. dec.
Req. SX = co. amp.
Formula as proved above.
Sin amp. = sin dec. sec lat.

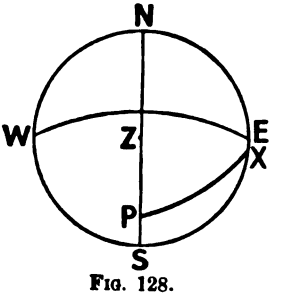


FIG. 128.

(2) August 1st, at 8ʰ 35ᵐ P.M. apparent time at ship, in lat. 61° 45′ S., long. 176° 30′ E., the sun's compass bearing was N. 66° 30′ W. If the variation from chart is 22° 15′ E., find compass error and deviation.

A.T.S. Aug.	1ᵈ 8ʰ 35ᵐ 0ˢ	176° 30′
Long. E.	− 11 46 0	4
A.T.G. July	31ᵈ 20 49 0	6,0)70,6 0
		11ʰ 46ᵐ
Dec. 1st noon	18° 11′ 45″ N.	H.V. 37″·26
Cor.	+ 1 59	3·2
	18 13 44 N.	7452
		11178
		6,0)11,9·232
		1′ 59″

Dec.	18° 13′ 44″	sin = 9·495286
Lat.	61 45 0	sec = 10·324845
True amp. W.	41 22 5 N.	sin = 9·820131
Comp. ,, W.	23 30 0 N.	
Err.	17 52 5 E.	
Var.	22 15 0 E.	
Dev.	4 22 55 W.	

The bearing in the question N. 66° 30′ W. is subtracted from 90° to reckon it from W., the same point as the true amp. Since true and comp. bearings are of same name N., they are subtracted for error, which is named E., because the true amp. reads to the right of the comp.; the difference of error and variation, because of same name E., is the dev., which is of contrary name to error, because error is subtracted from variation.

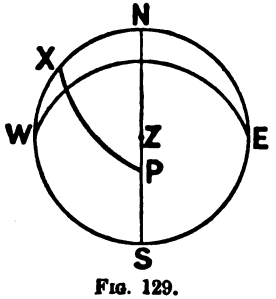


Fig. 129.

In figure X is position of sun.
In triangle SPX, S = 90°.
Given—

SP = lat.
PX = pol. dist. or co. dec.
Req. SX = co. amp.

Exercises.

1. August 2nd, at 7ʰ 10ᵐ P.M. mean time at ship, in lat. 42° 20′ S., long. 65° E., the sun's comp. bearing was N.N.W. ¾ W., the variation being 28° 30′ W. Find comp. err. and dev.

2. September 28th, at 5ʰ 50ᵐ A.M. mean time at ship, in lat. 50° 25′ S., long. 81° W., the sun rose by compass N. 64° 30′ E. Find true amp. and comp. err.; also if var. is 24° 15′ E., find dev.

3. August 9th, at 5ʰ 15ᵐ A.M. mean time at ship, in lat. 33° 51′ N., long. 125° W., the sun's comp. bearing was E. by N. ¼ N. Find true amp. and comp. err.; and if var. is 11° 20′ E., find dev.

4. September 30th, at 6ʰ 23ᵐ P.M. A.T.S., in lat. 48° 19′ S., long. 76° E., the sun set by compass W. ½ S. Find true amp. and comp. err.; the var. being 29° 45′ W., find the dev.

5. August 4th, at 5ʰ 6ᵐ A.M., M.T.S., in lat. 37° 15′ N., long. 20° 30′ E., the sun's comp. bearing was E. by N. ½ N. Find true amp., comp. err., and deviation, the var. being 11° 30′ W.

6. September 26th, at 6ʰ 5ᵐ P.M. M.T.S., in lat. 50° 15′ N., long. 126° 45′ W., the sun's comp. bearing was S. 69° 30′ W. Find true amp., comp. err., and dev., the var. being 22° 40′ E.

7. August 7th, about 6ʰ A.M., in lat. 20° 24′ S. long. 134° W., when a chron. correct for M.T.G. showed 3ʰ 20ᵐ, the sun's comp. bearing was E. Find the true amp., comp. err., and dev., the var. being 7° 15′ E.

8. September 7th, about 7ʰ P.M., in lat. 60° 14′ N., long. 166° W., when a chron. correct for M.T.G. showed 6ʰ 4ᵐ, the sun's comp. bearing was W. Find the true amp., comp. err., and dev., the var. being 12° 20′ E.

9. August 3rd, about 3ʰ 10ᵐ A.M., in lat. 65° 22′ N., long. 170° 15′ W., when a chron. correct for M.T.G. showed 2ʰ 35ᵐ, the sun's comp. bearing was N. ¼ W. Find the true amp., comp. err., and dev., the var. being 22° 40′ E.

10. September 23rd, at 6ʰ A.M. A.T.S. at ship, in lat. 0°, long. 50° W., the sun's comp. bearing was E. 10° S. Find the true amp., comp. err., and dev., the var. being 0°.

11. September 25th, at 6ʰ 5ᵐ P.M. A.T.S. at ship, in lat. 13° 15′ S., long. 95° E., the sun's bearing by comp. was N. 76° 30′ W. Find the true amp., comp. err., and dev., the var. being 2° W.

12. September 23rd, about 6^h P.M. at ship, in lat. 39° N., long. 78° 45′ W., when a chron. correct for M.T.G. showed $11^h\ 15^m\ 23^s$, the sun's comp. bearing was S. 88° W. Find true amp., comp. err., and dev., the var. being 6° 30′ E.

13. August 30th, at $5^h\ 59^m$ A.M., in lat. 0°, long. 180° W., the sun's comp. bearing was N.E. by E. ½ E. Find the true amp., comp. err., and dev., the var. being 9° 30′ E.

14. September 24th, about 6^h A.M., in lat. 2° 20′ N., long. 101° 11′ E., the sun's comp. bearing was E., when the chron. showed $11^h\ 15^m\ 14^s$ correct M.T.G. Find the true amp., comp. err., and dev., the var. being 1° 15′ E.

15. Find the time of sunrise on August 16th, in lat. 53° 20′ N., 4° W. long.

16. If a star in the equinoctial rises at $7^h\ 12^m\ 18^s$ by chronometer, what should the chron. indicate when star is on meridian.

17. At a certain place the sun rose E. 25° S. by compass, and set W. 15° N. by same compass. Find deviation, having given variation 22° W.

18. At a certain place a star when on the prime vertical has its altitude equal to its amplitude when setting. Find lat.

19. Find the greatest amplitude the sun can have in lat. 53° 20′ N.

20. In lat. 50° S. when sun's declination is 18° 40′ S.: find his altitude when on the prime vertical, and the approx. interval between then and setting.

CHAPTER XVI.

COMPASS ERROR BY AZIMUTHS.

ART. 76.—Another method of finding compass error and deviation is by Azimuths; that is, by observing the bearing of any celestial object when out of the horizon. This method has the advantage of being available at all times of the day or night when the object is visible. When an altitude is observed for time, by taking the bearing very little extra trouble is involved in working the Azimuth from the altitude; generally, however, the time is accurately taken, and the Azimuth taken from tables calculated for the purpose. Burdwood's Tables are used for latitudes 30° to 60°, and Davis's Tables for Latitudes 0° to 30°; both for declinations from 0° to 23°.

ART. 77.—The Azimuth of a Celestial Object—that is, its bearing reckoned from true North or South—is the angle at the Zenith between the Meridian of the observer and a vertical circle passing through the object, and is measured on the Horizon.

The Data required in the Calculation are the **True Altitude, Latitude, and Polar Distance.**

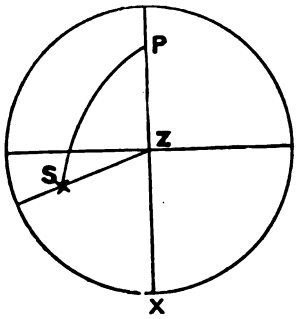


FIG. 130.

Let S be the object, PS its polar distance (p), PZ the co-lat. ($90° - l$), ZS the zenith distance ($90° - a$). The angle PZS is the azimuth from N., and its supplement SZX is the azimuth from S.

PZS is a spherical triangle, of which the three sides are given to find an angle PZS.

The general formula is—

$$\cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$$

whence in the figure—

$$\cos \text{PZS or} - \cos \text{SZX} = \frac{\cos \text{PS} - \cos \text{ZS} \,.\, \cos \text{PZ}}{\sin \text{ZS} \,.\, \sin \text{PZ}}$$

Putting the letters Z, a, l, p for the azimuth, altitude, latitude, and polar distance respectively—

$$-\cos Z = \frac{\cos p - \cos (90° - a) . \cos (90° - l)}{\sin (90° - a) . \sin (90° - l)}$$
$$= \frac{\cos p - \sin a . \sin l}{\cos a . \cos l}$$

adding 1 to each side of the equation—

$$1 - \cos Z = 1 + \frac{\cos p - \sin a . \sin l}{\cos a . \cos l}$$
$$\therefore 2 \sin^2 \frac{Z}{2} = \frac{(\cos a . \cos l - \sin a . \sin l) + \cos p}{\cos a . \cos l}$$
$$= \frac{\cos (a + l) + \cos p}{\cos a . \cos l}$$
$$\sin^2 \frac{Z}{2} = \frac{\cos \frac{1}{2}(a + l + p) \cos \frac{1}{2}(a + l - p)}{\cos a . \cos l}$$
$$\therefore \sin^2 \frac{Z}{2} = \frac{\cos s . \cos (s - p)}{\cos a . \cos l}$$
$$\text{where } s = \tfrac{1}{2}(a + l + p)$$
$$i.e. \sin^2 \frac{Z}{2} = \cos s . \cos (s - p) . \sec a . \sec l$$
$$\text{or } \sin \frac{Z}{2} = \sqrt{\cos s . \cos (s - p) . \sec a . \sec l}$$

from which is derived the rule for computing the azimuth of the *opposite name* to the *latitude*, namely, add together the log secants of the altitude and latitude, and the log cosines of the half-sum and remainder; divide by two to get the square root, which gives the log sine of half the azimuth.

ART. 78.—To find azimuth from altitude proceed as follows:—

(1) Find Greenwich time; correct declination and find polar distance.

(2) Correct altitude.

(3) Add altitude, latitude, and polar distance, divide by two for half-sum; difference between half-sum and polar distance is remainder.

(4) Add sec alt., sec lat., cos half-sum, and cos remainder; divide sum of logs by two, and take back sine; this angle multiplied by two is the true azimuth which mark contrary name to lat., E. if A.M. at ship, W. if P.M. at ship. If true and comp. azimuth are different in name N. or S., subtract true from 180° to make of same name as comp. bearing.

(5) The *difference* of the true and comp. bearings when of *same* name E. or W., or their *sum* when of *different* names, gives the comp. error which is marked E. or W., according as the true reads to the right or left of the comp. bearing.

(6) Find deviation as in amplitude.

Example.—

September 23rd, at 3ʰ 48ᵐ 27ˢ M.T.S. P.M., in lat. 56° 20′ S., long. 67° 35′ W., the obs. alt. of the Sun's U.L. was 16° 54′ 30″; ind. cor. + 1′ 1″; height of eye, 27 ft. The Sun's comp. bearing was W. ¼ S. Required the true azimuth, and comp. error; and, supposing the variation to be 22° E., what is the deviation?

M.T.S. Sept.	23ᵈ 3ʰ 48ᵐ 27ˢ		67° 35′
Long. W.	+ 4 30 20		4
M.T.G.	23ᵈ 8 18 47		6,0)27,0 20
			4ʰ 30ᵐ 20ˢ
O.A. ⊙	16° 54′ 30″	Dec. noon	0° 10′ 58″·1 N.
I.C.	+ 1 1	Cor.	8 5·1
	16 55 31		0 2 53 N.
Dip.	− 5 5		
	16 50 26	P.D.	90° 2′ 53″ N.
S.D.	− 15 58		
	16 34 28		
R. and P.	− 2 58		
T.A.	16 31 30	sec =	10·018319
Lat.	56 20 0	sec =	10·256208
P.D.	90 2 53		
	2)162 54 23		
	81 27 11	cos =	9·172076
	8 35 42	cos =	9·995095
			2)19·441698
		sin =	9·720849
			31° 43′ 30″
			2
		T. Az. N.	63 27 0 W.
			180
		S.	116 33 0 W.
		Comp. bearing S.	87 11 15 W.
		Comp. error.	29 21 45 E.
		Var.	22 E.
		Dev.	7 21 45 E.

The true azimuth is marked N., contrary to name of lat., and W. since it is P.M. at ship. It is subtracted from 180° to change the name to S., which reckons the angle from the same point as the comp. bearing; the difference is the error, since both are named W.; and the error is named E., because the true reads to the right of the comp. bearing.

Draw circle with $\frac{9}{10}$ inch radius to represent the rational horizon; NESW; vertical diameter NZS represents meridian of observer.

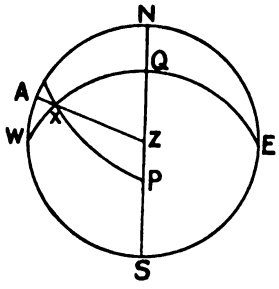


FIG. 131.

Mark off on scale SP = ZQ = lat. 56° 20′. Since declination is so small object may be considered as on the equinoctial EQW; with zenith distance as radius = 73° describe an arc to westward cutting EQW in X, the position of sun; through X draw vertical circle ZXA, and hour circle PX to complete.

Then Alt. is AX; Lat. is ZQ; Pol. dist. is PX.

Hence azimuth is PZX, the formula employed gives its supplement NZX.

From the figure.

In figure X is position of object, very nearly on the equinoctial.

In triangle PZX.

Given co-lat. = PZ ...	... ...	33°	40′ 0″
Pol. dist. = PX ...	... ...	90	2 53
Zen. dist. = ZX ...	... ...	73	28 30
Req. azimuth = Z.			

$$\cos\frac{Z}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - PX)}{\sin PZ \,.\, \sin ZX}}$$

where $s = \frac{1}{2}(PX + PZ + ZX)$

PX	90°	2′	53″		
PZ	33	40	0	cosec =	10·256208
ZX	73	28	30	cosec =	10·018319
2)197	11	23			
s	98	35	41	sin =	9·995095
$s - PX$	8	32	48	sin =	9·172062
					2)19·441684
	58	16	34	cos =	9·720842
			2		
True az. S.	116	33	8	W.	
Comp. bearing S.	87	11	15	W.	
Comp. error	29	21	53	E.	
var.	22			E.	
Dev.	7	21	53	E.	

EXERCISES.

1. August 4th, about 8ʰ 30ᵐ A.M., in lat. 38° 47′ S., long. 92° 12′ E.; when a chron. correct for M.T.G. showed 2ʰ 37ᵐ 35ˢ, the obs. alt. of Sun's L.L. was 16° 42′ 10″; ind. cor., + 2′ 15″; eye, 29 ft.; bearing by compass, N. 49° 30′ E. Find the true azimuth, comp. error, and deviation, the variation being 18° W.

2. August 19th, at 4ʰ 25ᵐ 37ˢ P.M. A.T.S., in lat. 41° 24′ N., long. 70° 20′ W., the obs. alt. of the Sun's U.L. was 26° 32′ 40″; bearing by compass, W.; ind. cor., − 2′ 7″; height of eye, 34 ft. Find the true azimuth and comp. error; also if the variation from chart is 9° 30′ W., find the deviation.

3. August 31st, about $9^h\ 20^m$ A.M., in lat. 52° 25′ N., long. 3° 17′ 30″ W.; when a chron., which was $38^m\ 19^s$ slow of M.T.G., showed $8^h\ 57^m\ 39^s$, the obs. alt. of the Sun's U.L. was 36° 26′ 10″; bearing by compass S., 28° 30′ E.; ind. cor., − 26″; eye, 41 ft. Find the true azimuth, comp. error., and deviation, the variation being 15° 40′ W.

4. September 1st, A.M. at ship, in lat. 44° 18′ N., long. 10° 15′ W.; when a chron. correct for M.T.G. indicated $9^h\ 22^m\ 37^s$, the obs. alt. of Sun's U.L., 34° 21′ 30″; ind. cor., + 4′ 15″; height of eye, 23 ft.; bearing by compass, S.E. ½ E. Required the true azimuth, and comp. error; and if variation from chart is 25° W., what is the deviation?

5. September $22^d\ 18^h\ 29^m\ 38^s$ (astronomical) M.T.G., in lat. 49° 40′ S., long. 135° 24′ E.; the Sun's comp. bearing, N. 61° 50′ W.; alt. of U.L., 22° 18′ 50″; height of eye, 20 ft. Find true azimuth, comp. error, and deviation, the variation being 0°.

6. September 30th, about $3^h\ 15^m$ P.M., in lat. 49° 37′ S., long. 179° 56′ E.; when a chron. showed $3^h\ 20^m\ 26^s$ correct M.T.G., the obs. alt. of Sun's L.L., 25° 3′ 20″; ind. cor., + 3′ 14″; height of eye, 37 ft.; bearing by compass, W.N.W. Find true azimuth, comp. error, and deviation, the variation from chart being 16° 30′ E.

ART. 79. **Time Azimuths of the Sun by the Tables.**—When the Apparent Time at Ship, the Lat., and the Sun's Dec. are known, the Sun's True Azimuth can be found by inspection in Burdwood's or Davis's Tables.

The same Tables will give the true Azimuth of a Star, a Planet, or the Moon, provided the Hour Angle and Declination are within the limits of the Tables.

A description of one of the books suffices, as the only difference is that Burdwood is used with lats. 30° to 60°, and Davis with lats. 0° to 30°.

The azimuths are calculated and tabulated for each degree of lat. and dec., and at intervals of 4 mins. in time. Interpolations must be made for odd minutes of time, lat., and dec. The degrees of lat. are found at the top of the page, underneath which is a remark showing whether lat. and dec. are the same or contrary in name. The columns are headed with the degrees of dec. from 0° to 23° consecutively; 0° to 11° on left hand page, 12° to 23° on right; and at the sides the times are entered to every 4 mins. A.M. time on the left-hand side of the page, P.M. time on the right. The method of using the Tables will be illustrated, and the following rules must be observed:—

(1) Find the apparent time at ship (civil time).

(2) Correct Sun's declination.

(3) Open the tables at the degree of lat. in question, and observe whether dec. and lat. are of same or contrary names. Find in the A.M. or P.M. column the next less entry in point of time to the A.T.S. in question, and in this line under the degree of declination will be found the azimuth, which has to be corrected for odd minutes of time, declination, and latitude by proportion, for the true azimuth.

(4) The true azimuth must be named the same name as the latitude—E. if A.M. at ship, W. if P.M. at ship.

Example.—

The apparent time at ship being $3^h\ 56^m\ 11^s$ P.M., the correct declination 0° 3′ N., and latitude 56° 20′ S. Find true azimuth from "Tables."

A.T.S.	...	...	$3^h\ 56^m\ 11^s$ P.M.
Dec.	...	...	0° 3′ N.
Lat.	...	...	56° 20′ S.

In Burdwood's Tables, with lat. 56°, contrary name to dec. opposite $3^h\ 56^m$, under dec. 0°, the Sun's azimuth is found to be 116° 28′; this has to be corrected for 11^s of time, 3′ of dec., and 20′ of lat.

Az. for $3^h\ 56^m$	116° 28′	Az. for 0° dec.	116° 28′	Az. for 56° lat.	116° 28′
,, 4	115 35	,, 1 ,,	117 1	,, 57 ,,	116 44
Diff. in 4^m	− 53	Diff. in 60′ ,,	+ 33	Diff. in 60′ ,,	+ 16
	·2		3		20
	4)10·6		6,0)9,9		6,0)32,0
,, 11^s	− 2·6	,, 3′ ,,	+ 1·6	,, 20′ ,,	+ 5·3

Azimuth from Tables 116° 28′

Correction − 2′·6 + 1′·6 + 5′·3 = + 4·3

True azimuth S. 116 32·3 W.

ART. 80. **Time Azimuth by Calculation.**—Here the hour angle ZPS, the polar distance PS, and the co-latitude PZ are given to find the angle PZS, which is the azimuth of the *same name* as the *latitude* (Fig. 130).

This is a case of two sides and the included angle being given to find one of the angles at the base. From the general formulæ—

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \tfrac{1}{2}(a - b)}{\cos \tfrac{1}{2}(a + b)} \cdot \cot \tfrac{1}{2}C$$

$$\tan \tfrac{1}{2}(A - B) = \frac{\sin \tfrac{1}{2}(a - b)}{\sin \tfrac{1}{2}(a + b)} \cdot \cot \tfrac{1}{2}C$$

by substituting the Z and S in the figure for A and B, and the letters h, p, l' for the H.A., polar distance, and co-latitude respectively, we have—

$$\tan \tfrac{1}{2}(Z + S) = \frac{\cos \tfrac{1}{2}(p - l')}{\cos \tfrac{1}{2}(p + l')} \cot \tfrac{1}{2}h$$

$$\tan \tfrac{1}{2}(Z - S) = \frac{\sin \tfrac{1}{2}(p - l')}{\sin \tfrac{1}{2}(p + l')} \cot \tfrac{1}{2}h$$

therefore, $\tan \tfrac{1}{2}(Z + S) = \sec \tfrac{1}{2}(p + l') \cdot \cos \tfrac{1}{2}(p - l') \cdot \cot \tfrac{1}{2}h$

and $\tan \tfrac{1}{2}(Z - S) = \operatorname{cosec} \tfrac{1}{2}(p + l') \cdot \sin \tfrac{1}{2}(p - l') \cdot \cot \tfrac{1}{2}h$

whence the following form for computation:—

$\tfrac{1}{2}(p + l')$	...	sec	...	cosec	
$\tfrac{1}{2}(p - l')$	...	cos	...	sine	
$\tfrac{1}{2}h$	...	cot	...	cot	
$\tfrac{1}{2}(Z + S)$	...	tan	...		
$\tfrac{1}{2}(Z - S)$	...	...	...	tan	

The *sum* of these angles is the *greater* of the two angles Z and S, and the *difference* is the *lesser* angle; and since the greater angle is opposite to the greater side, the azimuth Z is the greater angle when p is greater than l', and the smaller angle when p is less than l'.

NOTE.—When $\frac{1}{2}(p + l')$ exceeds 90°, the sec is negative, and therefore tan $\frac{1}{2}$(Z + S) is negative, in which case the supplement must be taken for $\frac{1}{2}$(Z + S).

Example.—

The apparent time at ship being $3^h\ 56^m\ 11^s$ P.M., correct dec. 0° 3′ N., and lat. 56° 20′ S., find true azimuth by calculation.

This figure is drawn similarly to Fig. 131, except that as the hour angle is given instead of the altitude, we must estimate an arc of the equinoctial equal to the hour angle $3^h\ 56^m$ to determine the point through which to draw the hour circle of the object: knowing that QW represents 6 hours: where this hour circle cuts the parallel of declination is the position of X.

In triangle PZX—

Given P = $3^h\ 56^m\ 11^s$ = hour angle
PZ = 33° 40′ = co-lat.
PX = 90° 3′ = pol. dist.
Req. Z = azimuth.

$$\tan \tfrac{1}{2}(Z + X) = \frac{\cos \frac{1}{2}(PX - PZ)}{\cos \frac{1}{2}(PX + PZ)} \cdot \cot \frac{P}{2}$$

$$\tan \tfrac{1}{2}(Z - X) = \frac{\sin \frac{1}{2}(PX - PZ)}{\sin \frac{1}{2}(PX + PZ)} \cdot \cot \frac{P}{2}$$

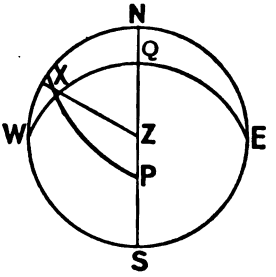


FIG. 132.

PZ =	33° 40′		
PX =	90 3		
PX + PZ =	123 43		
PX − PZ =	56 23		
$\frac{P}{2}$ =	$1^h\ 58^m\ 5^s{\cdot}5$	cot = 10·246953	cot = 10·246953
$\frac{1}{2}$(PX + PZ) =	61° 51′ 30″	sec = 10·326377	cosec = 10·054638
$\frac{1}{2}$(PX − PZ) =	28 11 30	cos = 9·945159	sin = 9·674331
$\frac{1}{2}$(Z + X) =	73 8 30	tan = 10·518489	
$\frac{1}{2}$(Z − X) =	43 24 30	... ...	tan = 9·975922
Z =	116 33 0		

True azimuth S. 116° 33′ W.

EXERCISES.

Under the following conditions find from the Tables or by calculation the True Azimuth, and hence the deviation:—

1. A.T.S. $2^h\ 48^m$ P.M.; dec. 10° 25′ S.; lat. 40 N.
2. A.T.S. $3^h\ 19^m$ P.M.; dec. 14° 33′ N.; lat. 49° N.
3. A.T.S. $9^h\ 14^m$ A.M.; dec. 21° 17′ N.; lat. 52° 25′ S.
4. September 1st, M.T.G. $31^d\ 15^h\ 54^m\ 42^s$, in lat. 51° 25′ S., long. 174° 25′ E., the Sun's comp. bearing was W. $\frac{3}{4}$ N.; variation 30° E.
5. August 12, A.M. at ship, in lat. 0°, long. 12° 50′ W.; when a chron. showed $8^h\ 16^m\ 20^s$ M.T.G., the Sun's comp. bearing was E. $\frac{3}{4}$ N.; var. 20° 30′ W.
6. September 3rd, at $7^h\ 57^m$ A.M. M.T.S., in lat. 13° 29′ N., long. 60° 30′ E., the Sun's comp. bearing was E. by N. $\frac{3}{4}$ N.; var. 1° 10′ W.

7. September 23rd, about $3^h\ 30^m$ P.M. in lat. 49° 40′ S., long. 135° 24′ E.; when a chron. showed $6^h\ 29^m\ 37^s$, the Sun's comp. bearing was N. 61° 50′ W.; var. 0°.

8. When the M.T.G. was August $31^d\ 21^h\ 22^m\ 37^s$, in lat. 44° 18′ N., long. 10° 15′ W.; the Sun's comp. bearing was S.E. ½ E.; var. 25° W.

9. About $4^h\ 15^m$ P.M. on September 23rd, in lat. 0° 15′ S., long. 150° 15′ E.; when a chron. showed $6^h\ 30^m$ M.T.G., the Sun's comp. bearing was W.; var. 9° E.

10. August 3rd, at $3^h\ 50^m$ P.M. A.T.S., in lat. 25° 16′ S., long. 92° 30′ E., the Sun bore by comp. N. 60° ½ W.; var. 8° 15′ W.

11. August 5th at $8^h\ 40^m$ A.M. M.T.S., in lat. 30° 21′ N., long. 51° 15′ E., the Sun's comp. bearing was S.E. by E. ¾ E.; var. 2° W.

12. August 9th at $4^h\ 25^m$ P.M. M.T.S., in lat. 37° 27′ N., long. 20° 14′ E., the Sun bore by comp. W. ¼ N.; var. 21° W.

13. August 23rd about $9^h\ 20^m$ A.M., in lat. 52° 49′ S., long. 53° 17′ W.; when a chron. showed $1^h\ 17^m\ 12^s$ G.M.T., the Sun bore by comp. N.E. by E.; var. 15° 30′ E.

14. August 24th about $10^h\ 40^m$ A.M., in lat. 64° 56′ N., long. 170° W.; when a chron. correct for M.T.G. showed $10^h\ 1^m\ 30^s$, the Sun's comp. bearing was S. ¾ W.; var. 24° E.

ART. 81. **Star Time Azimuths.**—As before noticed, providing the declination of the star is less than 23°, the Tables may be used for obtaining the true bearing. The Hour Angle of the star must, however, be calculated instead of the A.T.S., and always found in the P.M. column on the right-hand side of the page, otherwise the method is precisely the same.

Example.—

September 10th at $1^h\ 36^m\ 20^s$ A.M. mean time at ship, in lat. 38° 20′ N., long. 63° 30′ W., the star α Orionis bore by compass E. by S. ½ S. Required the comp. error, and deviation, the variation being 9° 15′ W.

M.T.S. Sept.	9^d	13^h	36^m	20^s	63° 30′	R.A.M.S. noon	11^h	10^m	$29^s{\cdot}8$
Long. W. +		4	14	0	4	Accel. for 17^h		2	47·6
						50^m			8·2
M.T.G. Sept.	9^d	17	50	20	6,0)25,4 0		11	13	25·6
					$4^h\ 14^m$	M.T.G.	13	36	20
Dec.*	7° 23′ 34″ N.								
Lat.	38 20 N.					R.A.M.	0	49	45·6
						R.A.*	5	50	4·9
						E.H.A.	5	0	19·3

In Burdwood's Tables, with lat. 38°, same name as declination, under 7° Dec., abreast 5^h on the right-hand side (always find H.A. of star on right-hand side), the star's azimuth is entered as 93° 43′, which requires correction for 19^s time, 24′ of Dec., and 20′ Lat.

Az. for	5^h	93°	43′	Az. for 7° dec.	93°	43′	Az. for 38° lat.	93°	43′
,,	$5^h\ 4^m$	93	5	,, 8° ,,	92	53	,, 39 ,,	94	0
Diff. for	4^m	−38		Diff. for 60′	−50		Diff. for 60′	+17	
		·3			24			20	
		4)11·4			6,0)120,0			6,0)34,0	
,,	19^s	−2·8		,, 24′	−20			+5·7	

	Az. from Tables	93° 43′
Correction	−2·8 − 20 + 5·7 =	−17
	True Az. N.	93 26 E.
		180 0
	S.	86 34 E.
Comp.	,, S.	73 7 E.
	,, err.	13 27 W.
	Var.	9 15 W.
	Dev.	4 12 W.

From the figure.

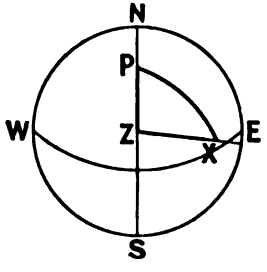


FIG. 133.

In triangle PZX, where X is position of star; given PZ = colat
PX = Pol. dist.
P = hour angle.
Req. Z = azimuth.

$$\text{Tan } \tfrac{1}{2}(Z + X) = \frac{\cos \tfrac{1}{2}(PX - PZ)}{\cos \tfrac{1}{2}(PX + PZ)} \cdot \cot \frac{P}{2}$$

$$\text{Tan } \tfrac{1}{2}(Z - X) = \frac{\sin \tfrac{1}{2}(PX - PZ)}{\sin \tfrac{1}{2}(PX + PZ)} \cdot \cot \frac{P}{2}$$

$P = 5^h\ 0^m\ 19^s$ $\quad PX = 82° 36′ 30″$
$\quad PZ = 51\ 40\ 0$

$PX + PZ = 134\ 16\ 30$
$PX - PZ = 30\ 56\ 30$

$\frac{P}{2}$ =	$2^h\ 30^m\ 9{\cdot}5^s$	Cot = 10·114398	Cot = 10·114398
$\frac{1}{2}(PX + PZ)$ =	67° 8′ 15″	Sec = 10·410586	Cosec = 10·035533
$\frac{1}{2}(PX - PZ)$ =	15 28 15	Cos = 9·983972	Sin = 9·426102
$\frac{1}{2}(Z + X)$ =	72 47 18	Tan = 10·508956	
$\frac{1}{2}(Z - X)$ =	20 38 35		Tan = 9·576033
Z =	93 25 53		

Tr. Az. N. 93° 25′ 53″ E.

EXERCISES.

1. August 18th, about $1^h\ 20^m$ A.M., in lat. 37° 50′ N., long. 50° W., when a chron. showed $4^h\ 37^m\ 40^s$ M.T.G., the comp. bearing of α Arietis was E. Find the true azimuth from the "Tables;" and if the variation is 14° 45′ W., what is the dev.?

2. September 14th, at $2^h\ 19^m$ A.M. M.T.S., in lat. 52° 49′ S., long. 21° 17′ W., the comp. bearing of Sirius was S.E. Find the true azimuth from the "Tables;" and if var. from chart is 33° 15′ W., find the dev.

3. September 6th, at $0^h\ 15^m$ A.M. M.T.S., in lat. 47° 43′ N., long. 48° 32′ W., the comp. bearing of Altair was N. 44° 30′ W. Find the true azimuth from the "Tables;" and the var. being 34° 40′ W., find the dev.

4. August 4th, at $11^h\ 25^m$ P.M. A.T.S., in lat. 38° 49′ S., long. 56° W., when the comp. bearing of Vega was N.N.W., calculate the true azimuth, comp. error, and dev., the var. being 17° 10′ E.

5. August 2nd, at $3^h\ 27^m$ A.M. A.T.S., in lat. 40° 50′ N., long. 50° 40′ W.,

the comp. bearing of α Orionis was E. ½ N. Using the "Tables," find the true azimuth, and comp. error; and the var. being 20° W., find the dev.

6. August 3rd, about $8^h\ 20^m$ P.M., in lat. 57° 19′ N., long. 3° 14′ E., when a chron., whose error on A.T.G. was $14^m\ 28^s$ fast, showed $8^h\ 44^m\ 32^s$, the comp. bearing of Arcturus was S. 65° 30′ W. Find from "Tables" the true azimuth; and var. being 25° W., what is the dev.?

7. September 11th, at $1^h\ 45^m$ A.M. M.T.S., in lat. 42° 35′ S., long. 85° 37′ E., the comp. bearing of Aldebaran was S. 50° E. Find true azimuth, comp. error, and dev., the var. being 20° W.

8. September 21st, when the M.T.G. by chron. was $20^d\ 20^h\ 38^m\ 20^s$, the comp. bearing of Fomalhaut, in lat. 14° 10′ N., long. 94° 50′ W., was S.W. ½ S. Find true azimuth, comp. error, and dev., the var. from chart. being 8° E.

9. September 21st, at $4^h\ 15^m$ A.M. A.T.S., in lat. 21° 17′ S., long. 87° 40′ E., the comp. bearing of Pollux was N. 40° 20′ E., var. 2° 15′ W. Find true azimuth, comp. error, and dev.

10. September 21st, about $11^h\ 30^m$ P.M., in lat. 48° 40′ S., long. 167° 30′ W., when a chron., whose error on M.T.G. was $19^m\ 24^s$ fast, indicated $11^h\ 17^m\ 24^s$, the comp. bearing of α^2 Centauri was S. ½ E. Find true azimuth, comp. error, and dev., the var. being 19° 20′ E.

11. August 2nd, at $6^h\ 40^m$ P.M. A.T.S., in lat. 5° 12′ S., long. 35° W., the comp. bearing of Dubhe (α Urs. Maj.) was N. 8° 30′ E.. The var. being 11° W., find true azimuth, comp. error, and dev.

12. August 1st, at $4^h\ 30^m\ 30^s$ A.M. M.T.S., in lat. 21° 30′ S., long. 92° 45′ E., the comp. bearing of Antares was S.W. by W. ¼ W. Find true azimuth, and comp. error; also, if var. is 9° W., what is the dev.?

13. If the declination of an object be greater than the latitude of the place, both being of the same name, find the greatest azimuth of the body, the time it will have its greatest azimuth, and its altitude.

14. In lat. of Greenwich 51° 28′ 40″ N., find greatest azimuth of α Urs. Maj. dec. 62° 16′ 50″ N.

15. Altitude of star when on the prime vertical is 25°, altitude when on the six o'clock hour circle 15°. Find its declination, and latitude of observer.

CHAPTER XVII.

LONGITUDE BY CHRONOMETER.

ART. 82. **Longitude by Chronometer.**—The time by chronometer when corrected for its original Error and accumulated Rate gives the time at Greenwich, whilst the time at ship can be calculated from an observed Altitude of a Celestial Object and the known Latitude. The difference converted into *Arc* is the Longitude.

The Longitude is E. when Greenwich time is *less* than ship time, and W. when *greater*.

The following are required in the Formula :—

(*a*) The true Altitude.

(*b*) The Polar Distance.

(*c*) The Latitude.

The **Polar Distance** is measured from the elevated Pole, which is the N. Pole in N. Lat., and the S. Pole in S. Lat. It is therefore 90° − Dec. when the Dec. is of the *same* name as the Lat., and 90° + Dec. when of a *different* name.

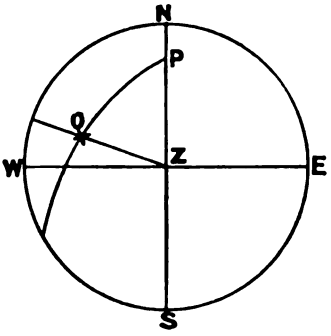


FIG. 134.

ART. 83. **To find the Hour Angle of a Celestial Object, the Altitude, Latitude, and Polar Distance being known.**—Let O be the object, then PO is its polar distance (p), ZO its zenith distance ($=90°-a$), and PZ the co-latitude ($=90°-l$); the angle ZPO is the hour angle. Therefore, in the spherical triangle PZO, three sides are given to find an angle ZPO.

The general formula for finding an angle is—

$$\cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$$

which, applied to the figure, gives—

$$\cos \text{ZPO} = \frac{\cos \text{ZO} - \cos \text{PO} \,.\, \cos \text{PZ}}{\sin \text{PO} \,.\, \sin \text{PZ}}$$

Let the letters a, l, p, and h stand for altitude, latitude, polar distance, and hour angle respectively;

$$\text{then } \cos h = \frac{\cos(90^\circ - a) - \cos p \,.\, \cos(90 - l)}{\sin p \,.\, \sin(90^\circ - l)}$$
$$= \frac{\sin a - \cos p \,.\, \sin l}{\sin p \,.\, \cos l}$$

Subtracting each side of the equation from 1—

$$1 - \cos h = 1 - \frac{\sin a - \cos p \,.\, \sin l}{\sin p \,.\, \cos l}$$
$$i.e.\ 2\sin^2\frac{h}{2} = \frac{\sin p \,.\, \cos l + \cos p \,.\, \sin l - \sin a}{\sin p \,.\, \cos l}$$
$$= \frac{\sin(p + l) - \sin a}{\sin p \,.\, \cos l}$$
$$= \frac{2\cos\frac{1}{2}(p + l + a) \,.\, \sin\frac{1}{2}(p + l - a)}{\sin p \,.\, \cos l}$$

Now let $s = \frac{1}{2}(p + l + a)$, then $(s - a) = \frac{1}{2}(p + l - a)$;

therefore—

$$\sin^2\frac{h}{2} = \frac{\cos s \,.\, \sin(s - a)}{\sin p \,.\, \cos l}$$
$$= \operatorname{cosec} p \,.\, \sec l \,.\, \cos s \,.\, \sin(s - a)$$
$$\text{and } \sin\frac{h}{2} = \sqrt{\operatorname{cosec} p \,.\, \sec l \,.\, \cos s \,.\, \sin(s - a)}$$

which is the formula used for computing the hour angle in the *longitude by chronometer* and other *time problems*.

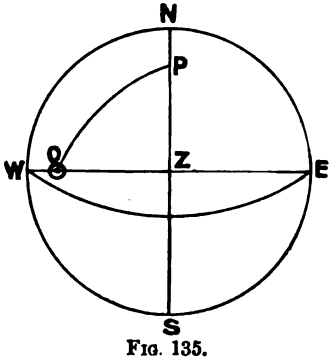


FIG. 135.

ART. 84. **To find the Time when the Sun or other Celestial Object is on the Prime Vertical.**—Let O be an object on the prime vertical; then PO is its polar distance, PZ is the co-latitude, and the angle P is the hour angle.

PZO is a spherical triangle, right angled at Z, of which the circular parts are PZ and ZO, and the complements of PO and the angles P and O.

Taking complement of P as middle part, PZ and the complement of PO are adjacent parts:

then sin (comp. of P) = tan PZ × tan (comp. of PO)
or cos P = tan PZ × cot PO
that is, cos H.A. = cot lat. × tan dec.

In the case of the Sun, the H.A. is the interval from apparent noon.

For any other object, add its R.A. to its H.A. *west* to find the R.A. of meridian, from which subtract the sun's R.A. to find the time.

ART. 85.—The following rules must be observed in Sun chronometers:—

(1) Find the correct G.M.T. by applying the error when last found, and accumulated rate, to the time shown by chronometer. As in other problems, great care must be taken to make the time shown by the chronometer agree with the rough Greenwich time deduced from D.R. long. and ship time; both as regards the *date* and *hours*.

(2) Correct declination, equation of time, and altitude. Find polar distance; and look at the heading of the equation of time column on page I., to see how it is to be applied.

(3) Add together true alt., lat., and pol. dist.; find the half sum, and from it subtract the alt.; this gives the chron. remainder. Add the secant lat., cosec pol. dist., cos half sum, and sin remainder, reject 10's, and look out result in Table XXXI., Norie; this is the hour angle.

NOTE 1.—Should the pol. dist. exceed 90°, use sec dec. for cosec pol. dist.
NOTE 2.—The lat. used must be the lat. at observation; if not given, it must be deduced from the lat. at noon.

(4) If P.M. at ship, the hour angle is the apparent time at ship of the same date as given in question.

If A.M. at ship subtract the hour angle from 24^h and date the day before day in question for app. time at ship. To the A.T.S. apply equation of time for M.T.S.

(5) The difference between the M.T.S. and M.T.G. is the longitude in time, which must be expressed in arc, for long. at obs. named west when G.M.T. is best, and east when G.M.T. is least.

Examples.—

(1) August 8th, about $3^h\ 50^m$ P.M. at ship, in lat. 56° 48′ N., long. D.R. 7° W.; when a chron. showed $4^h\ 29^m\ 34^s$, the obs. alt. of the Sun's L.L. was 30° 57′ 10″; ind. cor., −2′ 10″; height of eye, 20 ft. Find the longitude, the chron. having been found $7^m\ 46^s{\cdot}2$ fast of G.M.T. on July 18th, and losing daily $2^s{\cdot}7$.

Approx. time at ship, Aug.	8^d	$3^h\ 50^m$	elapsed time	21·2 days
Long. W.		28	rate	2·7
,, ,, Greenwich	8^d	4 18		1484
				424
			acc. loss	57·24

Time by chron., Aug.	8d 4h 29m 34s	Dec. 16° 20′ 28″·7 N	H.V. 42·16
Fast.	− 7 46·2	Cor. − 3 5·5	4·4
	4 21 47·8	16 17 23·2	16864
acc. loss	+ 57·2	90	16864
M.T.G.	8d 4 22 45	N.P.D. 73 42 37	6,0)18,5·504
			3′ 5″·5

Eq. T.	5m 33s·2	·307s
Cor.	− 1·4	4·4
	5 31·8	1228
		1228
		1·3508

Obs. alt.	30° 57′ 10″	
I.C.	− 2 10″	
	30 55 0	
Dip.	− 4 23	
	30 50 37	
S.D.	+ 15 48	
	31 6 25	
R. and P.	− 1 27	
T.A.	31 4 58	
Lat.	56 48 0	sec = 10·261566
P.D.	73 42 37	cosec = 10·017794
	2)161 35 35	
half sum	80 47 47	cos = 9·203967
remainder	49 42 49	sin = 9·882423
W.H.A.	3h 50m 26s	XXXI. = 9·365750
A.T.S., Aug.	8d 3h 50m 26s	
Eq. T.	+ 5 32	
M.T.S.	8d 3 55 58	
M.T.G.	8d 4 22 45	
Long. in time	26 47	
Long.	6° 41′ 45″ W.	

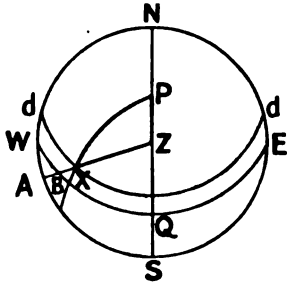


FIG. 136.

Construction of Fig. 136.—Draw circle NESW representing rational horizon, and NZS meridian. Measure NP = ZQ = 57° lat.; set off dec. = 16 to north, and draw EQW for equinoctial, and *dd* parallel of declination. With centre Z and radius = zen. dist. = 59° draw an arc cutting *dd* in X for position of sun, draw hour circle PXB, and vertical circle ZXA.

ZQ = lat. PZ = co. lat.
AX = alt. ZX = zen. dist.
BX = dec. PX = pol. dist.
ZPX = Hour angle.

To find H.A. from figure.

In triangle PZX; given PZ = co-lat., PX = pol. dist., ZX = zen. dist. Req. P = Hour angle.

$$\text{Cos}\,\frac{P}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - ZX)}{\sin PZ \,.\, \sin PX}}$$

where $s = \frac{1}{2}(PZ + ZX + PX)$.

ZX = 58° 55′ 2″		
PZ = 33 12 0	cosec	= 10·261566
PX = 73 42′ 37	cosec	= 10·017794
2)165 49 39		
s = 82 54 49	sin	= 9·996670
s − ZX = 23 59 47	sin	= 9·609252
		2)19·885282
$\frac{P}{2}$ = 1ʰ 55ᵐ 13ˢ	cos	= 9·942641
2		
W.H.A. 3 50 26		

(2) September 1st, about 9ʰ 15ᵐ A.M., in lat. at noon 46° 1′ 30″ S., D.R. long. 27° E.; when a chron. showed 7ʰ 19ᵐ 43ˢ, and had been found to be fast 3ᵐ 11ˢ·5 on May 3rd, and slow 12ˢ on June 27th, the alt. of Sun's L.L. was 24° 14′ 40″; ind. cor., − 3′ 15″; eye, 29 ft. Find long. at sights; and if run from obs. to noon was S. 70° E. 30 miles, find long. at noon.

Approx. time at ship, Aug. 31ᵈ 21ʰ 15ᵐ
Long. E. − 1 48
,, ,, Greenwich 31ᵈ 19 27

May 3rd, fast 3ᵐ 11ˢ·5
June 27th, slow 12
Loss in 55 days 3 23·5

55)203·5(3ˢ·7
165
385
385
...

Time by chron., Aug. 31ᵈ 19ʰ 19ᵐ 43ˢ
slow + 12
31ᵈ 19 19 55
Acc. rate + 4 3
M.T.G. Aug. 31ᵈ 19 23 58

Rate 3ˢ·7
elapsed days 65·8
6,0)24,3·46
4ᵐ 3ˢ

Dec. 8° 31′ 39·1″ N. H.V. 54·19″ Eq. T. 0ᵐ 9ˢ·1 H.V. ·78
Cor. + 4 9 4·6 3·6 4·6
8 35 48·1 N. 6,0)24,9·274 0 12·7 3·588
90 4′ 9″
S.P.D. 98 35 48 Lat. at noon 46° 1′ 30″ S.
S. 70° E., 30′ D. lat. 10 18 N.
Lat. at obs. 45 51 12 S.

Mid. Lat. 45° 56′ 21″	sec	= 10·157752
S. 70° E., 30′ = dep. 28·2	log	= 1·450249
D. long. 40·55	log	= 1·608001

Obs. alt.	24° 14′ 40″	
I.C.	−3 15	
	24 11 25	
Dip.	−5 16	
	24 6 9	
S.D.	+15 53	
	24 22 2	
R. and P.	−1 59	
T.A.	24 20 3	
Lat.	45 51 12	sec = 10·157080
P.D.	98 35 48	cosec = 10·004907
	2)168 47 3	
half sum	84 23 31·5	cos = 8·989985
remainder	60 3 28·5	sin = 9·937784
E.H.A.	2h 44m 13s XXXI.	= 9·089756
	24	
A.T.S. Aug. 31d	21h 15m 47s	
Eq. T.	+13	
M.T.S. 31d	21 16 0	
M.T.G. 31d	19 23 58	
	1 52 2	
	60	
	4)112 2	
Long. at obs.	28° 0′ 30″ E.	
D. long.	40 33 E.	
Long. at noon	28 41 3 E.	

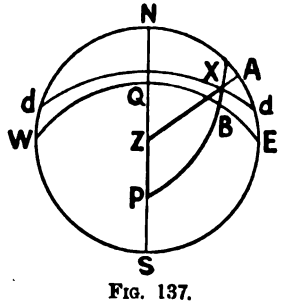


FIG. 137.

To find H.A. from figure.

In triangle PZX, given PZ = co-lat., PX = pol. dist., ZX = zen. dist. Req. P = Hour angle.

$$\text{Cos}\,\frac{P}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - ZX)}{\sin PZ \,.\, \sin PX}}$$

where $s = \frac{1}{2}(PZ + ZX + PX)$.

ZX =	65° 39′ 57″	
PZ =	44 8 48	cosec = 10·157080
PX =	98 35 48	cosec = 10·004907
	2)208 24 33	
s =	104 12 16·5	sin = 9·986515
$s - ZX$ =	38 32 19·5	sin = 9·794518
		2)19·943020
$\frac{P}{2}$ =	1h 22m 6s·5	cos = 9·971510
	2	
E.H.A.	2 44 13	

In Fig. 137, drawn as before explained.

ZQ = lat. PZ = co. lat.
AX = alt. ZX = zen. dist.
BX = dec. PX = pol. dist.
ZPX = Hour angle

EXERCISES.

1. August 4th, about $8^h\ 40^m$ A.M., in lat. at noon 38° 15′ S., D.R. long. 92° E. ; when a chron. showed $1^h\ 15^m\ 5^s$, which had been found slow $1^h\ 17^m\ 9^s$ on June 20th for G.M. noon, and slow $1^h\ 18^m\ 57^s$ on July 5th, the obs. alt. of the Sun's L.L. was 16° 42′ 10″ ; ind. cor., + 2′ 15″ ; eye, 29 ft. Find long. at obs. ; and if run since obs. was N. 24°, E. 35 miles, find long. at noon.

2. August 19th, about $4^h\ 30^m$ P.M., in lat. at noon 42° 3′ N., D.R. long. 70° W.; when a chron. showed $8^h\ 58^m\ 5^s$, which had been found slow $15^m\ 8^s{\cdot}3$ on June 27th for G.M. noon, and slow $14^m\ 22^s{\cdot}7$ on July 12th, the obs. alt. of the Sun's U.L. was 26° 32′ 40″ ; ind. cor., − 2′ 7″ ; eye, 34 ft., run since noon, S. 32° E. 46 miles. Find long. at obs. and noon.

3. August 31st, about $9^h\ 20^m$ A.M., in lat. at noon 52° 36′ N., D.R. long. 3° 15′ W. ; when a chron. showed $9^h\ 47^m\ 53^s$, which had been found fast $12^m\ 18^s{\cdot}8$ on August 1st for G.M. noon, and fast $12^m\ 14^s$ on August 7th, the obs. alt. of the Sun's U.L. was 36° 26′ 10″ ; ind. cor., − 26″ ; eye, 41 ft., run since obs. N. 64° W. 25 miles. Find long. at obs. and noon.

4. September 1st, about $8^h\ 40^m$ A.M., in lat. at noon 44° 42′ N., long. 11° W. ; when a chron. showed $9^h\ 34^m\ 3^s$, which was slow 4^s on April 17th for G.M. noon, and on June 6th was $4^m\ 8^s$ fast ; run since sights, N.W. ½ N. 31 miles, the obs. alt. of Sun's U.L. was 34° 21′ 30″ ; ind. cor., + 4′ 15″ ; eye, 23 ft. Find long. at sts. and noon.

5. September 23rd, about $3^h\ 38^m$ P.M., lat. at noon 49° 40′ S., long. 135° E. ; ship becalmed ; obs. alt. of Sun's U.L. 22° 18′ 50″ ; eye, 20 ft. ; time by chron., $6^h\ 27^m\ 28^s$, which was slow for G.M. noon on July 10th $2^m\ 10^s$, and has since kept mean time ; course and distance since obs., *nil.* Find long. at obs. and noon.

6. September 30th, about $3^h\ 30^m$ P.M., lat. at noon 49° 30′ 12″ S., long. 180° E., the obs. alt. of Sun's L.L. was 25° 3′ 20″ ; ind. cor., + 3′ 14″ ; eye, 37 ft. ; time by chron., $3^h\ 54^m\ 7^s$, which had been found $8^m\ 16^s{\cdot}4$ fast on May 6th, and fast $20^m\ 24^s{\cdot}4$ on July 15th for G.M. noon ; run since noon, S. 73° ½ W. 24 miles. Find long. at obs. and noon.

ART. 86.—Rules for Longitude by Star Chronometers :—

(1) Find G.M.T. as before.

(2) Take out star's R.A. and Dec., find pol. dist., and correct altitude, and R.A.M.S. (sidereal time).

(3) Find H.A. as before.

(4) If H.A. is west, add H.A. to R.A. of star for R.A.M. If H.A. is east, subtract H.A. from R.A. of star for R.A.M. From the R.A.M. subtract the R.A.M.S., this gives the M.T.S., which is of same date as question if P.M. at ship, and the day before date of question if A.M. at ship.

(5) Finish as before for longitude.

Example.—August 1st, about $2^h\ 39^m$ A.M. at ship, in lat. 30° 26′ N., long. 133° E. by D.R. ; the obs. alt. of Vega was 33° 38′ 50″ ; time by chron., $4^h\ 41^m\ 58^s$, which was slow $1^h\ 2^m\ 53^s$ of G.M. noon on June 26th, and losing $0^s{\cdot}4$ daily ; ind. cor., − 2′ 29″ ; eye, 23 ft. Find the long.

Approx. ship time July	$31^d\ 14^h\ 39^m$	Elapsed time	35·2
Long. E.	− 8 52	rate	·4
„ Greenwich time July	$31^d\ 5\ 47$		14·08

Time by chron., July 31d	4h	41m	58s
slow	+ 1	2	53
31d	5	44	51
Acc. loss			+ 14
M.T.G. 31d	5	45	5
R A. Vega	18h	33m	45s·4

R.A.M.S.	8h	36m	44s·2 for Aug. 1st.
Cor.	− 2	59·9	
	8	23	44·3
Dec.	38°	41′	45″ N.
	90		
N.P.D.	51	18	15

In Fig. 138, constructed as before.

ZQ = lat. PZ = co. lat.
AX = alt. ZX = zen. dist.
BX = dec. PX = pol. dist.
ZPX = Hour angle.
∠ZX = Azimuth.

Obs. alt.*	33°	38′	50″		
I.C.	− 2	29			
	33	36	21		
Dip.	− 4	42			
	33	31	39		
Ref.	− 1	26			
T.A.	33	30	13		
Lat.	30	26	0	sec =	10·064382
P.D.	51	18	15	cosec =	10·107641
	2)115	14	28		
half sum	57	37	14	cos =	9·728778
remainder	24	7	1	sin =	9·611299
W.H A.*	4h	38m	8s	XXXI. =	9·512100
R.A.*	18	33	45		
R.A.M.	23	11	53		
R.A.M.S.	8	33	44		
M.T.S. 31d	14	38	9		
M.T.G. 31d	5	45	5		
	8	53	4		
	60				
	4)533	4			
Long.	133°	16′	E.		

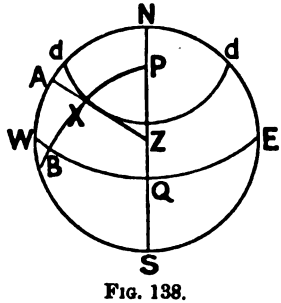


FIG. 138.

To find H.A. from figure.

In triangle PZX, X is object, PZ = co-lat., PX = pol. dist., ZX = zen. dist. Req. P = Hour angle.

$$\text{Cos}\,\frac{P}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - ZX)}{\sin PZ \,.\, \sin PX}}$$

where $s = \frac{1}{2}(ZX + PZ + ZX)$.

ZX =	56° 29′ 47″	
PZ =	59 34 0	cosec = 10·064382
PX =	51 18 15	cosec = 10·107641
	2)167 22 2	
s =	83 41 1	sin = 9·997355
s − ZX =	27 11 14	sin = 9·659820
		2)19·829198
$\frac{P}{2}$ =	2ʰ 19ᵐ 4ˢ	cos = 9·914599
	2	
W.H.A.*	4 38 8	

EXERCISES.

1. September 10th, about 4ʰ 30ᵐ A.M., in lat. 35° 47′ 30″ S., long. D.R. 111° E.; the obs. alt. of Achernar west of meridian was 59° 28′ 40″; ind. cor., + 2′ 12″; eye, 35 ft.; when a chron. showed 9ʰ 7ᵐ 12ˢ, which had been found fast 1ᵐ 3ˢ on May 12th of G.M. noon, and on August 20th was slow 40ˢ. Find the long.

2. August 24th, about 4ʰ 40ᵐ A.M., in lat. 10° 14′ 20″ N., long. D.R. 75° W.; the obs. alt. of Canopus east of the meridian was 12° 15′ 40″; ind. cor., − 3′ 13″; height of eye, 48 ft.; time shown by chron., 9ʰ 21ᵐ 52ˢ, which had been found slow of G.M. noon 17ᵐ 52ˢ·7 on July 13th, and on July 31st was slow 17ᵐ 4ˢ·1. Find the long.

3. August 1st, P.M. at ship, in lat. 24° 19′ S., long. by acct. 14° 30′ E.; when a chron. showed 8ʰ 15ᵐ 4ˢ, which had been found fast 28ᵐ 19ˢ on June 24th, and losing 1ˢ·2 daily; the obs. alt. of Altair east of the meridian was 42° 5′ 30″; ind. cor., + 3′ 54″; height of eye, 42 ft. Required the long.

4. September 11th, P.M. at ship, in lat. equator, long. of Greenwich; the obs. alt. of Vega west of the meridian was 26° 18′; ind. cor., − 1′ 55″; eye, 31 ft.; when a chron. showed 11ʰ 10ᵐ 54ˢ, which had been found 12ᵐ 29ˢ fast of G.M. noon on July 15th, and on August 9th was 13ᵐ 29ˢ fast. Find the long.

5. September 21st, about 1ʰ 30ᵐ A.M., in lat. 20° 30′ N., long. by acct. 31° W.; the obs. alt. of Capella east of meridian was 37° 14′ 30″; ind. cor., + 3′ 49″; eye, 19 ft.; when a chron. showed 3ʰ 21ᵐ 26ˢ, which had been found 3ˢ fast of G.M. noon on August 7th, and on August 31st was 3ᵐ 21ˢ slow. Find the long.

6. August 24th, about 1ʰ 15ᵐ A.M., in lat. 18° 12′ 30″ N., long. D.R. 91° E.; when a chron. showed 9ʰ 24ᵐ 17ˢ, which had been found fast 11ᵐ 43ˢ·8 on June 4th for G.M. noon, and on July 12th was fast 12ᵐ 18ˢ; the obs. alt. of Rigel east of meridian was 27° 26′; ind. cor., − 3′ 5″; eye, 46 ft. Find the long.

7. September 8th, about 5ʰ A.M., in lat. 43° 24′ S., long. by acct. 110° 45′ E.; when a chron. showed 9ʰ 41ᵐ 51ˢ, which had been found fast of G.M. noon 28ˢ·8 on August 23rd, and on August 31st showed G.M.T.; the obs. alt. of Procyon was 22° 48′ 50″; ind. cor., + 54″; height of eye, 30 ft. Find the long.

8. August 30th, about 10ʰ P.M., in lat. 35° 15′ N., long. D.R. 169° 30′ W.; when a chron. showed 9ʰ 12ᵐ 34ˢ, which had been found slow of G.M. noon on June 14th 6ᵐ 54ˢ·1, and on July 25th was correct; the obs. alt. of Fomalhaut was 15° 54′ 10″; ind. cor., — 1′ 26″; eye, 25 ft. Find the long.

9. September 2nd, about 10ʰ 15ᵐ P.M., in lat. 15° 14′ S., long. D.R. 50° E.; when a chron. showed 6ʰ 50ᵐ 41ˢ, which had been found correct for G.M.

noon on July 4th, and on August 18th was slow 3m 9s; the obs. alt. of Antares was 25° 21′; no ind. cor.; eye, 27 ft. Find the long.

10. August 14th, about 2h A.M., in lat. 40° 27′ 15″ N., long. by acct. 180° W.; the obs. alt. of Aldebaran was 22° 8′; ind. cor., — 22″; eye, 38 ft.; when a chron. showed 2h 7m 58s, which had been found slow 2m 33s·6 on June 25th, and on Aug. 4th was 4m 10s fast. Find the long.

11. August 28th, P.M. at ship, in lat. by mer. alt. of α Ophiuchi in the art. horizon on shore, 15° 55′ S., long. by cross-bearings 4° 45′ W.; the obs. alt. of Arcturus west of the meridian was 35°; ind. cor., — 4′; height of eye, 15 ft.; time by chron., 5h 58m 9s, which had been found slow of G.M. noon 1h 10m 15s on June 29th, and slow 1h 3m 33s on Aug. 8th. Find the long.

12. September 28th, about 6 P.M., in lat. 50° 17′ N. by mer. alt. of Vega, long. D.R. 161° E.; the obs. alt. of α Cygni was 68° 30′; eye, 22 ft.; time by chron., 7h 25m 21s·2, which was slow 2s·4 of G.M. noon on June 29th, and fast 25s·6 on August 3rd. Find the long.

ART. 87. **Longitude and Azimuth.**—As the data used in the computation of *Apparent Time at Ship* and the *Azimuth* are the same, and the Formulæ very similar, it is most convenient to find the Azimuth at the same time as the longitude by chronometer.

Example.—

Using data as in star chronometer example (Fig. 138), the logs may be conveniently arranged for chron. az. as follows:—

True alt.	33° 30′ 13″		sec = 10·078911
Lat.	30 26 0	sec = 10·064382	sec = 10·064382
P.D.	51 18 15	cosec = 10·107641	
	2)115 14 28		
	57 37 14	cos = 9·728778	cos = 9·728778
Chron. remdr.	24 7 1	sin = 9·611299	
Az. remdr.	6 18 59		cos = 9·997355
W.H.A.	4h 38m 8s	XXXI. = 9·512100	2)19·869426
		59° 21′ 52″	sin = 9·934713
		2	

True az. S. 118 43 44 W.

EXERCISES.

1. August 31st, about 5h P.M., at ship in lat. at noon 0° 21′ S., long. D.R 136° W.; the obs. alt. of the Sun's L.L. was 13° 13′ 40″; ind. cor., + 7′ 21″; eye, 17 ft.; bearing by compass W. ¼ N.; time shown by chron., 2h 5m 44s, which was slow 25m 19s on April 12th for G.M. noon, and on July 1st was 11m 55s slow. Find the long. at sights and noon, if the run since noon was N.E., 58 miles. Also find true azimuth, comp. error, and dev., the var. being 5° 40′ E.

2. October 1st, about 7h 15m A.M. at ship, in lat. 39° 30′ S., long. by acct. 17° W.; the obs. alt. of the Sun's L.L. was 16° 40′ 30″; ind. cor., + 6′ 12″;

eye, 20 ft.; bearing by compass S.E. by E. ¼ E.; time by chron., 8ʰ 22ᵐ 40ˢ·4, which had been found fast 1ᵐ 41ˢ·1 for G.M. noon on March 30th, and on June 1st was 2ᵐ 25ˢ·2 fast. Find the long., true bearing, and dev., the var. being 12° 15′ W.

3. September 1st, about 3ʰ 30ᵐ P.M. at ship, in lat. at noon 51° S., long. D.R. 175° E.; the obs. alt. of the Sun's L.L. was 14° 52′ 20″; ind. cor., − 7′ 10″; eye, 15 ft.; bearing by compass N. 50° 30′ W.; time shown by chron., 3ʰ 44ᵐ 40ˢ, which had been found 13ᵐ 0ˢ·3 slow for G.M. noon on February 15th, and on August 1st was 10ᵐ 30ˢ slow. Find long. at sights and noon, the run since noon being S.W. by W. ½ W. 53 miles. Also find true bearing, comp. error, and dev., the var. being 16° 10′ E.

4. September 23rd, about 7ʰ A.M. at ship, in lat. at noon 32° S., long. D.R. 55° E.; the obs. alt. of the Sun's L.L. was 10° 36′; ind. cor., − 2′ 10″; eye, 19 ft.; bearing by compass East; time shown by chron., 3ʰ 0ᵐ 35ˢ, which had been found 5ᵐ 45ˢ·5 slow for G.M. noon on March 31st, and on June 14th was slow 4ᵐ 45ˢ·5; run since sights, N. 59° W. 58 miles. Find long. at sights and noon. The var. from chart being 15° 40′ W., find true bearing, comp. error, and dev.

5. September 1st, about 1ʰ P.M. at ship, in lat. 11° 34′ 20″ S., long. D.R. 144° E.; the obs. alt. of the Sun's L.L. was 65° 2′; ind. cor., − 7′ 17″; eye, 27 ft.; bearing by compass N. 40° W.; time shown by chron., 2ʰ 32ᵐ 48ˢ, which had been found slow 1ʰ 0ᵐ 10ˢ for G.M. noon on April 1st, and on May 1st was 58ᵐ 40ˢ slow. Find the long. and the var. being 5° 45′ E., find comp. error and dev.

6. September 22nd, about 8ʰ 50ᵐ A.M., when the chron. showed 0ʰ 20ᵐ 32ˢ; the obs. alt. of the Sun's U.L. was 30° 59′ 30″; ind. cor., 2′ 5″ off the arc; height of eye, 30 ft.; bearing by compass S.E. ¼ E.; var., 11° 30′ W.; lat. at obs., 42° 16′ N.; long. by acct., 70° 40′ W.; on June 16th the chron. was slow 1ʰ 14ᵐ 30ˢ for G.M. noon, and on August 5th it was 1ʰ 11ᵐ 8ˢ slow. Required long., true azimuth, and dev.

7. September 23rd, about 1ʰ 45ᵐ P.M., lat. at noon, 40° 18′ S.; the obs. alt. of the Sun's L.L. was 42° 55′ 40″; eye, 7 ft.; long by acct., 148° E.; time by chron., 3ʰ 53ᵐ 29ˢ, which had been found fast 16ᵐ 20ˢ for G.M. noon on May 4th, and on August 2nd was 10ᵐ 47ˢ fast; run from noon to obs., W.N.W. 21 miles; the sun's comp. bearing was N.W., and the var. 10° 45′ E. Find the long., true bearing, and dev. for the direction of the ship's head.

8. August 5th, about 2ʰ 30ᵐ P.M., in lat. at noon 30° 4′ 45″ S., long. by acct. 94° E.; the obs. alt. of the Sun's U.L. was 31° 32′; ind. cor., − 4′ 44″; height of eye, 19 ft.; bearing by compass, N.W. ½ N.; the chron. showed 8ʰ 19ᵐ 41ˢ, and had been found to be fast of G.M.T. on May 23rd 7ᵐ 33ˢ, and losing daily 2½ˢ. Find the long. at sights and noon; and if the var. from chart was 14° 20′ W., find true azimuth and dev. for the direction of the ship's head. Run since noon, S. 40° E. 30 miles.

9. September 1st, about 4ʰ P.M., in lat. 2° 10′ N., long. D.R. 55° 45′ E.; the chron. showed 10ʰ 57ᵐ 5ˢ; the obs. alt. of the Sun's U.L. was 29° 11′ 40″; ind. cor., − 50″; eye, 22 ft.; bearing by compass N. 88° 30′ W.; on March 24th the chron. was slow of G.M.T. 56ᵐ 34ˢ, and on May 4th it was slow 1ʰ 3ᵐ 27ˢ. Find the long., true azimuth, and dev., the var. being 4° 15′ W.

10. September 1st, about 4ʰ 45ᵐ P.M., in lat. at noon 0° 36′ N.; long. D.R. 93° W.; the obs. alt. of the Sun's L.L. was 18° 3′; height of eye, 32 ft.; ind. cor., − 3′ 27″; bearing by compass S. 87° 15′ W.; the chron. showed 10ʰ 57ᵐ 27ˢ, and had been found correct for M.T.G. on June 30th, and slow 4ᵐ 50ˢ on July 27th; run from noon to obs., S. 35° W. 44 miles. Find the long. at sights and noon; also if the var. is 2° 30′ E., find true azimuth and dev.

11. August 20th, about 8ʰ 20ᵐ A.M., in lat. at noon 12° 31′ S., long. by acct. 168° W., when a chron. showed 7ʰ 19ᵐ 19ˢ, which had been found 18ᵐ 19ˢ slow for G.M. noon on June 8th, and gaining daily ·9ˢ; the obs. alt. of the Sun's L.L. was 29° 46′ 10″; ind. cor., − 2′ 22″; eye, 45 ft.; run since obs.,

S. 62° W. 34 miles; Sun's comp. bearing was N. 68° 30′ E.; var., 9° 15′ E. Find long. at obs. and noon, true azimuth, and dev.

12. September 23rd P.M., in lat. at noon 8° 44′ N., long. by acct. 174° W.; the obs. alt. of the Sun's U.L. was 53° 29′ 20″; ind. cor., − 3′ 19″; eye, 41 ft.; bearing by compass, W. by S. ½ S.; var., 8° 40′ E.; time by chron., $1^h\ 47^m\ 27^s$, which had been found $1^m\ 58^s$ slow for G.M. noon on August 18th, and losing daily $1^s.1$; run from noon to obs., N. 28° E. 26 miles. Find long. at sights and noon, true azimuth, and dev.

CHAPTER XVIII.

SUMNER'S METHOD BY PROJECTION.

ART. 88.—At any given moment the Sun is vertical over some place on the Earth's surface, and if a circle be drawn with this point as the centre, the Sun would have the same altitude from all points on the circumference of this circle.

Advantage is taken of this fact to find the Ship's Position by Projection on a chart.

Two points on the circumference of the circle can be found from the Sun's altitude, and two assumed latitudes; the projection on the chart of the arc of the circle of equal altitudes passing through these two positions would pass through the actual position of the ship.

After an interval of time another altitude is observed, and with the same assumed latitudes, two other points on a new circle of position are found, and projected on the chart in the same way. The intersection of the two projected lines gives the ship's position.

In practice, the longitudes computed are marked on Mercator's Chart on the parallels of assumed latitudes, and connected by straight lines called *Lines of Position*. The point of intersection of these lines gives the latitude and longitude of the ship.

The Sun's true bearing is at right angles to the line of position at each observation — towards the east if A.M., and towards the west if P.M.

If the ship changes place during the interval, the first line of position has to be moved (parallel to itself) according to the course and distance sailed.

Thus in figure, NESW is plane of observer's horizon, NZS the celestial meridian, X the position of object at first observation, and Y its position at second observation; Z the zenith, ZX and ZY the zenith distances of object. With X as centre and XZ as radius, describe a circle. This will be a circle of position for first observation, because the zenith of all places having the object at X would be situated on its circumference. Similarly, with Y as centre and YZ as radius, draw a second circle of

position cutting the first in the points Z and C. Hence either Z or C is the position of the observer. With suitably selected observations, these points are so wide apart that the true one may be chosen by comparison with the latitude by account. The lines ZXA, ZYB are portions of vertical circles, showing the bearings of the objects, and are seen to cut the circumferences of the circles of position at right angles, hence the bearing of the object is always at right angles to the line of position, which is a tangent to the circle of position at Z.

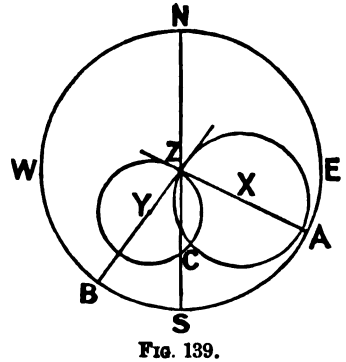


FIG. 139.

ART. 89. Rules for working Sumner problem—

(1) Find G.M.T.'s.

(2) Find correct declination for each time, and polar distances, correct observed altitudes.

(3) With the first altitude and polar distance, using the two assumed latitudes, work as in chronometer problem for two-hour angles, and hence two longitudes. These when projected on the chart and joined would give the first line of position.

With the second altitude and polar distance, using the two assumed latitudes, find as before two longitudes. These positions are projected on the chart and joined for the second line of position.

The direction of a line of position is taken from the chart, like a course, and may be named in either of the two ways the line trends.

The true bearings are at right angles to the lines of position, and are obtained by subtracting the direction of each line from 90°; and marked towards the east for E.H.A.'s; and towards the west for W.H.A.'s.

The run is laid off from any point of the first line, and where a parallel line to the first line, through the end of the run, cuts the second line is the position of the ship.

Example.—

August 5th, about $9^h\ 15^m$ A.M., in long. by account 55° 15′ W.; when a chron. correct for M.T.G. showed $1^h\ 2^m\ 21^s$, the obs. alt of the Sun's L.L. was 44° 14′ 20″; ind. cor., − 2′ 7″; eye, 30 ft.; and again P.M. at ship on the same day, when the chron. showed $7^h\ 5^m\ 23^s$, the obs. alt. of the Sun's L.L. was 39° 21′ 25″; run in the interval, S. ½ W. 63 miles. Find the line of position and Sun's true bearing at the first obs., and position of the ship at the second obs. by Sumner's method, by projection, assuming lats. 47° and 47° 45′ N.

First M.T.G., August $5^d\ 1^h\ 2^m\ 21^s$.			Second M.T.G., August $5^d\ 7^h\ 5^m\ 23^s$.		
Dec. 17° 9′ 52″·4 N.		H.V. 40·14	Dec. 17° 9′ 52″·4 N.		H.V. 40·14
	41·3	1·03		4 45	7·1
17 9 11		41·3442	17 5 7		284·994
90			90		
N.P.D. 72 50 49			N.P.D. 72 54 53		
Eq. T. $5^m\ 52^s·6$		H.V. ·23	Eq. T. $5^m\ 52^s·6$		H.V. ·23
	·2	1		1·6	7·1
5 52·4		·23	5 51		1·633
Obs. alt. ⊙ 44° 14′ 20″			Obs. alt. ⊙ 39° 21′ 25″		
I.C.	− 2 7		I.C.	− 2 7	
	44 12 13			39 19 18	
Dip	− 5 22		Dip	− 5 22	
	44 6 51			39 13 56	
S.D.	+ 15 48		S.D.	+ 15 48	
	44 22 39			39 29 44	
Cor.	− 53		Cor.	− 1 2	
T.A.	44 21 46	44° 21′ 46″		39 28 42	39° 28′ 42″
Lat.	47 0 0	47 45 0		47 0 0	47 45 0
P.D.	72 50 49	72 50 49		72 54 53	72 54 53
	164 12 35	164 57 35		159 23 35	160 8 35
½ Sum	82 6 18	82 28 48		79 41 47	80 4 17
Remdr.	37 44 30	38 7 0		40 13 5	40 35 35
Secs	10·166217	10·172394		10·166217	10·172394
Cosecs	10·019760	10·019760		10·019602	10·019602
Cos	9·137854	9·116849		9·252525	9·236589
Sine	9·786824	9·790471		9·810030	9·813369
	9·110655	9·099474		9·248374	9·241954
H.A.'s	$2^h\ 48^m\ 24^s$	$2^h\ 46^m\ 9^s$		$3^h\ 19^m\ 8^s$	$3^h\ 17^m\ 34^s$
A.T.S.	$4^d\ 21\ 11\ 36$	$4^d\ 21\ 13\ 51$		$5^d\ 3\ 19\ 8$	$5^d\ 3\ 17\ 34$
Eq. T.	+ 5 52	+ 5 52		+ 5 51	+ 5 51
M.T.S.	$4^d\ 21\ 17\ 28$	$4^d\ 21\ 19\ 43$		$5^d\ 3\ 24\ 59$	$5^d\ 3\ 23\ 25$
M.T.G.	$5^d\ 1\ 2\ 21$	$5^d\ 1\ 2\ 21$		$5^d\ 7\ 5\ 23$	$5^d\ 7\ 5\ 23$
	3 44 53	3 42 38		3 40 24	3 41 58

Long. (*a*) 56° 13′ 15″ W.; (*b*) 55° 39′ 30″ W.; (*c*) 55° 6′ 0″ W.; (*d*) 55° 29′ 30″ W.

In Fig. 140 NESW is Ratl. Horizon.

NZS is Meridian.

EQW is Equinoctial.

P, pole. Z, zenith. X and Y positions of sun. Complete figure and

also draw circles of position with X and Y as centres and XZ and YZ as radii.

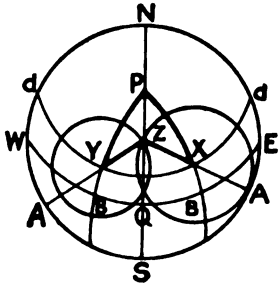


FIG. 140.

AX and AY are alts. ZQ is lat.
BX and BY are decs.
PX and PY are pol. dist.
ZPX and ZPY are hour angles.
SZX and SZY are azimuths.

Projection.—Mark the longitudes (*a*) and (*b*) on their respective latitudes. The line *ab* is the first line of position. Next set off 63 miles on a S. ½ W. course from *any* point of this line, and through the end of the run draw a parallel line to *ab*. This is the corrected position of the first line in consequence of the ship's change of place during the interval. At the time of taking the second observation the ship was somewhere on this line.

Then mark the longitudes (*c*) and (*d*) on their latitudes; *cd* is the second line of position, and at the time of taking the second observation the ship was somewhere on *cd*, consequently the intersection of *cd* with the parallel line drawn before is the position of the ship.

To obtain the sun's bearing, take off the corresponding line of position, subtract from 90°, and mark.

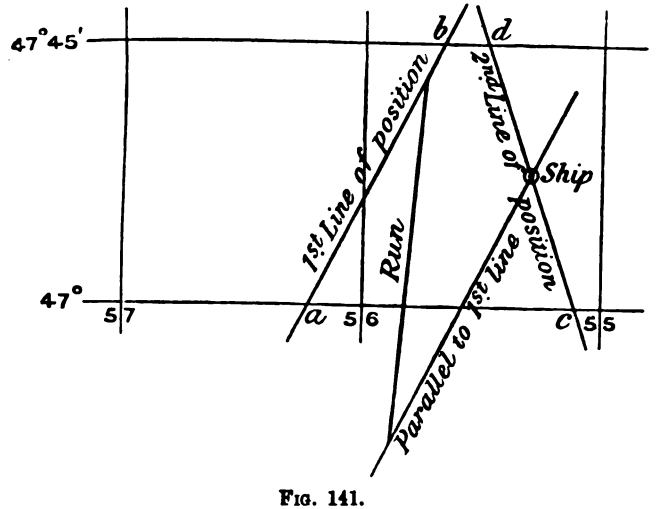


FIG. 141.

Ans. Direction of first line of position, N. 28° E. or S. 28° W.; Sun's true bearing, S. 62° E.; ship's position, lat. 47° 23′ N., long. 55° 18′ W.

EXERCISES.

1. August 4th, about $8^h\ 15^m$ A.M. at ship, long. by account 64° E.; when a chron. showed $4^h\ 11^m\ 45^s$, the obs. alt. of the Sun's L.L. was 36° 15′ 40″; and again P.M. at ship on the same day, when a chron. showed $10^h\ 36^m\ 45^s$, the obs. alt. of the Sun's L.L. was 43° 36′ 40″; chron. correct for G.M.T.; ind. cor., + 2′ 10″; height of eye, 39 ft.; the ship made a true course of W. ½ S. for 32 miles during the interval. Find the first line of position, the Sun's true azimuth, and position of the ship at the time of taking the second observation, by Sumner's method by projection on the chart, assuming lats. 48° 30′ N. and 49° N.

2. August 9th, about $8^h\ 50^m$ A.M. at ship, long. by account 4° W.; when a chron. which was correct for M.T.G. showed $9^h\ 8^m\ 39^s$, the obs. alt. of the Sun's U.L. was 38° 47′ 10″; and again P.M. of the same day, when the chron. showed $3^h\ 16^m\ 0^s$, the obs. alt. of the Sun's U.L. was 40° 40′ 30″; height of eye, 40 ft.; ind. cor., + 2′ 17″; the ship having made 72 miles S. 60° E. by compass, var. 20° W., dev. 10° W. in the interval. Find the lines of bearing and azimuths at both observations, also ship's position at second observation by Sumner's method by projection on the chart, assuming lats. 49° 20′ N. and 50° N.

3. August 24th, about $7^h\ 40^m$ A.M. at ship, long. Greenwich; when a chron. $7^m\ 40^s$ slow of G.M.T. showed $7^h\ 30^m\ 4^s$, the obs. alt. of the Sun's U.L. was 24° 33′ 50″; and again A.M. on the same day, when the same chron. showed $10^h\ 54^m\ 1^s$, the obs. alt. of the Sun's U.L. was 49° 9′ 20″; ind. cor., − 1′ 15″; height of eye, 27 ft.; the ship's true course in the interval was W. by N., 47 miles. Required the line of position and Sun's azimuth at the first observation, and position of ship at the second observation, by Sumner's method, assuming lats. 49° 50′ N. and 50° 20′ N.

4. September 1st, long. by account 180° E., with following observations find, by Sumner's method, the first line of position, the Sun's true bearing, and position of ship at second observation, assuming lats. 48° 40′ N. and 49° 20′ N.

Approx. ship time.	Time by chron.	Obs. alts.	
$0^h\ 30^m$ P.M.	$12^h\ 42^m\ 57^s$	$\overline{\odot}$ 48° 51′ 0″	ind. cor., + 1′ 10″
4 40 P.M.	4 50 20	$\overline{\odot}$ 19 30 20	eye, 17 ft.

Run during interval W.S.W., 6½ knots per hour. The chron. was fast $5^m\ 13^s$ for G.M.T. on May 12th, and gaining daily 2^s.

5. September 23rd, about $8^h\ 40^m$ A.M., long. D.R. 60° W.; when a chron. showed $12^h\ 37^m\ 55^s$, the obs. alt. of the Sun's U.L. was 27° 33′; and again P.M. on the same day, when a chron. showed $6^h\ 29^m\ 59^s$, the obs. alt. of the Sun's U.L. was 32° 53′ 40″; ind. cor., − 3′ 57″; eye, 25 ft.; the ship having made 23 miles on a true W. by S. ¼ S. course in the interval. Required the line of position and true bearing of the Sun at the first observation, and ship's position at second observation, by Sumner's method by projection on the chart, assuming lats. 46° 10′ S. and 46° 50′ S.

6. September 1st, about $8^h\ 45^m$ A.M., long. by account 110° W.; when a chron. showed $4^h\ 4^m\ 29^s$ G.M.T.; the obs. alt. of the Sun's L.L. was 18° 16′ 40″; and again P.M. on the same day, when the chron. showed $9^h\ 24^m\ 43^s$, the obs. alt. of the Sun's L.L. was 25° 52′ 50″; ind. cor., + 40″; eye, 12 ft.; the ship having made 42 miles on a true S.E. by S. ½ S. course in the interval. Find the line of position and sun's true bearing when the first altitude was taken, and ship's position at the second observation, by Sumner's method by projection on the chart, assuming lats. 49° 15′ S. and 49° 45′ S.

7. August 1st, about $5^h\ 30^m$ A.M., long. by account 166° E.; when a chron. correct for G.M.T. showed $7^h\ 39^m\ 17^s$, the obs. alt. of the Sun's L.L. was 20°

59′ 40″, and again A.M. on the same day, when the chron. showed 12ʰ 7ᵐ 57ˢ, the obs. alt. of the Sun's L.L. was 57° 6′ 10″ ; ind. cor., − 3′ 7″ ; eye, 36 ft.; the ship having made a true N. by W. ½ W. course 40 miles in the interval. Find the first line of position, the Sun's true bearing, and ship's position at the second observation by Sumner's method by projection on the chart, assuming lats. 49° 50′ N. and 50° 20′ N.

8. September 29th, in long. by account 180° E.; with following observations, assuming lats. 47° N. and 47° 50′ N., find by Sumner's method first line of position and Sun's true bearing; also position of ship at second observation.

Approx. ship time.	Time by chron.	Obs. alt. Sun's L.L.	
7ʰ 30ᵐ A.M.	7ʰ 29ᵐ 59ˢ	10° 53′ 20″	eye, 16 ft.
3 30 P.M.	3 26 39	26 48 20	ind. cor., + 1′ 15″

Run between observations, W. by S. ¾ S. 86 miles. The chron. was 22ᵐ 12ˢ fast of G.M.T. on Sept. 1st, and gaining 2ˢ·5 daily.

CHAPTER XIX.

THE CHART.

ART. 90.—Charts used by Navigators are all constructed on Mercator's principle (except those of Harbours, Bays, etc.). Their characteristic is that the spaces between the Parallels of Latitude increase towards the Poles, whilst the Meridians of Longitude are also parallel instead of converging as they do on a globe. The reason for this construction is explained in Chap. I.

Latitudes are marked on the *Graduated Meridians* at the sides of the chart, and longitudes on the *Graduated Parallels* at the top and bottom.

Soundings in fathoms for Mean Low Water of Ordinary Spring Tides are given in small figures, and Roman numerals at various places give the Time of High Water, "Full and Change." The nature of the bottom is also shown by small letters and abbreviations.

Since 1911 the pattern of **Compasses** adopted on Admiralty Charts has been altered to show both *Magnetic* and *True*. The outer or true circle being graduated from 0° to 360°. The inner or magnetic circle is divided into quadrants, each one reading from 0° to 90°, and to quarter points. The variation is marked in the circle for a given indicated year, and its decrease per annum. True north is indicated by a *star*, magnetic north by a *fleur de lis*. Bearings are given both True and Magnetic, thus 161° (S. 12° E. mag.).

Variation Charts are charts of the oceans or of the globe with lines of equal variation drawn on them at intervals of one or two degrees. From these charts the variation at ship can be found, provided the latitude and longitude are known.

When out of sight of land, the ship's position is fixed by the latitude and longitude found from observation of heavenly bodies. Near the coasts the position can be obtained from *soundings* and from *bearings* of lights and other known objects.

ART. 91.—Using the following Deviation Tables it will now be shown how to convert the course from the chart into the corresponding compass course.

Ship's head by compass.	Card A. Deviation.	Card B. Deviation.
N.	11 E.	36 W.
N. by E.	16 E.	36 W.
N.N.E.	21 E.	30 W.
N.E. by N.	27 E.	27 W.
N.E.	31 E.	24 W.
N.E. by E.	31 E.	21 W.
E.N.E.	28 E.	17 W.
E. by N.	26 E.	12 W.
E.	24 E.	6 W.
E. by S.	20 E.	2 E.
E.S.E.	15 E.	6 E.
S.E. by E.	9 E.	12 E.
S.E.	4 E.	17 E.
S.E. by S.	1 E.	22 E.
S.S.E.	2 W.	27 E.
S. by E.	5 W.	32 E.
S.	8 W.	36 E.
S. by W.	11 W.	38 E.
S.S.W.	14 W.	37 E.
S.W. by S.	18 W.	35 E.
S.W.	21 W.	32 E.
S.W. by W.	24 W.	27 E.
W.S.W.	27 W.	20 E.
W. by S.	30 W.	12 E.
W.	26 W.	5 E.
W. by N.	23 W.	3 W.
W.N.W.	21 W.	10 W.
N.W. by W.	19 W.	16 W.
N.W.	16 W.	22 W.
N.W. by N.	11 W.	27 W.
N.N.W.	4 W.	31 W.
N. by W.	4 E.	34 W.

It is seen that the deviations are given for the compass directions of the ship's head; when the deviation is applied to the corresponding ship's head by compass, the magnetic direction of the ship's head is obtained: thus—

Card A. Comp. co. N.N.E. dev. 21° E. gives mag. co. N. 44° E.
Card B. „ N.N.E. „ 30° W. „ „ N. 7° W.

To obtain the deviation and compass course for a given magnetic course, two consecutive compass courses must be selected from the Table, so that when converted into magnetic

courses, one will be greater and the other less than the given magnetic course, then by proportion the deviation, and hence the compass course may be obtained.

Thus using Card A: Find the Compass courses for mag. courses. (*a*) N. 70° E. (*b*) S. 12° W. (*c*) N. 15° W.

EXAMPLES.

(*a*) N. 70° E.

Comp. co.	Dev.	Mag. co.
N.E. by N.	27° E.	N. 61° E.
N.E.	31 E.	N. 76 E.
(Diff. of dev.)	4	15 (Diff. of courses)
(Diff. of N. 61° E. and N. 70° E.)	9	

15)36

2° to add to 27° E.
Hence deviation is 29° E.

Comp. co. N. 70° E.
Dev. 29 E.

Mag. co. N. 41° E.

(*b*) S. 12° W.

Comp. co.	Dev.	Mag. co.
S.S.W.	14° W.	S. 9° W.
S.W. by S.	18 W.	S. 16 W.
(Diff. of dev.)	4	7 (Diff. of courses)
(Diff. of S. 9° W. and S. 12° W)	3	

7)12

2° to add to 14° W.
Hence deviation is 16° W.

Mag. co. S. 12° W.
Dev. 16 W.

Comp. S. 28° W.

(*c*) N. 15° W.

Comp. co.	Dev.	Mag. co.
N.N.W.	4° W.	N. 27° W.
N. by W.	4 E.	N. 7 W.
(Diff. of dev.)	8	20 (Diff. of courses)
(Diff. of N. 27° W. and N. 15° W.)	12	

20)96

5° to subtract from 4° W.
Hence deviation is 1° E.

Mag. co. N. 15° W.
dev. 1 E.

Comp. co. N. 16° W.

NOTE 1.—When the chart course is *true*, the *variation* must first be applied to find the magnetic course. The bearings must also be corrected for variation.

NOTE 2.—The deviation can generally be found by inspection, as the difference of deviation between two adjacent points is usually small. In the tables the deviations are purposely increased to illustrate the method.

EXERCISES.

1. Using Card A.: find Comp. co. corresponding to following Mag. co.: (*a*) N. 79° E. (*b*) East. (*c*) S.E. (*d*) S. 28° E. (*e*) S. 85° W.
2. Using Card B.: find the courses to steer by compass to make good following chart courses: (*a*) North. (*b*) East. (*c*) N. 83 W. (*d*) West.

ANSWERS.

1. (*a*) N. 48° E. (*b*) N. 60° E. (*c*) S. 52 E. (*d*) S. 27 E. (*e*) N. 73° W.
2. (*a*) N. 26° E. (*b*) S. 86½ E. (*c*) N. 83 W. (*d*) S. 80° W.

ART. 92.—When a vessel is steaming along a given course on the chart at a known rate, her estimated or *expected D.R.* position on that course can always be set down as the distance run is known, the *observation* position can be determined by station pointer, cross bearings, or two bearings and the run between, the course and distance from the *D.R. to* the *obs.* position *is* the *set* and *drift* of the current experienced. Thus in Fig. 142 vessel

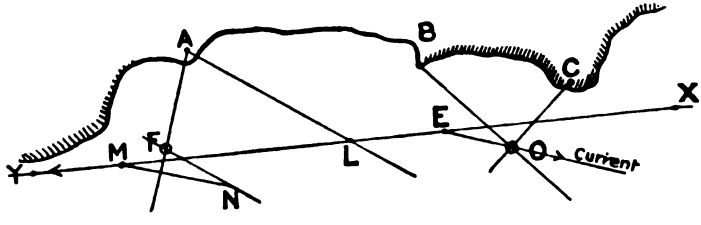


FIG. 142.

proceeding from X to Y, at the end of a given time expects to be at E; B and C are two conspicuous objects, at this time the compass bearings of B and C are taken and corrected, and then laid on the chart in the directions BO and CO, O the point of intersection of these bearings is the observed position of the ship, EO represents the set and drift of the current.

Again, at a given time let the bearing of an object A be taken, corrected and laid off as AL, after a known interval another bearing of A is taken, corrected and laid off as AF, supposing the current found to be still acting, the position of the ship at the end of the time may be found thus: Measure LM along LY equal to the distance the ship steams in the interval, set off MN in the direction of the current, and equal to the drift of the current in the interval, through N draw a parallel line to the first bearing, AL: to cut the second bearing in F. F is the position of the ship allowing for the current.

To set a *course* to *counteract* a given *current* is another problem of great importance to the navigator. In the previous paragraph the effect of a known current on the position of the ship at the end of the run has been considered; now it is required to find what course must be steered by a ship through a known current in order that she may make a given position.

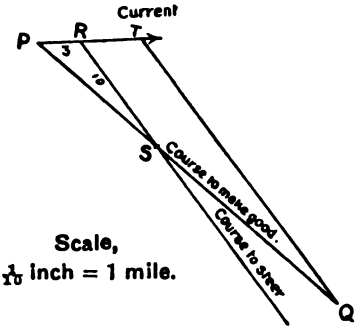


FIG. 143.

In Fig. 143 the vessel is at P and wishes to get to Q 30·5 miles away: her speed is 10 knots; the current sets in the direction PR at 3 knots. Measure 3-tenths inch along PR to R, and with R as centre and 10-tenths as radius cut the line PQ in S, then RS is the direction to steer to counteract the current, that is for the ship to get to point S on the line PQ, and PS is the distance made good in one hour: this measured gives 11·8 miles.

$$\text{Hence time taken to arrive at Q} = \frac{30{\cdot}5}{11{\cdot}8} \text{ hours}$$
$$= 2{\cdot}5 \text{ hours.}$$
$$\text{Total distance for ship to steam} = 2{\cdot}5 \times 10 \text{ miles}$$
$$= 25 \text{ miles.}$$

Or by drawing a line through Q parallel to RS to cut direction of current in T, and measuring TQ, will also give total distance for ship to steam = 25·5 miles.

The problem may be solved trigonometrically, as in triangle PRS are given PR, RS, and angle SPR, to find angle RSP by "rule of sines." This angle shows how much nearer south the course to steer is than the direction of PQ.

N.B.—Find the deviation for the new course, and apply to it to obtain the compass course.

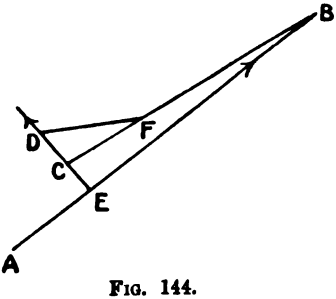


FIG. 144.

Again, in Fig. 144 let the vessel be going along the line AB, and at the end of a given time, say 1½ hours, finds her position to be at C, find the course she must steer to arrive at B. As the speed of the ship is known, her DR position would be at a fixed point on AB, say E,

consequently the set and drift of the current acting is the direction and length of EC in 1½ hours. Join CB, this is the course required to be made good. Produce EC, make CD = CE, this represents the current for 1½ hours, with D as centre, radius equal to AE the distance the ship steams in the same time, describe an arc cutting CB in F. Then the direction of DF is the required course, and CF is the distance made good in 1½ hours, whence by simple proportion, the time taken to arrive at B, or the distance towards B made in any required time, may be found.

N.B.—When laying off a course to counteract a current, always allow the current to act, and the ship to steam for the *same* time, 1 hour, 2 hours, etc., as may be selected.

Exercises.

Using St. George's Channel Chart, Devia. Card B.

1. (*a*) Find the course to steer by compass from Bell Buoy, St. Tudwall Rds., to New Quay Lt. Also the deviation, variation, and distance.

(*b*) With the ship's head on the above compass course, Pengarn Mt. bore by compass N. 56° E. and Aberdovey Lt. bore S. 84° E. by the same compass. Find the ship's position, and set and drift experienced since course was set, ship steaming 6 knots and taking 2 h. 40 m. to reach position.

(*c*) With the ship's head as above, Aberystwith Lt. bore by compass S. 65° E. and after continuing 3 h. 10 m. on the above-named course it bore N. 49° E. by the same compass. Find the ship's position and distance from New Quay Lt., making allowance for current found in (*b*).

(*d*) Find the course to steer by compass from Bell buoy to New Quay in the current found in (*b*) vessel making 6 knots: and the distance she then makes good towards New Quay in 3 hours 50 minutes. How far would ship have to steam to reach New Quay?

(*e*) Horizontal angles, find position by station pointer: St. Tudwall Lt., 86½°; Aberystwith Lt., 62°; New Quay Lt.

(*f*) What do the small numbers indicate that are placed about the chart and at what time of tide?

(*g*) What do the Roman numerals indicate that are occasionally seen near the coast and in harbours?

(*h*) How would you find approximately the time of high water at any place, the Admiralty tables not being at hand, nor any other special tables available?

2. (*a*) Find the courses to steer by compass and distance on each course from Dunmore Lt., Waterford Hr., to Q off Sth. Arklow Lt. V. on G. line 3¼ miles from Lt. V.

Note.—Alter course when Coningbeg Lt. bears 360°, distance 3 miles, and again when Tuskar Rk. Lt. bears 288°, distance 2 miles.

(*b*) With ship's head on the 2nd compass course, Sth. Pt. S. Saltee bore by compass N. 61° W., and Carnsore Pt. bore N. 46° E. by the same compass. Find the ship's position, set and drift experienced, if the expected position was with Barrel's Lt. V. bearing 251°, distance off 3 miles.

(*c*) With the ship's head on the 3rd compass course, Lucifer Lt. bore by compass N. 17° E., and after continuing 56 minutes on the above-named course, it bore N. 62° W. by the same compass. Find the ship's position and distance from Blackwater Lt., making due allowance for a current which set N.E. mag. 3 knots, vessel steaming 15 knots.

(*d*) Find the courses to steer by compass from Dunmore Lt. to Q in a current which set east mag. 2½ knots, the vessel making 12 knots, and the distance she then makes good towards Q in 3 hours 45 minutes.

(*e*) Horizontal angles, find position by station pointer, Hook Pt. Lt., 58°; Coningbeg Lt. V., 24½°; Tuskar Lt.

3. (*a*) Set the courses to steer by compass from X, lat. 51° 35′ N. long. 5° 18′ W., to Y., lat. 52° 24½′ N. long. 5° 2′ W., ship to pass 3 miles off S. Bishop Lt.; then alter course to reach Y, with Cardigan Bay L.V. bearing 90°, distance 1¼ miles; maintaining your distance of 3 miles from S. Bishop Lt., and give distance run on each course.

(*b*) With the ship's head on 1st compass course, St. Anne's Hd. Lt. bore by compass S. 17° E., and Small's Lt. bore N. 58° W. by the same compass. Find the ship's position, set and drift experienced, the ship having run 49 minutes on the course, steaming 11 knots.

(*c*) With ship's head on 2nd compass course, Strumble Hd. bore by compass S. 39° E., and after continuing 1 hour 38 minutes on above course, it bore S. 31° W. by same compass. Find the ship's position and distance from Cardigan Bay L.V., allowing for a current which set S. 63° W. mag. 2½ knots, vessel steaming 11 knots.

(*d*) After running on second course 1 hr. 22 mins., ship making 11 knots, her position was found to be lat. 52° 2′ N., long 5° 25′ W. Find course to steer by compass from this point to Y, allowing for a similar current to that experienced during earlier part of run; and distance made good in 2 hrs. 50 mins. towards Y. How long would she take to reach Y.?

CHAPTER XX.

USE OF NAPIER'S DIAGRAM.

ART. 93.—In the list of questions proposed to candidates for Masters' Certificates in the Board of Trade Regulations, Nos. 11, 12, 13, and 14 refer to the use of Napier's Diagram.

11) Sum of S. & E. bearings } 79°
Sum of S. & W. bearings } 5
Difference 8)55

Corr. Mag. bearing } S. 8° E.
14) Comp. bearing } S. 79° W.
Dev. 12 E.

S. 91 W.
Cor. Mag. or N. 89 W.

Comp. bearing } N. 8° W.
Dev. 12° E.

Corr. Mag. N. 4 E.

Example.—

No. 11. Having taken the following compass bearings of a distant object, find its *Correct Magnetic Bearing*, and thence the deviation.

Correct magnetic bearing required: S. 8° E.

Ship's head by standard compass.	Bearing of distant object.	Deviation required.	Ship's head by standard compass.	Bearing of distant object.	Deviation required.
N.	S. 4° E.	4° W.	S.	S. 13° E.	5° E.
N.E.	S.	8° W.	S.W.	S. 22° E.	14° E.
E.	S. 4° W.	12° W.	W.	S. 20° E.	12° E.
S.E.	S. 1° W.	9° W.	N.W.	S. 11° E.	3° E.

No. 12. With the deviation as above, construct a curve of deviation on a Napier's Diagram, and give the courses you would steer by the Standard Compass to make the following correct magnetic courses:—

Magnetic courses: S.S.W. W.N.W. N.N.E. E.N.E. S.S.E.
Compass courses required: S. 14° W. N. 77° W. N. 29½° E. N. 79° E. S. 20°½ E.

No. 13. Supposing you have steered the following courses by the Standard Compass, find the correct magnetic courses from the above curve of deviations:—

Compass courses: W.S.W. N.N.W. E.N.E. S.S.E.
Magnetic courses required: S. 82° W. N. 23° W. N. 57° E. S. 25° E.

No. 14. You have taken the following bearings of two distant objects by your Standard Compass as above, with the ship's head at W. ½ S. by compass: find the correct magnetic bearings.

Bearings by compass: W. by S. N. ¾ W.
Magnetic bearings required: N. 89° W. N. 3° E.

Explanation.—In No. 11 the *Correct Magnetic Bearing* is the *mean* of the eight observed bearings.

(*a*) When the bearings all read one way from S. or N., the *mean* is found by *adding* the eight bearings together and dividing the *sum* by 8.

(*b*) When the bearings read different ways, viz. some towards E. and some towards W., then the *mean* is found by summing up the bearings of the *same Name* separately (as in the above example), and dividing the *difference of the totals* by 8.

(*c*) The deviations are the differences between the correct magnetic bearing and each of the eight bearings by Compass, it being E. dev. when the correct magnetic reads to the right of the compass bearing, and W. when to the left.

ART. 94.—**Napier's Diagram and Curve of Deviations.**

The deviations found for the eight equidistant courses are taken with a pair of dividers from the middle line, which is graduated like a compass card, and set off from the graduated line on the dotted lines for the given courses—to the left for W. dev., and to the right for E. dev. A curve is then drawn, as in the diagram (Fig. 145). The deviation for any course (compass or magnetic) may now be found by simply measuring the part of the line (dotted or plain) intercepted by the curve.

For No. 12, place one foot of the dividers on the given course, and extend the other on the *plain* line to the curve. Keeping one foot on the middle line, turn the other *up* or *down* according as the line inclines *upwards* or *downwards*, and read off the required compass course.

For No. 13, proceed exactly in the same way as in No. 12, except that the *dotted* lines only are to be used.

NOTE.—The advantage of the diagram and curve is that it affords a ready *mechanical* way of changing magnetic courses to compass courses, and *vice versâ*, and so avoids the risk of error in applying deviation by adding or subtracting.

For No. 14, find the *Deviation* for the given direction of ship's head, by measuring the part of the *dotted* line intercepted by the curve, and apply it in the usual way to the bearings by compass, viz. E. to right, and W. to left.

Example.—(*a*) Magnetic bearing required : S. 53° 45′ W.

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	S. 75° W.	21° 15′ W.	S.	S. 34° W.	19° 45′ E.
N.E.	S. 64° W.	10° 15′ W.	S.W.	S. 31° W.	22° 45′ E.
E.	S. 56° W.	2° 15′ W.	W.	S. 49° W.	4° 45′ E.
S.E.	S. 50° W.	3° 45′ E.	N.W.	S. 71° W.	17° 15′ W.

8mm
8)430°
53° 45′

NOTE.—The magnetic bearing required is the mean of all the bearings by compass, found by adding them and dividing by eight. The deviations are then marked on the dotted lines of a Napier's diagram, and a curve drawn.

(*b*) Magnetic courses : N.N.W. S.S.E. W.N.W. E.S.E.
Compass courses required : N. 1° W. S. 31° E. N. 53° W. S. 68° E.

DIAGRAM.

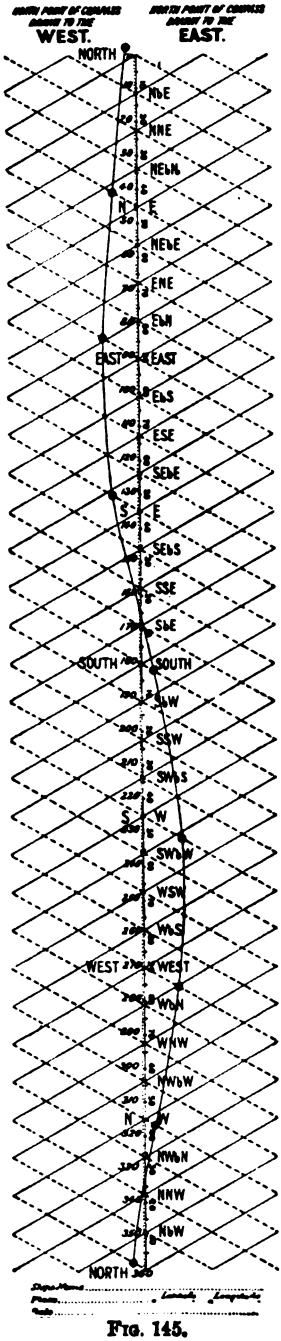


FIG. 145.

Explanation.—Place one leg of the compasses on the given course, and extend the other along the *plain* line to the curve, and turn upwards if the line tends upwards, but downwards if it tends downwards, and read off the required course.

(*c*) Compass courses given: N.N.E. E.N.E. S.S.W. W.S.W.
Magnetic courses required: N. 6° E. N. 62° E. S.W. S. 83° W.

Explanation.—Place one leg of the compasses on the given course, and extend the other along the *dotted* lines to the curve, and turn up or down, as the line tends.

(*d*) Ship's head by compass: N.E. ½ E.
Compass bearings: S.E. by S. N.N.W.
Magnetic bearings required: S. 44° E. N. 32½° W.

Explanation.—Find the deviation on the given compass course from the dotted line, and apply it to both bearings (east to right, west to left).

Exercises on Napier's Diagram and Curve.

1. (*a*) Correct magnetic bearing required :—

Ship's head by compass.	Bearing of distant object.	Deviation.	Ship's head by compass.	Bearing of distant object.	Deviation.
N.	N. 83° E.		S.	N. 50° E.	
N.E.	S. 86° E.		S.W.	N. 34° E.	
E.	E.		W.	N. 40° E.	
S.E.	N. 75° E.		N.W.	N. 61° E.	

Find the correct magnetic bearing and the deviations.

(*b*) Correct magnetic courses: N.E. ½ E. E. by N. S.E. by S. ½ S. N. ½ W.
Required courses to steer.

(*c*) Courses steered: N.W. ¾ W. S. E. by S. N. by E.
Required correct magnetic courses.

(*d*) Ship's head by compass: W.S.W.
Bearings of distant object: S.S.W. and N.E.
Required the correct magnetic bearing.

2. (*a*) Correct magnetic bearing required :—

Ship's head by compass.	Bearing of distant object.	Deviation.	Ship's head by compass.	Bearing of distant object.	Deviation.
N.	S. 85° W.		S.	N. 73° W.	
N.E.	S. 79° W.		S.W.	N. 68° W.	
E.	S. 88° W.		W.	N. 74° W.	
S.E.	N. 76° W.		N.W.	N. 79° W.	

Required the correct magnetic bearing and the deviations.

(*b*) Correct magnetic courses: E. ½ N. E.S.E. S. ¾ W. N.W.
Required courses to steer.

(*c*) Courses steered: N. S.E. ¼ S. S.W. ½ W. N. 35° W.
Required correct magnetic courses.

(*d*) Ship's head S. 65° E. Bearings by compass, W. 10° S. N. 40° E. Required the deviation and the correct magnetic bearings.

3. (*a*) Correct magnetic :—

Ship's head by compass.	Bearing of distant object.	Deviation.	Ship's head by compass	Bearing of distant object.	Deviation.
N.	S. 4° W.		S.	S. 34° W.	
N.E.	S. 1° E.		S.W.	S. 32° W.	
E.	S.		W.	S. 11° W.	
S.E.	S. 22° W.		N.W.	S. 9° W.	

(*b*) Correct magnetic courses : N. $\frac{1}{2}$ W. W. by S. $\frac{3}{4}$ S. S. 70° E. E. 24° N.
Courses to steer by compass.

(*c*) Courses steered by compass : W.N.W. $\frac{1}{2}$ N. N.E. $\frac{1}{2}$ N. S. $\frac{3}{4}$ E. N. 40° W.
Correct magnetic courses.

(*d*) Ship's head at W. by N. by compass.
Bearings by compass (of distant objects) : S.W. $\frac{3}{4}$ W. N.W. by W. $\frac{1}{2}$ W.
Correct magnetic bearings.

4. (*a*) Correct magnetic.

Ship's head by compass.	Bearing of distant object.	Deviation.	Ship's head by compass.	Bearing of distant object.	Deviation.
N.	N. 32° W.		S.	N. 4° E.	
N.E.	N. 27° W.		S.W.	N. 4° E.	
E.	N. 6° E.		W.	N. 10° W.	
S.E.	N. 2° E.		N.W.	N. 15° W.	

(*b*) Correct magnetic courses : S. 50° W. N. E. 5° S. E. by N.
Courses to steer by compass.

(*c*) Courses steered by compass : W. by S. N. by E. $\frac{1}{2}$ E. S.E. $\frac{3}{4}$ S. S. by W.
Correct magnetic courses.

(*d*) Ship's head at N.N.W. by compass.
Bearings by compass (of distant objects) : W.N.W. S.S.W.
Correct magnetic bearings.

5. (*a*) In the following table give the correct magnetic bearing, and thence the deviation.

Correct magnetic :—

Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.
N.	N. 62° E.		S.	E. 15° S.	
N.E.	E. 24° N.		S.W.	S. 81° E.	
E.	E. 7° N.		W.	E. 5° N.	
S.E.	S. 77° E.		N.W.	N. 67° E.	

(*b*) With the deviations as above, give the courses you would steer by the Standard Compass to make the following courses correct magnetic :—
Magnetic courses : N.E. ½ E. E. ¼ S. S. ½ W. N. ½ W.
Compass ,,

(*c*) Supposing you have steered the following courses by the Standard Compass, find the correct magnetic courses.
Compass courses : N.E. ½ N. S.E. ½ S. W. ¼ N. N.W. ½ N.
Magnetic.

(*d*) You have taken the following bearings of two distant objects by your Standard Compass as above : with the ship's head at N. ¼ W., find the bearings correct magnetic.
Compass bearings : S.S.E. ¼ E. E. ½ N.
Correct magnetic.

6. (*a*) In the following table give the correct magnetic bearing of the distant object, and thence the deviation :—
Correct magnetic :—

Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.
N.	S. 32° 0′ E.		S.	S. 6° 50′ W.	
N.E.	S. 4° 45′ E.		S.W.	S. 26° 0′ E.	
E.	S. 16° 0′ W.		W.	S. 48° 0′ E.	
S.E.	S. 21° 30′ W.		N.W.	S. 44° 35′ E.	

(*b*) With the deviations as above, give the courses you would steer by the Standard Compass to make the following courses correct magnetic :—
Magnetic : N. by E. ¾ E. E. ½ N. S.S.W. ¼ W. N.W. ½ W.
Courses to steer.

(*c*) Supposing you have steered the following courses by the Standard Compass, find the correct magnetic courses.
Standard : N.N.E. ¾ E. E. by N. ½ N. S.S.E. ¼ E. N.W. ¼ N.
Magnetic.

(*d*) You have taken the following bearings of two distant objects by your Standard Compass as above : with the ship's head at S.E. ½ S., find the bearings correct magnetic :—
Compass bearings : N.E. ½ E. N. by W. ¼ W.
Correct magnetic.

7. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	N. 56° E.		S.	N. 55° E.	
N.E.	N. 48° E.		S.W.	N. 64° E.	
E.	N. 45° E.		W.	N. 68° E.	
S.E.	N. 49° E.		N.W.	N. 63° E.	

(*b*) Magnetic courses : N. ¼ E. S. 70° E. S. 55° W. N. 8° W.
Compass courses required.
(*c*) Compass courses : S.S.W. N.W. ½ W. N. S. 82° E.
Magnetic courses required.
(*d*) Ship's head by compass : S.W.
Compass bearings : E. ¼ N. N. by E.
Magnetic bearings required.

8. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	N. 44° W.		S.	S. 79° W.	
N.E.	N. 47° W.		S.W.	S. 89° W.	
E.	N. 80° W.		W.	N. 69° W.	
S.E.	S. 76° W.		N.W.	N. 52° W.	

(*b*) Magnetic courses: S. ¼ E. S.W. ¼ S. N. 80° W. N. 75° E.
Compass course required.
(*c*) Compass courses: N.N.W. ¾ W. W. by S. ¼ S. S. N. 80° E.
Magnetic courses required.
(*d*) Ship's head by compass: W.
Bearings by compass: E. 11° N. N. 6° W.
Magnetic bearings required.

9. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	N. 32° W.		S.	N. 20° E.	
N.E.	N. 20° W.		S.W.	N. 9° E.	
E.	N. 5° E.		W.	N. 15° W.	
S.E.	N. 17° E.		N.W.	N. 31° W.	

(*b*) Magnetic courses: S. 65° W. N.W. N. 32° E. E.
Compass courses required.
(*c*) Compass courses: N. N.E. by N. E. by S. S.W. ½ S.
Magnetic courses required.
(*d*) Ship's head by compass E.
Bearings by compass: W. ¼ N. S. by W. ¾ W.
Magnetic bearings required.

10. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	S. 43° W.		S.	W.	
N.E.	S. 50° W.		S.W.	S. 83° W.	
E.	S. 65° W.		W.	S. 65° W.	
S.E.	S. 80° W.		N.W.	S. 45° W.	

(*b*) Correct magnetic courses: S.E. by E. N.N.E. N.W. ½ W. S. 10° W.
Compass courses required.
(*c*) Compass courses: N.W. by W. N. by W. N.E. ½ E. S. 50° E.
Magnetic courses required.
(*d*) Ship's head: E. ½ N.
Bearings by compass: S.E. ¾ E. N.E. by N.
Magnetic bearings required.

11. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	S. 2° W.		S.	S. 29° E.	
N.E.	S. 7° W.		S.W.	S. 26° E.	
E.	S. 6° E.		W.	S. 15° E.	
S.E.	S. 16° E.		N.W.	S. 3° E.	

(*b*) Magnetic courses: W. N. 30° E. S. 56° E. S. 3° W.
Compass courses required.
(*c*) Compass courses: E. N.E. by N. ¼ N. N.W. by W. S. 3° E.
Magnetic courses required.
(*d*) Ship's head by compass: S.E.
Bearings by compass: N. E. by N. ¾ N.
Magnetic bearings required.

12. (*a*) Magnetic bearing required—

Ship's head by compass.	Bearing by compass.	Deviation.	Ship's head by compass.	Bearing by compass.	Deviation.
N.	N.		S.	N. 21° W.	
N.E.	N. 7° E.		S.W.	N. 17° W.	
E.	N. 10° W.		W.	N. 18° E.	
S.E.	N. 13° W.		N.W.	N. 16° E.	

(*b*) Magnetic courses: E.S.E. S.S.E. W. by N. N.E. ¼ N.
Compass courses required.
(*c*) Compass courses: N. ½ W. W. ¾ N. N.E. ¾ N. E.
Magnetic courses required.
(*d*) Ship's head by compass: N.E.
Bearings by compass: S.W. ¼ S. S.E ¾ S.
Magnetic bearings required.

CHAPTER XXI.

GREAT CIRCLE SAILING.

ART. 95.—A circle is really a plane surface bounded by a line called the circumference, but it is common to speak of this line as the circle.

A great circle of the globe is one which divides it in halves. Hence the Equator is a great circle, and as all great circles bisect one another, every other great circle must intersect the Equator in two opposite points (Fig. 146). A meridian of longitude cuts the Equator at right angles, but other great circles cross it more or less obliquely, and go to equal distances north and south. The two points on the Great Circle which are farthest from the Equator are called the Vertexes, and the meridian of longitude which passes through the Vertex is called the Meridian of Vertex.

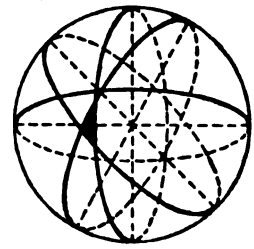


FIG. 146.

Now, the shortest distance between any two places on the globe is the arc of the Great Circle passing through them. Therefore, when this route is practicable, the navigator would save time in a sailing vessel, and coals as well as time in a steamship, by following the Great Circle track.

Illustration (Fig. 147).—Let A be the starting-point, and B the destination, AVB the arc of the Great Circle passing through the two places, P the north pole, PA and PB the meridians of longitude through A and B, and V the highest latitude reached on the parallel LL_1 and meridian of vertex PVM; then—

The ∠ APB is the Difference of Longitude of A and B.
The ∠ PAB is the Initial Course.
The ∠ PBA „ Final Course.
AB „ Distance on the Great Circle.

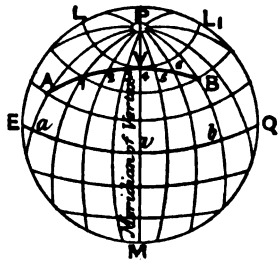


FIG 147.

vV is the Latitude of Vertex.
The ∠ APV (or arc av) is the Longitude of A from Vertex.

The ∠ BPV (or arc *bv*) is the Longitude of B from Vertex.
PA and PB, Co-latitudes of A and B.
PV, Co-latitude of Vertex.
1, 2, 3, 4, 5, 6, points on the Great Circle.

N.B.—The Meridian of Vertex is at right angles to the Great Circle.

Inspection of the figure shows that the course from A would first be northerly until the Vertex is reached, where it would be due east, and then southerly to B. From B to A these directions would be reversed.

In proceeding along the Great Circle the course would change at each instant, but as this would not be possible in navigation, Great Circle sailing in practice consists in fixing the latitude and longitude of a number of points on the circle, and then shaping courses from point to point by Mercator's sailing. The greater the number of points chosen the nearer would the ship's track approximate to the Great Circle.

The co-latitudes PA and PB and the distance AB are three sides of a spherical triangle, of which two sides (the co-latitudes) and the included angle (difference of longitude) are known. From these data all the other details can be found by the principles of spherical trigonometry.

These details are—

(1) The initial and final courses.
(2) The distance on the Great Circle.
(3) The latitude and longitude of Vertex.
(4) The longitude from Vertex of A and B.
(5) The latitudes and longitudes of points on the Great Circle.
(6) The courses and distances by Mercator from point to point.

ART. 96. **Great Circle Sailing** (Fig. 148).—Let AB be the arc of a great circle from the starting-point (A) to the destination (B), and CA, CB arcs of meridians of longitude; then the angles A and B are the *initial* and *final* courses, the angle C is the difference of longitude, and CA, CB the co-latitudes of A and B.

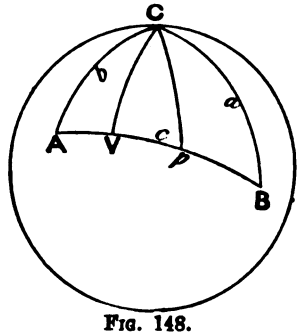


FIG. 148.

Also, let the arc CV be drawn at right angles to AB; then V is the vertex, and CV the co-latitude of vertex, putting the letters *a*, *b*, *c* for the sides respectively opposite the angles A, B, C, and using the formulæ for oblique-angled spherical triangles when two sides and included angle are known, we have—

$$\text{Tan } \tfrac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cdot \cot \tfrac{1}{2}C$$

$$\text{Tan } \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} \cdot \cot \tfrac{1}{2}C$$

$$\text{The greater angle (A)} = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$$

$$\text{„ lesser „ (B)} = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$$

$$\cos \tfrac{1}{2}c = \frac{\cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(A + B)} \cdot \sin \tfrac{1}{2}C$$

$$= \cos \tfrac{1}{2}(a + b) \cdot \sec \tfrac{1}{2}(A + B) \cdot \sin \tfrac{1}{2}C$$

The latitude of vertex, longitude from vertex, and the latitudes of points on the great circle are found by applying the rules for the solution of right-angled spherical triangles.

1. To *find "Latitude of Vertex."*—In the right-angled triangle ACV are known the angle A and the side AC.

Taking CV (co-latitude of V) as middle part, the opposite parts are comp. of A and comp. of AC;

$$\text{therefore sin CV} = \cos \text{(comp. of A)} \times \cos \text{(comp. of AC)}$$
$$= \sin A \times \sin AC$$

2. *To find "Longitude from Vertex" of* A, *viz. the angle* ACV.—In the same triangle, taking comp. of AC as middle part, comp. of ACV, and the comp. of A are adjacent parts;

$$\therefore \sin \text{(comp. of AC)} = \tan \text{(comp. of ACV)} \cdot \tan \text{(comp. of A)}$$
$$\text{Hence } \cos AC = \cot ACV \cdot \cot A$$
$$\text{or } \cot ACV = \cos AC \cdot \tan A$$

Similarly, the latitude of V and "longitude from V" of B can be found from the triangle BCV.

The longitude of the vertex can be found by applying the "longitude from vertex" to the longitude of A or B.

3. *To find the Latitude of a Point* (*p*) *whose "Longitude from Vertex"* (*is assumed*) (*viz. the angle* *p*CV).—In the right-angled triangle *p*VC, the side CV and the angle *p*CV being known—

$$\sin \text{(comp. of } pCV) = \tan CV \times \tan \text{(comp. of } Cp)$$
$$\text{or } \cos pCV = \tan CV \times \cot Cp$$
$$\text{transposing, } \cot Cp = \frac{\cos pCV}{\tan CV} = \cos pCV \times \cot CV$$

Similarly, the latitudes of other points on the great circle track can be found. The courses and distances from point to point are calculated as in Mercator's sailing.

ART. 97. **Naming the Courses.**—When the Vertex falls on the portion of the G.C. arc joining the two places, it is said to be "in;" when the Vertex is not situated on this arc, but on the arc produced, it is said to be "out." This can be decided

when the initial and final courses have been computed; for if they are on the same side of 90° the vertex is "in," but if one is greater and the other less than 90° it is "out," and is in every case on the same side of the Equator as the greater latitude.

The first course is always of the same name, N. or S., as the Pole used in the calculation; the last course of contrary name to this; both are named E. or W. like a Mercator course.

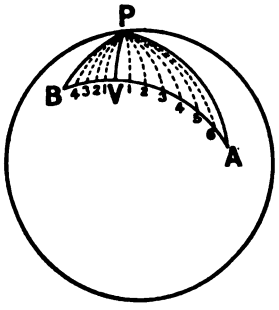


FIG. 149.

Example.—VERTEX "IN."

Required the initial and final courses and the distance on the Great Circle from (A) San Lucas to (B) Yokohama; also the position of the Vertex, and position of points on G.C. differing 10° in longitude.

A lat. 22° 52′ N. ∴ PA = 67° 8′ long. 109°53′ W.
B „ 35 26 N. ∴ PB = 54 34 „ 139 39 E.

PA + PB =	121 42	249 32
½(PA + PB) =	60 51	360 0
PA − PB =	12 34	P = 110 28
½(PA − PB) =	6 17	P/2 = 55 14

In triangle PAB, given PA, PB, and P, required A, B, and AB.

$$\text{Tan}\,\frac{B+A}{2} = \frac{\cos\frac{1}{2}(PA-PB)}{\cos\frac{1}{2}(PA+PB)}\cdot\cot\frac{P}{2}$$

$$\text{Tan}\,\frac{B-A}{2} = \frac{\sin\frac{1}{2}(PA-PB)}{\sin\frac{1}{2}(PA+PB)}\cdot\cot\frac{P}{2}$$

$$\text{Cos}\,\frac{AB}{2} = \frac{\cos\frac{1}{2}(PA+PB)}{\cos\frac{1}{2}(B+A)}\cdot\sin\frac{P}{2}$$

½(PA+PB) = 60° 51′	sec = 10·312384	cosec = 10·058813	cos =	9·687616
½(PA−PB) = 6 17	cos = 9·997383	sin = 9·039197		
P/2 = 55 14	cot = 9·841457	cot = 9·841457	sin =	9·914598
½(B+A) = 54° 46′ 45″	tan = 10·151224			sec = 10·239029
½(B−A) = 4 58 30	... ...	tan = 8·939467		
B = S. 59 45 15 W. Final course		46° 4′ 2″	cos =	9·841243
A = N. 49 48 15 W. Initial „		2		
		AB = 92 8 4 = 5528·1 miles.		

Since A and B are of same affection, vertex is "in."

To find position of vertex "V."

In triangle PAV, V = 90°. Given PA and A, required PV = co-lat. V. P = *d* long. of V from A.

Sin PV = cos (co. A) . cos (co. PA) and sin (co. PA) = tan (co. P) . tan (co. A)
= sin A . sin PA or cos PA = cot P . cot A
∴ cot P = cos PA . tan A

A = 49° 48′ 15″	sin = 9·883004	...	...	tan = 10·073174	
PA = 67 8	sin = 9·964454	...	...	cos = 9·589489	
PV = 44 44	sin = 9·847458	...	65° 18′	cot = 9·662663	
∴ Lat. V = 45° 16′ N.		Long. A 109° 53′ W.			
		∴ Long. V 175 11 W.			

To find "points" on the G.C. *differing* 10° *in long.*—Since no particular meridian is stated in the question as the starting-point, the simplest way is to calculate from the vertex, using the angles at P or differences of long. as 10°, 20°, 30°, etc. As the total d long. between A and V is 65° 18′, the number of points will be $\frac{65}{10} = 6$. The latitudes of points equally distant from the vertex are the same; hence, these lats. will do for either side of vertex.

Thus, if 1, 2, 3, 4, 5, 6 represent points, respectively 10°, 20°, 30°, 40°, 50°, 60° from the vertex, there are a series of triangles, PV1, PV2, PV3, etc., to solve, all right angled at V; PV is co-lat. V, and angles at P are 10°, 20°, 30°, etc., to find P1, P2, P3, etc.—

$$\text{Sin (co. P)} = \tan PV \,.\, \tan (\text{co. P1})$$
$$\therefore \cot P1 = \cos P \,.\, \cot PV.$$

PV = 44° 44′	cot = 10·004043	...	...	cot = 10·004043	
P = 10	cos = 9·993351	...	... 20°	cos = 9·972986	
45 10½	cot = 9·997394	...	46° 31′	cot = 9·977029	
PV = 44° 44′	cot = 10·004043	...	...	cot = 10·004043	
P = 30	cos = 9·937531	...	... 40°	cos = 9·884254	
48 50½	cot = 9·941574	...	52° 17½′	cot = 9·888297	
PV = 44° 44′	cot = 10·004043	...	...	cot = 10·004043	
P = 50	cos = 9·808067	...	... 60°	cos = 9·698970	
57 1½	cot = 9·812110	...	63° 13½′	cot = 9·703013	

These are the co-lats. of the points which may now be arranged in order from A to B, thus—

A San Lucas	lat. 22° 52′	N.	long 109° 53′	W.
6	,, 26 46½	N.	,, 115 11	W.
5	,, 32 58½	N.	,, 125 11	W.
4	,, 37 42½	N.	,, 135 11	W.
3	,, 41 9½	N.	,, 145 11	W.
2	,, 43 29	N.	,, 155 11	W.
1	,, 44 49½	N.	,, 165 11	W.
Vertex	,, 45 16	N.	,, 175 11	W.
1	,, 44 49½	N.	,, 174 49	E.
2	,, 43 29	N.	,, 164 49	E.
3	,, 41 9½	N.	,, 154 49	E.
4	,, 37 42½	N.	,, 144 49	E.
B Yokohama	,, 35 26	N.	,, 139 39	E.

These positions set off on a chart and joined by a curve, give the G.C. track.

As it is impossible to keep a ship on the track, points are taken differing very little in long., say 2°, and courses steered from point to point by Mercator. Should a vessel be driven far from the track by stress of weather, the navigator should strike a *new* G.C. from the *actual position* of the ship, and not work back to the original one.

Example. VERTEX "OUT."

Required the initial and final courses, and distance on the G.C. from Start to Barbadoes; also position of the vertex, and position of a succession of points differing 5° in long. commencing where the 5° W. meridian cuts the track.

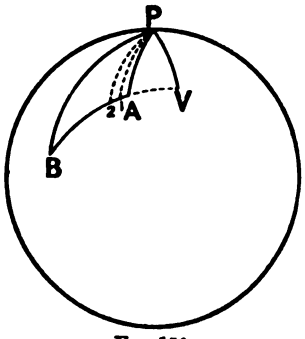


FIG. 150.

Start lat. 50° 13′ N. AP = 39° 47′ long. 3° 39′ W.
Barb. „ 13 3 N. BP = 76 57 „ 57 37 W.
BP + AP = 116 44 P = 53 58
½(BP + AP) = 58 22 P/2 = 26 59
BP − AP = 37 10
½(BP − AP) = 18 35

To solve triangle APB, having given PA, PB, and P.

$$\text{Tan } \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(BP-AP)}{\cos \frac{1}{2}(BP+AP)} \cdot \cot \frac{P}{2}$$

$$\text{Tan } \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(BP-AP)}{\sin \frac{1}{2}(BP+AP)} \cdot \cot \frac{P}{2}$$

$$\text{Cos } \frac{AB}{2} = \frac{\cos \frac{1}{2}(BP+AP)}{\cos \frac{1}{2}(A+B)} \cdot \sin \frac{P}{2}$$

½(BP+AP) = 58° 22′	sec = 10·280270	cosec = 10·069855	cos =	9·719730
½(BP−AP) = 18 35	cos = 9·976745	sin = 9·503360		
P/2 = 26 59	cot = 10·293146	cot = 10·293146	sin =	9·656799
	tan = 10·550161		sec =	10·566751
½(A+B) = 74° 15′ 57″				
½(A−B) = 36 19 14		tan = 9·866361		
A = N. 110 35 11 W.	or S. 69° 24′ 49″ W.	1st Co. 28° 39′	cos =	9·943280
B = S. 37 56 43 W.	Final Course.	2		

Distance 57 18 = 3438 miles.

Since A and B are of contrary affection, vertex is "out."

To find position of vertex "V."
In triangle APV, V = 90°. Given AP and A, to find PV and P.

Sin PV = cos (co. AP) . cos (co. A) and sin (co. AP) = tan (co. A) . tan (co. P)
= sin AP . sin A or cos AP = cot A . cot P
∴ cot P = cos AP . tan A

AP = 39° 47′ sin = 9·806103	... ...	cos =	9·885627
A = 69 25 sin = 9·971351	... ...	tan =	10·425340
PV = 36 48 sin = 9·777454	26° 2½′ E.	cot =	10·310967
Lat. V = 53 12 N.	Long. A 3 39 W.		
	Long. V 22 23½ E.		

To find a succession of points.
First point where 5° W. meridian cuts the track.

Long. from V of 1 = 22° 23½′ + 5° = 27° 23½′
In triangle PV1; V = 90° given PV and P; to find P1
sin (co. P) = tan (co. P1) . tan PV
or cos P = cot P1 . tan PV
∴ cot P1 = cos P . cot PV

P = 27° 23½′	cos =	9·948355	...	32° 23½′	cos =	9·926551
PV = 36 48	cot =	10·126043	...	...	cot =	10·126043
P1 = 40 7	cot =	10·074398	P2 = 41 32½		cot =	10·052594

Points.—1, lat. 49° 53′ N. long. 5° W.
2, ,, 48 27½ N. ,, 10 W.
and so on.

ART. 98. **Composite Great Circle and Parallel Sailing.**—When the Great Circle track would take the ship into too high a latitude, then a combination of Great Circle sailing and Parallel sailing is used to obtain the shortest route.

The highest practicable latitude is made the latitude of vertex for the arcs of *two* Great Circles, one passing through the starting-point, and the other through the destination. The route would then be along the first arc to the Vertex, next on the parallel of latitude to the Vertex of the other arc, and finally along this arc to the destination.

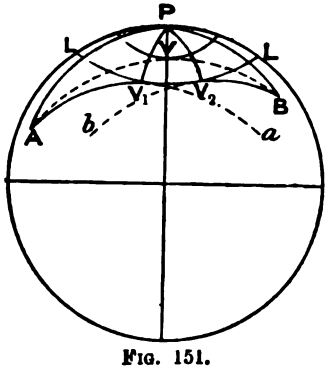


FIG. 151.

Illustration (Fig. 151).—Let A be the point of departure, and B the destination, and AVB the arc of the Great Circle whose vertex is V. The parallel LL_1 is assumed to be the highest practicable latitude. AV_1 and BV_2 are arcs of two great circles touching the parallel, V_1V_2 is the arc of the parallel of latitude or meridian distance, PV_1 and PV_2 are arcs of the meridians of vertexes and make right angles with AV_1 and BV_2. The solutions, therefore, depend on right-angled spherical trigonometry and parallel sailing.

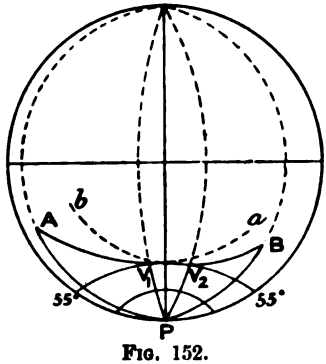


FIG. 152.

Example (Fig. 152).—Required the distance on the composite track from Rio to Cape Leeuwin, taking 55° as the limit of S. latitude; also the initial and final courses.

Lat. of Rio 23° 4′ S.
Long. of Rio 43° 8′ W.
Lat. of C. Leeuwin 34° 19′ S.
Long. of C. Leeuwin 115° 6′ E.

(1) To find the distances AV_1 and BV_2.

In triangle PAV_1, $V_1 = 90°$ given AP and PV_1
sin (co. AP) = cos AV_1 . cos PV_1
$\therefore$ cos AV_1 = cos AP . sec PV_1
Similarly, cos BV_2 = cos BP . sec PV_2

Co-lat. of A	66°	56′	...	cos 9·593067			
,, B	55	41	...	...	...	...	cos 9·751099
Co-lat. of V_1 and V_2	35	0	...	sec 0·086635		...	sec 0·086635
AV_1 =	61	25½	...	cos 9·679702			
BV_2 =	46	30½	...	...	...	...	cos 9·837734
Sum	107	56					
	60						

6476 geographical miles.

(2) To find the differences of long. APV_1 and BPV_2, or longitudes from Vertex.

$$\text{Sin (co. P)} = \tan PV_1 \,.\, \tan \text{(co. AP)}$$
$$\therefore \cos P = \tan PV_1 \,.\, \cot AP$$
$$\text{and } \cos P = \tan PV_2 \,.\, \cot BP$$

Co-lat. of A	66°	56′	...	cot 9·629255			
,, B	55	41	...	...		...	cot 9·834154
Co-lat. of V_1 and V_2	35	0	...	tan 9·845227		...	tan 9·845227
D. long of A and V_1	72	39	...	cos 9·474482			
,, B and V_2	61	27	...	...	...	...	cos 9·679381
Sum	134	6					
Total D. long	158	14					

D. long. V_1V_2 24 8 = 1448 geographical miles.

(3) To find the meridian distance between V_1 and V_2.

D. long.	1448	...	...	log 3·160769
Lat.	55°	...	...	cos 9·758591
Mer. dist.	830·5	...	...	log 2·919360
Arcs of Great Circles	6476			
Total dist.	7306·5			

NOTE.—The distance on the Great Circle between A and B is 7131 miles.

(4) To compute the initial and final courses.

$$\text{Sin } PV_1 = \cos \text{(co. AP)} \,.\, \cos \text{(co. A)}$$
$$= \sin AP \,.\, \sin A$$
$$\therefore \sin A = \sin PV_1 \,.\, \text{cosec } AP$$
$$\text{and } \sin B = \sin PV_2 \,.\, \text{cosec } BP$$

Co-lat. of A	66°	56′	...	cosec 0·036189			
,, B	55	41	...	...	...	...	cosec 0·083054
Co-lat. of V_1 and V_2	35	0	...	sine 9·758591		...	sine 9·758591
Initial co. S.	38	34 E.	...	sine 9·794780			
Final co. N.	43	59 E.	...	...	...	...	sine 9·841645

(5) To find the longitudes of V_1 and V_2.

Long. of A	43°	8′	W.	Long. of B	115°	6′ E.
D. Long. to V_1	72	39	E.	D. long. to V_2	61	27 W.
Long. of V_1	29	31	E.	Long. of V_2	53	39 E.

Various points on the arcs AV_1 and BV_2 can be fixed as in G.C.S. either by assuming intervals of longitude from V_1 towards A, and from V_2 towards B, or by working from a fixed meridian to destination.

ART. 99. **Windward Great Circle Sailing.**—What is called Windward Great Circle sailing applies only to sailing vessels, and consists in putting the ship on the tack on which her course would be nearest to the Great Circle track without reference to the Rhumb or Chart course, which in some cases would differ four or five points or more from the former.

Illustration (Fig. 153).—Let O be the starting-point, and

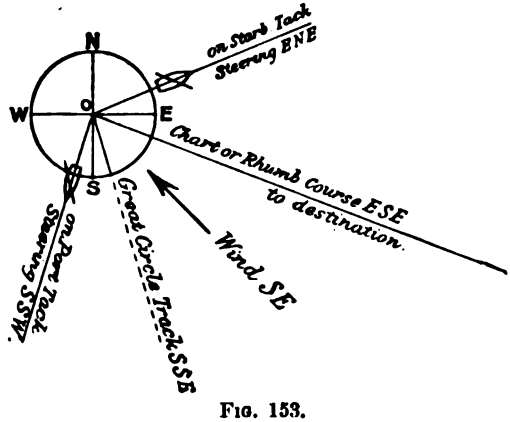


FIG. 153.

suppose destination to bear E.S.E. by Chart, the Great Circle course from O being S.S.E.

With the wind at S.E., the course on the starboard tack would be E.N.E. and on the port tack S.S.W. It is evident that on the port tack the ship's head would only be four points off the shortest route, whilst on the starboard tack she would be steering at right angles to it. In sailing 100 miles on the former course the Traverse Table shows that she would make 70 miles towards her destination, but on the latter she would be no nearer to it, having sailed on a line at right angles to the direct route on the globe.

With the wind at S.E. by S., and steering S.W. by S. (apparently away from destination, according to chart), she would still gain 36 miles in 100 miles sailed, whilst on the starboard tack, steering E. by N., she would make good only 19 miles.

Other still more striking cases might be given, but the above are sufficient to show the advantage of the method.

All problems in Great Circle sailing, Composite sailing, and Windward Great Circle sailing can best be understood from a

globe. A pair of 12-inch globes should be in every ship's equipment.

Exercises.—Great Circle Sailing.

1. Find the first and final courses from Cape of Good Hope, lat. 34° 22′ S., long. 18° 30′ E., to Rio Janeiro, lat. 22° 55′ S., long. 43° 9′ W.; also find the distance, latitude, and longitude of Vertex, and a succession of points for every five degrees of longitude.

2. Find the first and last courses, and distance from San Francisco, lat. 37° 49′ N., long. 122° 29′ W., to Yokohama, lat. 35° 26′ N., long. 139° 39′ E., the latitude and longitude of Vertex, and a succession of points for every twenty degrees of longitude.

3. Find the initial and final courses, distance on the Great Circle, latitude and longitude of Vertex, and a succession of points differing 10° in longitude.

From A, lat. 33° 51′ S., long. 151° 16′ E.
To B, ,, 37 47 N., ,, 122 30 W.

4. Find the initial and final courses, distance on the Great Circle, latitude and longitude of Vertex, and a succession of points differing 20° in longitude.

From A, lat. 22° 16′ N., long. 114° 10′ E.
To B, ,, 12 4 S., ,, 77 16 W.

5. Find the initial and final courses, distance on the Great Circle, latitude and longitude of Vertex, and a succession of points differing 5° in longitude.

From A, lat. 52° 12′ N., long. 6° 12′ W.
To B, ,, 13 7 N., ,, 59 25 W.

6. Find the initial and final courses, distance on the Great Circle, latitude and longitude of Vertex, in sailing.

From R, lat. 43° 40′ N., long. 150° 37′ E.
To Q, ,, 43 40 N., ,, 150 37 W.

Also find latitude of point where the meridian of 155° E. cuts the track, and a succession of points differing 5° in longitude, from thence to destination.

7. Find the initial and final courses, distance on the Great Circle, position of Vertex, and a succession of points starting in long. 120° W., and differing 5° in longitude to Equator.

From X., lat. 38° 27′ N., long. 100° 26′ W.
To Y., ,, 38 27 S., ,, 150 47 E.

8. A ship sails from Rio de Janeiro, lat. 22° 54′ S., long. 43° 6′ W., to Madeira, lat. 32° 43′ N., long. 16° 40′ W. Find longitude of point where the track crosses the Equator.

9. The point of "Maximum Separation in Latitude" on a Great Circle, is that point where the Great Circle course is the same as the Mercator course. Find the latitude of such a point in sailing from lat. 40° 30′ N., long. 120 E., to lat. 45° N., long. 172° E.

10. One Vertex of a great circle is in lat. 50° N. 70° W.: what is position of the other Vertex? A ship sailing on a Great Circle crosses the Equator: show that the course at that time = complement lat. Vertex.

11. In sailing from lat. 34° 51′ S., long. 20° E., to lat. 34° 19′ S., long. 115° 6′ E., on a Great Circle arc, how far would the vessel be from Kerguelen island, lat. 49° 54′ S., long. 70° 12′ E., when upon same meridian as island?

12. A ship starts from A on the course N. 64° 12′ E., and sails on a Great Circle arc to B; she increases her distance from the Equator till she gets into long. 19° 26′ 30″ W. on parallel 51° 50′ 30″ N., and then diminishes it. Find lat. and long. of A and B, their diff. long. being 43° 34′.

Exercises.—Composite Sailing.

1. Find the first and last courses and distance from Bahia Blanca, lat. 38° 59′ S., long. 61° 39′ W., to Cape Lewin, lat. 34° 31′ S., long. 115° 6′ E.; easting to be run on the parallel of 45° S. Find also the longitudes of points of arrival at and departure from the parallel.

2. On a composite track from Cape S. Mary, lat. 16° 40′ S., long. 50° 4′ E., to Mont Video, lat. 34° 43′ S., long. 56° 16′ W., the westing to be run on the parallel of 40° S. Required the first and last courses, total distance, and longitudes of points of arrival at and departure from the given parallel.

3. Find the first and last courses, distance, and longitudes of arrival at and departure from parallel of 50° N. when sailing on a composite track from Yokohama, lat. 35° 26′ N., long. 139° 39′ E., to Cape Flattery, lat. 48° 23′ N., long. 124° 44′ W.

4. Find the first and last courses and distance when sailing on a composite track from Valparaiso, lat. 33° 2′ S., long. 71° 28′ W., to Port Lyttelton, lat. 43° 36′ S., long. 172° 50′ E., the westing being run on the latitude of Port Lyttelton; also the position of a succession of points from long. 73° W., and every 5° to destination.

CHAPTER XXII.

LATITUDE BY DOUBLE ALTITUDES.

ART. 100. **Double Altitudes.**—This is a problem for determining the latitude from—

(*a*) Two different altitudes of the same object (Sun, Moon, Star, or Planet).

(*b*) Simultaneous altitudes of two different objects.

The data required for the computation are—

(1) The polar angle.

(2) The two altitudes.

(3) The declination.

Definition.—The **polar angle** is the angle at the Pole between the hour circles passing through the two positions of the object or objects.

(1) For the Sun, the polar angle is the interval between the apparent times of observation; therefore, if mean times are given, the equation of time must be applied to each, or the change of equation of time applied to the mean time interval.

The rule for applying this correction is—

If Eq. of Time be *additive* to mean time and *increasing* add.
" " *subtractive* from mean time and *decreasing* add.
" " *additive* to mean time and *decreasing* subtract.
" " *subtractive* from mean time and *increasing* subtract.

Thus, if the mean interval on August 28th is $4^h\ 30^m\ 18^s$, what is the apparent interval?

H.V. Eq. of T. ·72		
Int. 4·5		
360	Mean Int.	$4^h\ 30^m\ 18^s$
288	Change in Int.	+ 3·2
3·240	App. Int.	4 30 21·2

The change being added because the Eq. of Time is subtractive from mean time and decreasing.

(2) A Star's polar angle is the interval of sidereal time

between the observations, and is found by accelerating the mean interval for the interval.

Thus, if the mean interval is $4^h\ 30^m\ 18^s$, what is the sidereal interval?

Mean Int.	$4^h\ 30^m\ 18^s$	
Accel. for	4^h	39·4
	30^m	4 9
	18^s	·1
Sid. Int.	4 31	2·4

(3) When simultaneous altitudes of two stars are observed, the polar angle is the difference between their right ascensions.

(4) For the Moon or a Planet, the polar angle is the sidereal interval, increased or decreased by the change in right ascension during the elapsed mean time, according as the R.A. of the object is decreasing or increasing.

ART. 101. **Correction for "Run."**—Both altitudes are supposed to be observed at the same place; therefore, if a ship changes her position during the interval, a correction depending on the course and distance run and the bearing of the object must be applied to one of the altitudes, in order to make it what it would be at the place where the other altitude is observed.

FIG. 154.

FIG. 155.

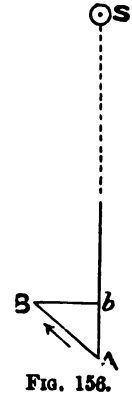


FIG. 156.

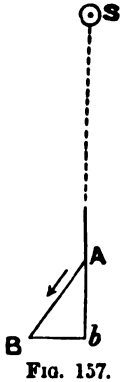


FIG. 157.

Thus, if the ship's course is *the same* as the bearing of the object at the first observation, that is, *directly towards* it, and the distance run ten miles, the first altitude would have to be *increased* by 10′; but if the course is *opposite* to the bearing, that is, *directly away from* the object, the first altitude will have to be *decreased* by 10′; whilst if the course were at right angles to the bearing, there would be no correction.

When the course is *obliquely towards or from* the object, the angle between the bearing and the course must be found.

With this angle as a course in the Traverse Table and distance run in the Distance column, the correction will be found, in the Latitude column, and will be *added* to or *subtracted* from the first altitudes according as the course is *towards* or *from* the Sun.

Illustration.—Let A and B be the ship's positions at the first and second observations respectively, and AS the direction of the object.

Fig. 154. Course directly towards the object; correction = AB+
„ 155. „ „ from „ ; „ = AB−
„ 156. „ obliquely towards „ ; „ = A*b*+
„ 157. „ „ from „ ; „ = A*b*−
„ 158. „ at right angles to bearing; „ = 0

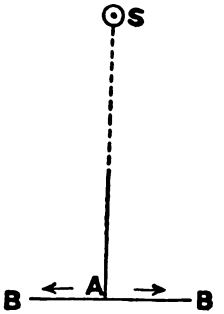


Fig. 158.

In Figs. 156 and 157 the angle at A is the course, and AB the distance in the Traverse Table, and A*b* the latitude, since B*b* is a perpendicular upon AS.

When the bearing is given at the *first* observation, the computation gives the latitude at the *second* observation, and *vice versâ*.

Therefore, if the bearing at the *second* observation is given, the "correction for run" must be applied to the *second* altitude, *reversing* the rule for applying it; that is, if the run is *towards* the object, *subtract* the correction from the *second* altitude, and if *from* the object, *add* the correction. The latitude found is the latitude at the *first* observation.

Of the various methods for solving the problem, the best known is "Ivory's method," but the latitude found requires a correction for the change of declination in half the elapsed time.

Art. 102. **Double Altitudes. Ivory's Method.**—Let S and S_1 be

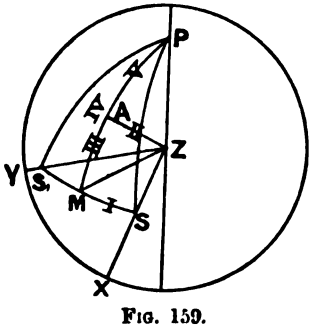


Fig. 159.

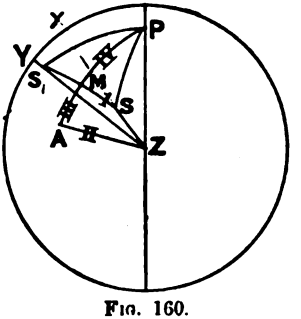


Fig. 160.

the two positions of the celestial body, and SS_1 the arc of a great

circle, bisected in M; PS or PS_1, the polar distance; ZS and ZS_1, the zenith distances; PZ, the co-lat.; and ZA, a perpendicular on PM. The angle MPS = ½ elapsed time, or ½ polar angle. MS is arc 1, ZA arc 2, AM arc 3, PM arc 4, and PA arc 5.

To find Arc 1 (MS).—In the right-angled triangle PMS are given PS (polar distance) and the angle MPS (½ polar angle), to find MS.

$$\text{Sin MS} = \cos(\text{comp. of MPS}) \times \cos(\text{comp. of PS})$$
$$\text{or } \sin \text{MS} = \sin \text{MPS} \times \sin \text{PS}$$

that is, sin arc 1 = sin ½ polar angle × cos declination
or cosec arc 1 = cosec ½ polar angle × sec declination

To find Arc 2 (ZA).—Applying the general formula to the triangle ZMS—

$$\cos \text{ZS} = \cos \text{ZMS} . \sin \text{ZM} . \sin \text{MS} + \cos \text{ZM} . \cos \text{MS} \quad . \quad (1)$$

Similarly, in the triangle ZMS_1—

$$\cos \text{ZS}_1 = -\cos \text{ZMS} . \sin \text{ZM} . \sin \text{MS} + \cos \text{ZM} . \cos \text{MS} \quad . \quad (2)$$

(because ZMS_1 is the supplement of ZMS, and MS_1 = MS). Subtracting (2) from (1)—

$$\cos \text{ZS} - \cos \text{ZS}_1 = 2(\cos \text{ZMS} . \sin \text{ZM} . \sin \text{MS})$$
$$i.e. \quad \sin \text{XS} - \sin \text{YS}_1 = 2(\sin \text{AMZ} . \sin \text{ZM} . \sin \text{MS}) \quad . \quad (3)$$

(because the alts. XS and YS_1 and the angle AMZ are the complements of ZS, ZS_1, and ZMS).

In the right-angled triangle AMZ—

$$\sin \text{AZ} = \sin \text{AMZ} . \sin \text{ZM}$$

Substituting sin AZ in equation (3)—

$$\sin \text{XS} - \sin \text{YS}_1 = 2 \sin \text{AZ} . \sin \text{MS}$$
$$\therefore 2 \sin \text{AZ} = \frac{\sin \text{XS} - \sin \text{YS}_1}{\sin \text{MS}}$$
$$\sin \text{AZ} = \frac{\cos \tfrac{1}{2}(\text{XS} + \text{YS}_1) . \sin \tfrac{1}{2}(\text{XS} - \text{YS}_1)}{\sin \text{MS}}$$

that is, sin arc 2 = cos ½ sum of alts. . sin ½ diff. of alts. . cosec arc 1

To find Arc 3 (AM).—Adding equations (1) and (2)—

$$\cos \text{ZS} + \cos \text{ZS}_1 = 2 \cos \text{ZM} . \cos \text{MS} \quad . \quad . \quad . \quad . \quad (4)$$

But in the right-angled triangle AMZ, cos ZM = cos AM . cos AZ;

$$\therefore \cos \text{ZS} + \cos \text{ZS}_1 = 2 \cos \text{AM} . \cos \text{AZ} . \cos \text{MS}$$
$$\text{or } \sin \text{XS} + \sin \text{YS}_1 = 2 \cos \text{AM} . \cos \text{AZ} . \cos \text{MS}$$
$$\therefore \cos \text{AM} = \frac{\sin \text{XS} + \sin \text{YS}_1}{2 \cos \text{AZ} . \cos \text{MS}}$$
$$= \frac{\sin \tfrac{1}{2}(\text{XS} + \text{YS}_1) \cos \tfrac{1}{2}(\text{XS} - \text{YS}_1)}{\cos \text{AZ} . \cos \text{MS}}$$

that is, cos arc 3 = sin ½ sum of alts. × cos ½ diff. alts. × sec arc 2 × sec arc 1

To find Arc 4 (PM).—In the right-angled triangle PMS—

$$\cos PS = \cos MS \,.\, \cos PM$$

$$\therefore \cos PM = \frac{\cos PS}{\cos MS} = \cos PS \,.\, \sec MS$$

that is, cos arc 4 = sin dec. × sec arc 1

To find Arc 5 (PA).—Arc 5 is the difference or the sum of 3 and 4 according to circumstances.

To find the Latitude.—In the right-angled triangle PZA—

$$\cos PZ = \cos PA \,.\, \cos ZA$$

that is, sin lat. = cos arc 5 . cos arc 2

and therefore cosec lat. = sec arc 5 . sec arc 2

The foregoing formulæ are arranged for computation as follows :—

	° ′			
½ pol. angle	— — —	†cosec		
Dec.	— — —	sec		sin
Arc 1,	— — —	cosec	sec	sec
½ sum alts.	— — —	cos	sin	
½ diff. „	— — —	sin	cos	
Arc 2,	— — —	†sin	*sec	
Arc 3,	— — —		cos	
Arc 4,	— — —			cos
Arc 5,	— — —	sec		
Arc 2,	— — —	*sec		
Lat.	— — —	cosec		

Logs marked thus * occur twice.

Logs marked thus † are used again for finding correction to latitude.

The Ambiguous Case.—When the sum of arcs 3 and 4 is less than 90°, it may be doubtful whether they should be *added* or *subtracted* to find arc 5. But if the lat. be first found by adding, and the lat. by account is not given, the correctness of the answer may be tested by computing the hour angle at the middle time, and seeing if it agrees with the known hour angle. If not, then *subtract* 3 from 4, and again compute the latitude.

To compute the Hour Angle (H_1) *at the Middle Time.*—In the right-angled triangle PZA—

$$\sin AZ = \sin PZ \,.\, \sin ZPA$$

$$\therefore \sin ZPA = \frac{\sin AZ}{\sin PZ} = \sin AZ \,.\, \operatorname{cosec} PZ$$

i.e. sin H.A. at mid. time = sin arc 2 . sec lat.

To correct the lat. found by "Ivory."

(1) Find change of declination in half elapsed time.

(2) Add following logs:—log change of dec., secant approx. lat., cosec half int., sine arc 2, reject 10′, and take back log. This is the correction.

(3) When the *second* altitude is the *greater*.

Allow it N, when the Sun is going N.

Allow it S, when the Sun is going S.

Reverse this when the *first* altitude is the greater.

ART. 103.—*Rule for finding latitude by "Ivory."*

(1) Find polar angle as directed, and the declination for the Middle Greenwich time.

(2) Correct the altitudes, applying correction for "Run" to alt. for which bearing of object is given, as directed. Find half sum and half difference of alts.

(3) *For Arc* 1, add logs of cosecant half polar angle and secant dec.; this gives cosecant arc 1.

For Arc 2, add logs cosecant arc 1, cosine half sum alts., sine half diff. alts.; this gives sine arc 2.

For Arc 3, add logs secant arc 1, sine half sum alts., cosine half diff. alts., secant arc 2; this gives cosine arc 3.

For Arc 4, add logs sine dec, and secant arc 1; this gives cosine arc 4.

For Arc 5. This arc is always the difference of arcs 3 and 4 when lat. and dec. are of different names, it can never exceed 90°, so should the sum of 3 and 4 be greater than 90°, their difference must also be used for 5; but if the sum is less than 90°, there may be some doubt, which can be decided by computing the H.A. at middle time from the formula

$$\sin \text{H.A.} = \text{sine arc } 2 \times \sec \text{lat.}$$

and selecting that lat. which gives the known H.A. at middle time.

For lat. add logs secant arc 5 and secant arc 2; this gives cosecant lat. to be corrected as directed for true lat.

Remarks.

(1) When dec. is 0, arc 4 is 90°, because sin 0 = 0, and therefore cos arc 4 = 0.

(2) When alts. are equal, diff. = 0, then arc 2 = 0, and arc 5 = co-lat.

(3) Arc 4 is of same affection as polar distance. Therefore when lat. and dec. are of different names supplement of angle found is arc 4.

Example.—August 5th, lat. N., long. 55° 17′ 30″ W. With the following observations find the latitude at second observation—

A.T.S. 9ʰ 15ᵐ 19ˢ A.M.	Obs. alt. $\underline{\odot}$ 44° 14′ 20″	Ind. cor., − 2′ 7″
,, 3 18 22 P.M.	,, $\underline{\odot}$ 39 21 25	Eye, 30 ft.

Sun's bearing at first observation, S. 62° E.; run during the interval, S. 6° W. 63 miles.

First A.T.S.	August 4ᵈ	21ʰ	15ᵐ	19ˢ	Dec.	17° 9′ 48″·4 N.	H.V. 40″·14
Second A.T.S.	,, 5	3	18	22	Cor.	− 2 39·4	3·97
Interval		6	3	3		17 7 9 N.	159·35
H.E.T.		3	1	31·5		H.V.	40″·14
Mid. time	5	0	16	50		H.E.T.	3
Long.		3	41	10			
Mid. A.T.G.	5	3	58	0		Ch. of dec. in H.E.T.	120·42

Bearing S. 62° E.
Run S. 6 W.

Inc. angle 68 dist. 63′
gives d. lat. 23·6

to be added to first alt., because angle is less than 90°.

First O.A.	44°	14′	20″				Second O.A.	39°	21′	25″		
I.C.	−	2	7					−	2	7		
	44	12	13					39	19	18		
Dip	−	5	22					−	5	22		
	44	6	51					39	13	56		
S.D.	+	15	48					+	15	48		
	44	22	39					39	29	44		
R. & P.		−	51						−	1	2	
	44	21	48					39	28	42		
Run	+	23	36					44	45	24		
	44	45	24					2)	5	16	42	
	39	28	42				Half diff.		2	38	21	
	2)84	14	6									
Half sum	42	7	3									
H.E.T.	3ʰ	1ᵐ	31ˢ·5	cosec	10·147644							
Dec.	17°	7′	9″	sec	10·019681						sin	9·468878
Arc 1.	42	51	50	cosec	10·167325		sec	10·134913			sec	10·134913
½ sum	42	7	3	cos	9·870270		sin	9·826498				
½ diff.	2	38	21	sin	8·663191		cos	9·999539				
Arc 2.	2	52	41	sin	8·700786		sec	10·000548				
Arc 3.	23	46	16	...	...	...	cos	9·961498				
Arc 4.	66	19	19	...	...	...	...	...	...		cos	9·603791
Arc 5.	42	33	3	sec	10·132723		H.E.T.	cosec	10·147644			
Arc 2.	2	52	41	sec	10·000548		...	...	sin	8·700786		
Approx. lat.	47	22	13	cosec	10·133271		...	...	sec	10·169246		
							Ch. in dec.	120·4	log	2·080626		
Cor.		+	13	...	...	...	...	12·54	log	1·098303		
True lat.	47	22	26	N.								

ART. 104. **Direct Method.**—In the triangle PSS_1 are given the two sides PS and PS_1 and the included angle at P, from which can be found the third side SS_1 and the angles PS_1S and PSS_1.

Then in the triangle ZSS_1 with the three sides given, the angle ZS_1S can be found. Also the angle $PS_1Z = PS_1S - ZS_1S$.

Lastly, in the triangle PZS_1 the two sides PS_1 and ZS_1 and the included angle PS_1Z are known, from which can be found the co-lat. PZ.

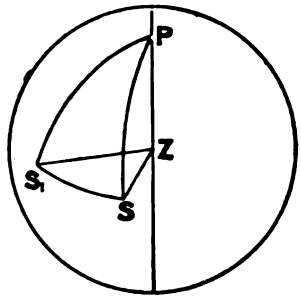


FIG. 161.

The latitude found by this method requires no correction for change of declination, because the actual polar distances are used, but it is subject to the same ambiguity as Ivory's method when SS_1 produced passes near the zenith. In the latter case the angle PS_1Z may be either the difference or the sum of PS_1S and ZS_1S.

Example.—The same problem as before worked from figure.

In figure X and Y are the two positions of the sun. P the polar angle = $6^h\ 3^m\ 3^s$

PX the 1st pol. dist.	=	72° 50′ 49″
PY the 2nd ,,	=	72 54 53
ZX the 1st zen. dist.	=	45 14 36
ZY the 2nd ,,	=	50 31 18

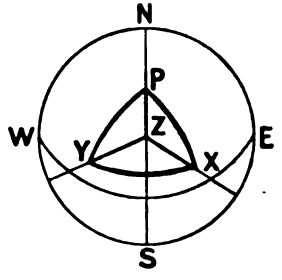


FIG. 162.

In triangle PXY, given two sides PX and PY, and included angle P, find X and XY.

$$\text{Tan } \tfrac{1}{2}(X+Y) = \frac{\cos \frac{1}{2}(PY-PX)}{\cos \frac{1}{2}(PY+PX)} \cdot \cot \frac{P}{2}$$

$$\text{Tan } \tfrac{1}{2}(X-Y) = \frac{\sin \frac{1}{2}(PY-PX)}{\sin \frac{1}{2}(PY+PX)} \cdot \cot \frac{P}{2}$$

$$\text{Cos } \frac{XY}{2} = \frac{\cos \frac{1}{2}(PY+PX)}{\cos \frac{1}{2}(X+Y)} \cdot \sin \frac{P}{2}$$

$\frac{P}{2} = 3^h\ 1^m\ 31^s{\cdot}5$	cot 9·994220	cot 9·994220	sin 9·852355
$\frac{1}{2}(PY+PX) = 72°\ 52'\ 51''$	sec 10·531122	cosec 10·019682	cos 9·468878
$\frac{1}{2}(PY-PX) = 0\ 2\ 2$	cos 10·000000	sin 6·771935	
$\frac{1}{2}(X+Y) = 73\ 23\ 24$	tan 10·525342		sec 10·543854
$\frac{1}{2}(X-Y) = 0\ 2\ 6$		tan 6·785837	
$X = 73\ 25\ 30$		42° 51′ 51″	cos 9·865087
		XY = 85 43 42	

In triangle ZXY, given the three sides, to find X.

$$\cos\frac{X}{2} = \sqrt{\frac{\sin s \,.\, \sin(s - ZY)}{\sin ZX \,.\, \sin XY}}$$

where $s = \frac{1}{2}(ZY + ZX + XY)$.

ZY = 50° 31′ 18″		
XY = 85 43 42	cosec	10·001208
ZX = 45 14 36	cosec	10·148678
2)181 29 36		
s = 90 44 48	sin	9·999963
s − ZY = 40 13 30	sin	9·810092
	2)19·959941	
17° 16′ 4·5″	cos	9·979970
∴ ZXY = 34 32 9		
PXY = 73 25 30		
Hence PXZ = 38 53 21		

Now in triangle PXZ, given two sides PX and ZX, and included angle PXZ, to find PZ = co-lat.

$$\text{Tan}\ \tfrac{1}{2}(Z + P) = \frac{\cos\frac{1}{2}(PX - ZX)}{\cos\frac{1}{2}(PX + ZX)} \cdot \cot\frac{X}{2}$$

$$\text{Cos}\ \frac{PZ}{2} = \frac{\cos\frac{1}{2}(PX + ZX)}{\cos\frac{1}{2}(Z + P)} \cdot \sin\frac{X}{2}$$

$\frac{X}{2}$ = 19° 26′ 40″	cot	10·452191	sin	9·522305
$\frac{1}{2}(PX + ZX)$ = 59 2 42	sec	10·288729	cos	9·711271
$\frac{1}{2}(PX - ZX)$ = 13 48 6	cos	9·987276		
$\frac{1}{2}(Z + P)$ = 73 24 32	tan	10·728196	sec	10·735659
	21° 18′ 45″		cos	9·969235
PZ = 42 37 30				
Lat. = 47 22 30 N.				

Art. 105. **Simultaneous Altitudes.**—The altitudes of two objects are observed at the same time; the problem may be worked either by Sumner's method, when the intersection of the two lines of position is the position of the ship, as there is no allowance for "Run," or by Direct Spherics.

Example.

September 9th, A.M. at ship in long. by account 128° W., when the G.M.T. by chron. was $8^d\ 20^h\ 52^m\ 15^s$; obs. alt. of α Ceti east of meridian was 26° 10′ 20″; obs. alt of α Ophiuchi west of meridian was 9° 59′ 30″; ind. cor., − 2′ 18″; height of eye, 30 ft. Required the ship's position, and true bearings of the stars, by Sumner's method, assuming lats. 51° and 51° 30′ N., or by any other method.

M.T.G. Sept. 8d 20h 52m 15s S.T. at noon 11h 6m 33s·2
3 17·1
Accel. 8·6
R.A.M.S. 11 9 58·9

O.A. α Ceti	26° 10′ 20″			O.A. α Ophiuchi			9° 59′ 30″				
I.C.	− 2 18						− 2 18				
	26 8 2						9 57 12				
Dip	− 5 22						− 5 22				
	26 2 40						9 51 50				
Ref.	− 1 56						− 5 20				
T.A.	26 0 44			26° 0′ 44″			9 46 30			9° 46′ 30″	
Lat.	51 0 0			51 30 0			51 0 0			51 30 0	
P.D.	86 16 44			86 16 44			77 22 20			77 22 20	
	163 17 28			163 47 28			138 8 50			138 38 50	
½ sum	81 38 44			81 53 44			69 4 25			69 19 25	
remainder	55 38 0			55 53 0			59 17 55			59 32 55	
Secs	10·201128			10·205850			10·201128			10·205850	
Cosecs	10·000916			10·000916			10·010635			10·010635	
Cos	9·162256			9·149150			9·552873			9·547885	
Sins	9·916687			9·917976			9·934417			9·935537	
	9·280987			9·273892			9·699053			9·699907	
E.H.A.'s	3h 27m 18s			3h 25m 30s		W.H.A.'s	6h 0m 2s			6h 0m 30s	
R.A.*	2 57 22			2 57 22			17 30 34			17 30 34	
R.A.M.	23 30 4			23 31 52			23 30 36			23 31 4	
R.A.M.S.	11 9 59			11 9 59			11 9 59			11 9 59	
M.T.S.	8d 12 20 5			8d 12 21 53			8d 12 20 37			8d 12 21 5	
M.T.G.	8 20 52 15			8 20 52 15			8 20 52 15			8 20 52 15	
	8 32 10			8 30 22			8 31 38			8 31 10	

Long. (*a*) 128° 2′ 30″ W. (*b*) 127° 35′ 30″ W. (*c*) 127° 54′ 30″ W. (*d*) 127° 47′ 30″ W.

By projection on a chart these longs. give—

Bearing of α Ceti S. 59° E. Lat. 51° 12′ N.
Bearing by α Ophiuchi N. 81 W. Long. 127 51 W.

Instead of projecting, the position of the ship may be found thus—

D. long. between "*a*" and "*c*" = 8′ E.; call this A.
" " "*a*" " "*b*" = 27′ " B.
" " "*b*" " "*d*" = 12′ W. " C.
D. lat. " lats. given = 30′ " D.

The cor. to lesser lat. $= \frac{A \times D}{A \pm C}$

" long (*a*) $= \frac{A \times B}{A \pm C}$

sum of A and C is taken when they are of contrary names (as in example)
diff. of A ,, C ,, ,, same ,,

$$\text{Cor. to lat.} = \frac{8 \times 30}{20} = 12' \qquad \text{Cor. to long} = \frac{8 \times 27}{20} = 10'\ 48''$$

Lesser lat.	51° 0′ N.	Long. "a"	128° 2′ 30″ W.
Cor.	12	Cor.	10 48 E.
Lat. in	51 12 N.	Long. in	127 51 42 W.

The azimuths may be found from the Traverse Tables thus—

1st obs. D. lat. = 30′
D. long. = 27′

This gives dep. = 17′, hence co. "a" to "b" from traverse or first line of position is N. 30° E.

∴ bearing of α Ceti is S. 60° E.

2nd obs. D. lat. = 30′
D. long. = 7′

This gives dep. = 4′·4, hence co. "c" to "d" from traverse or second line of position is N. 9° E.

∴ bearing of α Ophiuchi is N. 81° W.

From the figure.—

In the figure X and Y are the positions of the two stars.

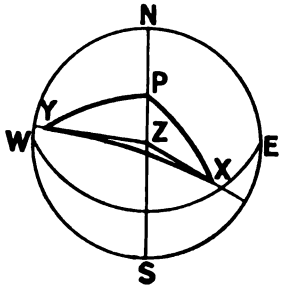


FIG. 163.

PX is pol. dist. of 1st star = 86° 16′ 44″
ZX is zen. ,, ,, = 63 59 16
PY is pol. ,, 2nd star = 77 22 20
ZY is zen. ,, ,, = 80 13 30
P is diff. of R.A.'s = 9ʰ 26ᵐ 48ˢ

In triangle PXY, given two sides PX and PY, and included angle P. Find X and XY.

$$\text{Tan } \tfrac{1}{2}(Y + X) = \frac{\cos \frac{1}{2}(PX - PY)}{\cos \frac{1}{2}(PX + PY)} \cdot \cot \frac{P}{2}$$

$$\text{Tan } \tfrac{1}{2}(Y - X) = \frac{\sin \frac{1}{2}(PX - PY)}{\sin \frac{1}{2}(PX + PY)} \cdot \cot \frac{P}{2}$$

$$\cos \frac{XY}{2} = \frac{\cos \frac{1}{2}(PX + PY)}{\cos \frac{1}{2}(Y + X)} \cdot \sin \frac{P}{2}$$

$\frac{P}{2}$	= 4ʰ 43ᵐ 24ˢ	cot	9·540653	cot	9·540653	sin	9·975277
$\frac{1}{2}(PX + PY)$	= 81° 49′ 32″	sec	10·847138	cosec	10·004435	cos	9·152862
$\frac{1}{2}(PX - PY)$	= 4 27 12	cos	9·998687	sin	8·890125		
$\frac{1}{2}(Y + X)$	= 67 40 20	tan	10·386478			sec	10·420325
$\frac{1}{2}(Y - X)$	= 1 33 39	...	...	tan	8·435213		
X	= 66 6 41	...	...		69° 17′ 41″	cos	9·548464

XY = 138 35 22

In triangle ZXY, given three sides ZX, ZY, and XY, to find angles Z and X.

$$\text{Cos}\,\frac{Z}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - XY)}{\sin ZX \,.\, \sin ZY}} \quad \text{and} \quad \cos\frac{X}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - ZY)}{\sin ZX \,.\, \sin XY}}$$

where $s = \frac{1}{2}(ZX + ZY + XY)$.

XY = 138° 35′ 22″			cosec 10·179503
ZY = 80 13 30		cosec 10·006351	
ZX = 63 59 16		cosec 10·046385	cosec 10·046385
2)282 48 8			
s = 141 24 4		sin 9·795090	sin 9·795090
s − XY = 2 48 42		sin 8·690667	
s − ZY = 61 10 34			sin 9·942555
		2)18·538493	
79 17 14		cos 9·269246	2)19·963533
Z = 158 34 28			cos 9·981766
		16° 29′ 13″	
		ZXY = 32 58 26	
		PXY = 66 6 41	
		∴ PXZ = 33 8 15	

In triangle PXZ, given two sides PX and ZX, and included angle X; to find PZ = co-lat., P = H.A. 1st star, Z = az. 1st star.

$$\text{Tan}\,\tfrac{1}{2}(Z + P) = \frac{\cos\frac{1}{2}(PX - ZX)}{\cos\frac{1}{2}(PX + ZX)} \,.\, \cot\frac{X}{2}$$

$$\text{Tan}\,\tfrac{1}{2}(Z - P) = \frac{\sin\frac{1}{2}(PX - ZX)}{\sin\frac{1}{2}(PX + ZX)} \,.\, \cot\frac{X}{2}$$

$$\text{Cos}\,\frac{PZ}{2} = \frac{\cos\frac{1}{2}(PX + ZX)}{\cos\frac{1}{2}(Z + P)} \,.\, \sin\frac{X}{2}$$

$\frac{X}{2}$ = 16° 34′ 7″·5	cot 10·526486	cot 10·526486	sin 9·455096	
$\frac{1}{2}(PX + ZX)$ = 75 8 0	sec 10·590793	cosec 10·014787	cos 9·409207	
$\frac{1}{2}(PX - ZX)$ = 11 8 44	cos 9·991730	sin 9·286237		
$\frac{1}{2}(Z + P)$ = 85 33 5	tan 11·109009		sec 11·110320	
$\frac{1}{2}(Z - P)$ = 33 54 35		tan 9·827510		
Z = 119 27 40		19° 23′ 45″	cos 9·974623	
P = 51 38 30		PZ = 38 47 30		
		∴ lat. 51 12 30 N.		

∴ E.H.A. 3ʰ 26ᵐ 34ˢ
R.A.ˢ 2 57 22

R.A.M. 23 30 48
R.A.M.S. 11 9 59

M.T.S. 8ᵈ 12 20 49
M.T.G. 8ᵈ 20 52 15

8 31 26

Long. 127° 51′ 30 W.

Azimuth of α Ceti = Z, or N. 119° 28′ E., which is S. 60° 32′ E.

$$\begin{array}{rr} \text{SZX} = & 60°\ 32' \\ \text{YZX} = & 158\ \ 34\tfrac{1}{2} \\ \hline \text{Azimuth of } \alpha \text{ Ophiuchi SZY} = & 98\ \ 2\tfrac{1}{2}, \text{ or N. } 81°\ 57\tfrac{1}{2}' \text{ W.} \end{array}$$

Exercises.

1. August 14th, in long. 73° 14′ E., lat. S.; with the following observations find the latitude at the second obs. :—

Approx. A.T.S., $9^h\ 20^m$ A.M.; time by chron., $4^h\ 26^m\ 18^s$; obs. alt. Sun's L.L., 30° 21′ 50″. Approx. A.T.S., $3^h\ 5^m$ P.M.; time by chron., $10^h\ 12^m\ 12^s$; obs. alt. Sun's L.L., 26° 38′ 10″.

Chron. showed correct A.T.G.; ind. cor., − 3′ 16″; height of eye, 30 ft.; Sun's bearing at first obs., N. 46° 30′ E.; run between sights, S. 57° 30′ E. 8 knots per hour.

2. August 18th, lat. N., long. 124° 23′ W.; with the following observations find the latitude at the second obs. :—

Approx. A.T.S., $10^h\ 0^m$ A.M.; chron., $6^h\ 21^m\ 46^s$; obs. alt. $\underline{\odot}$, 50° 32′ 50″; ind. cor., + 1′ 39″. Approx. A.T.S., $2^h\ 50^m$ P.M.; chron., $11^h\ 9^m\ 34^s$; obs. alt. $\underline{\odot}$, 42° 46′ 10″; eye, 35 ft.; chron. correct for M.T.G.; Sun's bearing at first obs., S. 49° 30′ E.; run between obs., S. 67° 30′ W. 10 knots per hour.

3. August 21st, lat. N., long. 130° 21′ W.; with the following observations find the latitude at the second obs. :—

M.T.S., $9^h\ 21^m\ 42^s$ P.M.; obs. alt. Vega, 75° 47′ 27″; bearing, S. 40° W. M.T.S., $1^h\ 52^m\ 50^s$ A.M.; obs. alt. Vega, 34° 53′ 50″; run, S. 50° E. 9 knots per hour; ind. cor., + 2′ 58″; eye, 29 ft.

4. August 27th, lat. S., long. 84° 27′ E.; with the following observations find the latitude at the second obs.; ind. cor., + 4′ 14″; eye, 41 ft.

M.T.S., $2^h\ 12^m\ 19^s$ A.M.; obs. alt. Aldebaran, 18° 10′ 50″; bearing, N. 61° E. M.T.S., $5^h\ 12^m\ 30^s$ A.M.; obs. alt. Aldebaran, 45° 40′ 10″; run, S. 6° E. 11 knots per hour.

5. August 17th, about $11^h\ 15^m$ P.M. at ship; lat. by D.R. 48° 15′ N., long. 179° 30′ W.; when a chron. $8^m\ 4^s$ fast of M.T.G. showed $11^h\ 28^m\ 17^s$, the obs. alt. of α Arietis was 27° 15′ 40″; ind. cor., − 1′ 13″; at the same time the obs. alt. of Altair was 47° 6′ 30″; ind. cor., + 2′ 20″; height of eye, 41 ft. Find the position of the ship and true bearings of the stars by Sumner's method, assuming lats. 15′ on each side of D.R. lat., or by any other method.

6. September 3rd, in long. 50° 25′ W., lat. N.; when the A.T.S. was $7^h\ 14^m\ 20^s$ A.M., the obs. alt. of Sun's U.L. was 18° 30′ 20″; and again at $11^h\ 0^m\ 25^s$ A.M., A.T.S., the obs. alt. of Sun's U.L. was 47° 26′ 20″; ind. cor., − 2′ 11″; eye, 28 ft.; run between observations, N. 9° E. 30 miles. If the Sun bore S. 81° E. at first sight, find lat. at second sight.

7. August 16th, lat. 24° N., long. 14° W.; with the following observations of Fomalhaut (α Pis. Aust.) find the lat. at second obs. :—

Approx. M.T.S., $9^h\ 45^m$ P.M.; chron., $10^h\ 42^m\ 22^s$; obs. alt., 15° 8′ 50″; bearing, East. Approx. M.T.S., $3^h\ 15^m$ A.M.; chron., $4^h\ 12^m\ 25^s$; obs. alt., 28° 14′ 0″; ind. cor., + 2′ 48″. Run between sights, N. 64° E. 10 knots per hour; eye, 45 ft.

8. September 15th, lat. S., long. 123° 34′ E., with the following observations find the lat. at second obs.; ind. cor., − 3′ 13″; eye, 36 ft. :—

Approx. A.T.S., $0^h\ 36^m$ P.M.; chron., $4^h\ 21^m\ 12^s$ M.T.G.; obs. alt. Sun's L.L., 38° 44′ 10″. Approx. A.T.S., $4^h\ 21^m$ P.M.; chron., $8^h\ 6^m\ 17^s$ M.T.G.; obs. alt. Sun's L.L., 14° 14′ 50″.

Bearing at first obs. N. by W.; run between sights, N. by W. 12 knots per hour.

9. September 21st., lat. S., long. 94° 26′ E.; with the following observations of Achernar (α Eridani) find lat. at second obs.; ind. cor., + 2′ 34″; eye, 33 ft.:—

Approx. M.T.S., 10ʰ 6ᵐ P.M.; chron., 3ʰ 40ᵐ 0ˢ; obs. alt., 55° 53′ 50″; bearing, S. 31° E. Approx. M.T.S., 5ʰ 46ᵐ A.M.; chron., 11ʰ 20ᵐ 20ˢ; obs. alt., 49° 13′ 30″; run, N. 31° W. 63 miles in the interval. The chron. was 8ᵐ 23ˢ slow of G.M.T.

10. August 27th, about 2ʰ 35ᵐ A.M. at ship, in long. D.R. 156° E.; when a chron. correct for M.T.G. showed 4ʰ 11ᵐ 28ˢ; the obs. alt. of Aldebaran was 35° 11′ 20″; no ind. cor.; at the same time the obs. alt. of Markab was 47° 49′ 40″; ind. cor., *nil*; eye, 45 ft. Find the position of the ship and true bearings of the stars by Sumner's method, assuming lat. 49° 30′ N. and 50° N., or by any other method.

11. September 10th, in long. 91° 14′ E., lat. D.R. 27° S.; with following observations find lat. at second obs.; ind. cor., *nil*; eye, 25 ft.:—

Approx. A.T.S., 10ʰ 40ᵐ A.M.; chron., 4ʰ 28ᵐ 3ˢ; obs. alt. Sun's L.L., 51° 52′ 10″. Approx. A.T.S., 3ʰ 50ᵐ P.M.; chron., 9ʰ 38ᵐ 8ˢ; obs. alt. Sun's L.L., 26° 25′ 40″.

Bearing at 1st obs., N. 35° E.; run during interval, N. 80° W. 12 knots per hour. The chron. was correct for G.M.T.

12. September 6th, lat. D.R. 13° S., long. 30° 25′ W.; with following observations of Altair, find lat. at second obs.; ind. cor., + 3′ 18″; eye, 25 ft.:—

Approx. M.T.S., 9ʰ 50ᵐ P.M.; G.M.T., 11ʰ 50ᵐ 2ˢ; obs. alt., 62° 13′ 40″; bearing, N. 36° W. Approx. M.T.S., 1ʰ 20ᵐ A.M.; G.M.T., 3ʰ 20ᵐ 15ˢ; obs. alt., 18° 4′ 50″.

Run during interval N. 36° W., 14 knots per hour.

13. September 16th, about 4ʰ 45ᵐ A.M., in long. by acct. 127° 30′ E.; when a chron. which was 12ᵐ 24ˢ fast of G.M.T. showed 8ʰ 23ᵐ 12ˢ, the obs. alt. of Rigel was 49° 28′ 10″; ind. cor., − 3′ 15″; and at the same time the obs. alt. of Fomalhaut was 27° 25′ 50″; ind. cor., + 2′ 11″; height of eye, 27 ft. Find the position of the ship and true bearings of the stars by Sumner's method, assuming lats. 46° 40′ S. and 47° 20′ S., or by any other method.

14. September 28th, in lat. N., long. 6° 15′ W.; with following observations of the Sun's L.L. find lat. at first obs.; ind. cor., − 2′ 12″; eye, 30 ft.:—

Approx. A.T.S., 11ʰ 10ᵐ A.M.; chron., 11ʰ 10ᵐ 25ˢ; obs. alt., 34° 29′ 10″. Approx. A.T.S., 4ʰ 0ᵐ P.M.; chron., 3ʰ 59ᵐ 49ˢ; obs. alt., 16° 34′ 0″; bearing, S. 50° W.; run during interval, N. 60° W. 10 knots per hour. The chron. was 14ᵐ 21ˢ slow of G.M.T.

15. September 1st, in lat. N., long. 128° 35′ W.; with following observations of α Cygni (Deneb) find lat. at first obs.; ind. cor., + 3′ 19″; eye, 28 ft.:—

Approx. M.T.S., 8ʰ 0ᵐ P.M.; chron., 4ʰ 40ᵐ 10ˢ; obs. alt., 66° 5′ 20″. Approx. M.T.S., 4ʰ 10ᵐ A.M.; chron., 12ʰ 50ᵐ 20ˢ; obs. alt., 21° 21′ 30″; bearing, N. 20° W. The chron. was 1ᵐ 20ˢ fast of M.T.G.; run during the interval, N. 34° E. 12 knots per hour.

CHAPTER XXIII.

FINDING ERROR OF CHRONOMETER.

ART. 106. **Error of Chronometer by a Single Altitude of the Sun or a Star.**—For this purpose it is necessary to know the correct latitude and longitude of the place of observation. The altitude would therefore be usually observed in port, using an artificial horizon.

The computation of the hour angle is exactly the same as in the problem of finding the longitude by chronometer. Thence the mean time at place and mean time at Greenwich are found, and the error of chronometer determined.

Example.—August 30th, at $9^h\ 3^m$ A.M. M.T.S., in lat. 40° 20′ N., long. 25° 30′ W.; the obs. alt. of the Sun's L.L. in an artificial horizon was 79° 10′ 20″; ind. cor., − 3′ 18″; time by chron., $10^h\ 45^m\ 15^s$. Required its error on M.T.G. and A.T.S.

M.T.S. Aug.	$29^d\ 21^h\ 3^m\ 0^s$		25° 30′	Dec.	9° 14′ 44″·3 N.	H.V.	53″·49
Long.	+ 1 42 0		4	Cor.	+ 1 6·9		1·25
M.T.G.	$29^d\ 22\ 45\ 0$	6,0)10,2	0		9 15 51·2		26745
					90		10698
			$1^h\ 42^m$				5349
				N.P.D.	80 44 8		
							66·8625
Obs. alt.	79° 10′ 20″			Eq. T.	$0^m\ 45^s·9$	H.V.	·75
I.C.	− 3 18			Cor.	+ ·9		1·2
	2)79 7 2				46·8		·9,00
	39 33 31						
S.D.	+ 15 52						
	39 49 23						
R. & P.	− 1 1						
T. A.	39 48 22						

T.A.	39	48	22	
Lat.	40	20	0	sec 10·117879
P.D.	80	44	8	cosec 10·005702
	2)160	52	30	
	80	26	15	cos 9·220432
	40	37	53	sin 9·813708
E.H.A.	2h	58m	16s	xxxi. = 9·157721
A.T.S.	29d	21	1	44
Eq. T.			+	46·8
M.T.S.	29d	21	2	30·8
Long.		+ 1	42	0
M.T.G.	29d	22	44	30·8
Chron.	29d	22	45	15
Fast				44·2 on M.T.G.
Fast		1	43	31 on A.T.S.

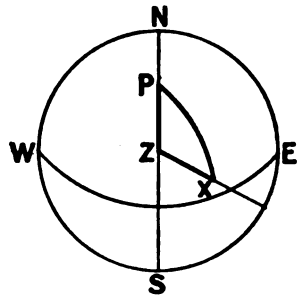


FIG. 164.

The H.A. might also have been determined, direct from the figure, as in the case of the chronometer. In the triangle PZX, the three sides are given to find angle P.

ART. 107. **Error of Chronometer by Equal Altitudes.**—(*a*) *Equal Altitudes of the Sun.*—If the times by chronometer when the Sun has equal altitudes east and west of the meridian be noted, the mean of these times, with a correction on account of the change of declination during the interval, gives the time shown by the same chronometer when the Sun's centre crosses the meridian, that is, *apparent noon.* This affords a means of finding the error of the chronometer.

The observation can best be made on shore with an artificial horizon, the latitude and longitude of the place of observation being accurately known.

If there were no change of declination, it is obvious that the hour angles E. and W. would be equal, and the mean of the times would be what the chronometer showed at apparent noon. But as the Sun's declination changes, the middle time would fall on one side or the other of apparent noon by a small interval called the "Equation of Equal Altitudes."

Thus, when the Sun's polar distance increases, the east H.A. would be greater than the west H.A., because it would take less time to fall to the same altitude, and the middle time would be a few seconds *before* apparent noon. Again, when the polar distance decreases, the west H.A. would be the greater, because it would take a longer time for the Sun to attain the same altitude, and so the middle time would then be a few seconds *after* apparent noon.

The object of the calculation is therefore to find the correction to be applied to the *middle time* to find the exact time by

chronometer when the Sun's centre was on the meridian, and thence the error of the chronometer.

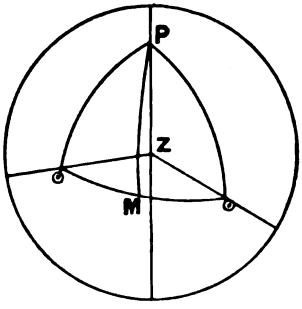


FIG. 165.

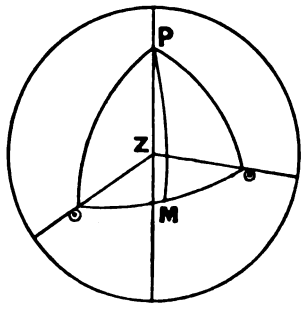


FIG. 166.

The working out of the formula gives the correction in two parts.

The first part is marked + or − according as the Sun's *polar distance* is *increasing* or *decreasing*.

The second part is marked + or − according as the *declination* is *increasing* or *decreasing*.

The *algebraical sum* of the two parts is the *Equation of Equal Altitudes* with the proper sign.

The elements required for the calculation are, the *Time Interval*, the *Sun's Declination*, and the *Equation of Time*, with the latitude and longitude of the place of observation.

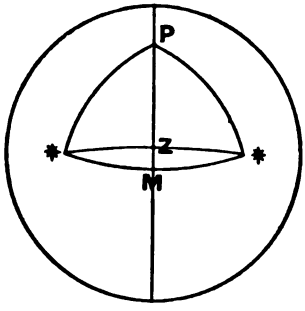


FIG. 167.

(*b*) *Equal Altitudes of a Star.*— The intervals from the time when a star crosses the meridian to the times when it has equal altitude east and west of the meridian are exactly equal, because its declination does not change. Hence the middle time by chronometer would be the time of meridian passage, and as the time of meridian passage can be found independently, the error of the chronometer can be determined.

Examples.

(1) August 1st, at Valparaiso, lat. 33° 2′ S., long. 71° 42′ W., the sun had equal alts. at—

$2^h\ 16^m\ 12^s$ A.M., and $7^h\ 20^m\ 42^s$ P.M., by chron.

Find its error on M.T.G. and A.T.S.

A.T.S. Aug.	1d	0h	0m	0s	71°	42′	
Long.		+4	46	48		4	
A.T.G.	1d	4	46	48	286°	48	

First obs. 2h 16m 12s A.M.
Second obs. 7 20 42 P.M.

Int. 5 4 30
½ Int. 2 32 15

Mid. Time 4 48 27

Dec. 18° 11′ 45″ N. H.V. 37″·26 Eq. T. 6m 10s·22 H.V. ·13 Ch. of Dec. in 2 days 30′ 24″

Cor. −2 59 4·8 Cor. −·63 4·8

18 8 46 N. 29808 6 9·59 104 60 1824
14904 52

6,0)17,8·848 ·634

2′ 59″

Int.	5h 4m 30s	...	...	log A,	7·7571	...	...	log B,	7·6533
Lat.	33° 2′	...	...	tan	9·8131				
Dec.	18 9	...	...	...	...	...	...	tan	9·5156
Two daily ch.	1824	...	...	log	3·2610	...	...	log	3·2610
				log	0·8312			log	0·4299
Pol. dist. dec. Part I.					−6″·78				
Dec. ,, ,, II.					−2·69				
Eq. of equal alts.					−9·47				
Mid. time by chron.				4h 48m 27s					
Watch showed at approx. noon				4 48 17·53					
				0 0 0					
Watch fast A.T.S.				4 48 17·53					

A.T.G. 4h 46m 48s
Eq. T. +6 9·59

M.T.G. 4 52 57·59
Watch 4 48 17·53

Slow M.T.G. 4 40·06

(2) September 30th, the following times by chron. were noted in lat. 7° 56′ S., long. 14° 26′ W., when Markab had equal alts.

Star E. 7h 34m 50s Star W. 13h 3m 53s

Find error of chron. on G.M.T.

R.A. Markab			23h	0m	4s·66		
R.A.M.S.			12	33	17·38		
Approx. M.T. transit		30d	10h	26m	47s·28		
Long. W.			+57	44			
M.T.G.	30d	11	24	31·28			
R.A.M.S.			12h	33m	17s·38		
				1	48·42		
Accel.					3·94		
					·08		
			12	35	9·82		
R.A.*			23	0	4·66		
M.T.S. of transit			10	24	54·84		
Long.				57	44		
M.T.G. of transit			11	22	38·84		
Mid. time			10	19	21·5		
Chron. slow			1	3	17·34 on G.M.T.		

First obs. 7h 34m 50s
Second obs. 13 3 53

Int. 5 29 3
½ Int. 2 44 31·5
Mid. time 10 19 21·5

EXERCISES.

1. August 4th, at 8^h 40^m A.M. A.T.S., in lat. 38° 47′ S., 92° 12′ E.; the obs. alt. of the Sun's L.L. was 16° 42′ 10″; ind. cor., + 2′ 15″; eye, 29 ft., when a chron. showed 2^h 45^m 10^s. Find its error on M.T.G. and A.T.S.

2. August 31st, at 9^h 22^m A.M. A.T.S., in lat. 52° 25′ N., long. 3° 20′ W.; the obs. alt. of the Sun's U.L. was 36° 26′ 10″; ind. cor., − 26″; eye, 41 ft.; if a chron. indicated 9^h 20^m 19^s, what was its error on M.T.G.

3. August 19th, at 4^h 26^m P.M. A.T.S., when a point in lat. 41° 35′ 30″ N., long. 70° 30′ 42″ W., bore N. 35° W., distant 14 miles; the obs. alt. of the Sun's U.L. in artificial horizon was 52° 53′ 10″; ind. cor., − 3′ 30″; if a chron. showed 9^h 10^m, find its error on M.T.G.

4. September 30th, at 3^h 30^m P.M. A.T.S., when a point in lat. 49° 42′ S., long. 179° 44′ 30″ E., bore S. 56° W., distant 9 miles; the obs. alt. of the Sun's L.L. in artificial horizon was 49° 57′ 10″; ind. cor., + 3′ 56″; eye, 50 ft. If a chron. showed 3^h 15^m 20^s, find its error on M.T.G.

5. August 16th, at 9^h 45^m P.M. M.T.S., when a point in lat. 24° 17′ 12″ N., long. 14° 9′ 30″ W., bore S. 27° E., distant 11 miles; the obs. alt. of Fomalhaut in artificial horizon was 31° 46′ 20″; ind. cor., + 2′ 12″, when a chron. showed 11^h. Find its error on M.T.G.

6. September 21st, at 10^h 6^m P.M. M.T.S., in lat. 43° 19′ S., long. 94° 26′ E.; the obs. alt. of Achernar was 54° 50′ 50″, when a chron. showed 3^h 40^m; ind. cor., + 2′ 34″; eye, 33 ft. Find error on M.T.S. and M.T.G.

7. August 22nd, at 1^h 53^m A.M. M.T.S., when a point in lat. 50° 23′ 24″ N., long. 130° 6′ 30″ W., bore N. 70° E., distant 10 miles; the obs. alt. of Vega in artificial horizon was 69° 44′ 20″; ind. cor., − 1′ 16″; a chron. showed 10^h 35^m, find its error on M.T.G.

8. August 22nd, in lat. 27° 10′ N., long. 56° 17′ E., the following times were observed when the Sun had equal alts. Find the error of the chron. for A.T.S. and M.T.G.

A.M. 4^h 19^m 27^s	P.M. 0^h 7^m 30^s
4 25 32	0 12 15

CHAPTER XXIV.

CALCULATION OF ALTITUDES.

ART. 108. **Computing an Altitude.** — In some problems of Nautical Astronomy it is necessary to compute the altitude of a celestial object. Especially is this the case when setting questions, so that the altitude shall be consistent with the other data.

The data required to be known are the hour angle of the object and the latitude and longitude of the place.

The following method has the advantage of requiring only one table of logarithms.

In the figure, S is the object, PS the polar distance, the angle

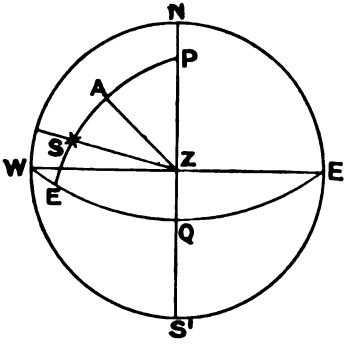


FIG. 168.

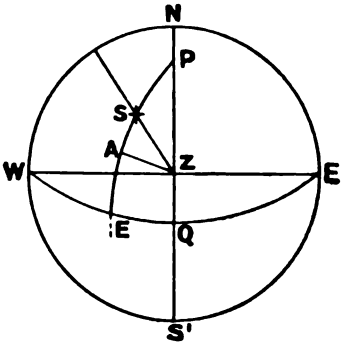


FIG. 169.

ZPS the hour angle, PZ the co-lat., and ZA an arc of a great circle perpendicular to PS.

Let PA = θ;

then SA = $(p \sim \theta)$

In the right-angled spherical triangle PAZ—

$$\cos h = \cot \text{PZ} \,.\, \tan \text{PA}$$
$$= \tan l \,.\, \tan \theta$$
$$\therefore \tan \theta = \cos h \,.\, \cot l \quad \ldots\ldots\ldots \quad (1)$$

In the same triangle—

$$\cos \text{PZ} = \cos \text{PA} \,.\, \cos \text{AZ}$$
$$i.e.\ \sin l = \cos \theta \,.\, \cos \text{AZ}$$
$$\therefore \cos \text{AZ} = \sin l \,.\, \sec \theta \quad \ldots\ldots\ldots \quad (2)$$

Also in the triangle SAZ—

$$\cos SZ = \cos AZ \,.\, \cos AS$$
$$i.e.\ \sin a = \sin l \,.\, \sec \theta \,.\, \cos (p \sim \theta)$$

by substituting the value of cos AZ from (2).

NOTE.—$(p \sim \theta)$ signifies the difference between p and θ, because one or the other may be the greater according as the perpendicular ZA falls within or without the triangle PZS.

For computation the following arrangement is convenient (l = lat., a = alt., p = polar distance, h = hour angle) :—

h	...	cos			
l	...	cot		...	sin
θ	...	tan		...	sec
p					
$p \sim \theta$	...	...	...	...	cos
a	...	...	...	...	sin

The altitude may also be calculated by the log rising method, by transposing formula proved in the ex-meridian chapter, which becomes—

sin alt. = cos zen. dist. − cos lat. cos dec. log rising hour angle, *or* by direct spherics.

ART. 109.—The first method may be stated as follows:—

(1) Find the hour angle of the object.

(2) Correct declination, find polar distance.

(3) Add cos H.A. and cot lat.; this gives tan arc 1. Difference between arc 1 and pol. dist. gives arc 2: add sin lat., sec arc 1, and cos arc 2; the result is sin true alt.

(4) Apply refraction and parallax to true alt. for app. alt.

Example.

September 7th, at $8^h\ 51^m\ 32^s$ A.M. apparent time at ship, in lat. 48° 10′ N., long. 149° 22′ 15″ W. Find true and apparent altitudes of the Sun.

A.T.S. Sept.	$6^d\ 20^h\ 51^m\ 32^s$	149° 22′ 15″	Dec. 6° 19′ 18″·6 N.
Long. W.	+ 9 57 29	4	Cor. − 6 22
A.T.G. Sept.	7^d 6 49 1	6,0)59,7 29 0	6 12 56·6 N.
			90
		$9^h\ 57^m\ 29^s$	N.P.D. 83 47 3
		H.V. 56·01	
		6·82	
	$24^h\ 0^m\ 0^s$		
	20 51 32	6,0)381·99	
E.H.A.	3 8 28	6′ 22″	

H.A. 3h 8m 28s	cos = 9·832833		
Lat. 48 10 0	cot = 9·951896	sin =	9·872208
Arc 1, 31 20 53	tan = 9·784729	sec =	10·068531
P.D. 83 47 3			
Arc 2, 52 26 10	... ...	cos =	9·785078
True alt. 32 7 58	... ...	sin =	9·725817
Par. − 8			
32 7 50			
Ref. + 1 31			
App. alt. 32 9 21			

Rules for log rising method.

(1) Find the hour angle of the object.

(2) Correct declination; find zenith distance by adding dec. and lat. if of contrary name, and subtracting them if of same name.

(3) Add cos lat., cos dec., and log rising hour angle, increasing index by 1; reject 10's and take back log, Table XXIV.; this gives "Nat. No."

Subtract "Nat. No." from Nat. cos. zen. dist., and take back Nat. sin true alt.

(4) Apply ref. and par. for app. alt.

Example.—

	H.A. 3h 8m 28s	log rising = 5·504460
	Lat. 48° 10′ 0 N.	cos = 9·824104
	Dec. 6 12 57 N.	cos = 9·997440
	211838	log = 5·326004
Z.D. 41° 57′ 3″	N. cos 743718	5926
T.A. 32 7 57	N. sin 531880	205)7800(38
R. and P. + 1 23		615
A.A. 32 9 20		1650
		1640
		10

From figure.

In figure X is the position of the object, AX is alt. of object.

In triangle PZX,

Given co-lat. = PZ
Pol. dist. = PX
Hour angle = P
Required zen. dist. = ZX

$$\text{Tan } \tfrac{1}{2}(Z + X) = \frac{\cos \frac{1}{2}(PX - PZ)}{\cos \frac{1}{2}(PX + PZ)} \cdot \cot \frac{P}{2}$$

$$\cos \frac{ZX}{2} = \frac{\cos \frac{1}{2}(PX + PZ)}{\cos \frac{1}{2}(Z + X)} \cdot \sin \frac{P}{2}$$

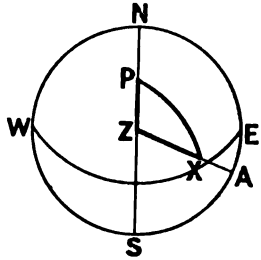


FIG. 170.

PX =	83° 47′ 3″	$\frac{P}{2}$ = 1ʰ 34ᵐ 14ˢ	cot = 10·360490	sin =	9·601715
PZ =	41 50 0				
PX+PZ =	125 37 3	½(PX+PZ) = 62° 48′ 32″	sec = 10·340122	cos =	9·659878
PX−PZ =	41 57 3	½(PX−PZ) = 20 58 32	cos = 9·970222		
		77 57 16	tan = 10·670834	sec =	10·680503
				cos =	9·942096

	28° 56′	3″
		2
ZX =	57	52 6
	90	
T.A. =	32	7 54
R. and P.		+ 1 23
A.A.	32	9 17

EXERCISES.

1. September 7th, at 11ʰ 14ᵐ 35ˢ A.M. apparent time at ship, in lat. 48° 10′ N., long. 149° 22′ 15″ W. Find true and appar. alts. of Sun.

2. September 29th, at 4ʰ 32ᵐ 46ˢ P.M. M.T.S., in lat. 46° 50′ N., long. 180° W. Find true and appar. alts. of Sun.

3. August 2nd, at 9ʰ 15ᵐ 20ˢ P.M. M.T.S., in lat. 23° 12′ N., long. 62° 35′ W. Find true and appar. alts. of Antares.

4. September 28th, at 4ʰ 5ᵐ 20ˢ A.M. M.T.S., in lat. 47° 42′ S., long. 84° 25′ E. Find true and appar. alts. of α Hydræ.

5. August 10th at 8ʰ 20ᵐ 41ˢ A.M. M.T.S., in lat. 37° 14′ N., long. 14° 26′ W. Find true and appar. alts. of the Moon.

6. August 12th, at 4ʰ 58ᵐ 48ˢ A.M. M.T.S., in lat. 25° 19′ S., long. 169° 37′ E. Find true and appar. alts. of Saturn.

7. August 30th, at 8ʰ 30ᵐ 25ˢ P.M. M.T.S., in lat. 10° 15′ S., long. 98° 27′ 15″ E. Find true and appar. alts. of the Moon.

8. August 9th, at 5ʰ 4ᵐ 2ˢ A.M. M.T.S., in lat. 5° 14′ N., long. 20° 27′ W. Find true and appar. alts. of Jupiter.

CHAPTER XXV.

LONGITUDE BY LUNAR DISTANCES.

ART. 110.—In a lunar observation the Greenwich time is found by measuring the distance between the moon and certain bright objects which lie in or near her path. The bodies chosen are the Sun, Venus, Mars, Jupiter, Saturn, and the bright stars α Arietis, Aldebaran, Pollux, Regulus, Spica, Antares, α Aquilæ, Fomalhaut, and α Pegasi.

In the N.A., pp. XIII. to XVIII.,[1] are given for each day the distances east or west, between the moon and certain of these objects at intervals of three hours of M.T.G. These true distances are the angles between the centres of the Moon and object selected, as measured at the Earth's centre. The M.T.G. corresponding to any other distance may be found by proportion, but owing to the variation in the uniformity of the motion a correction for "second differences" found in the N.A. must be applied before the *correct* G.M.T. is known. Before the invention of chronometers the "Lunar Problem" was almost the only means available at sea for finding the Greenwich time, on which the determination of longitude depends. It may still be of great use in the event of chronometers "running down" or becoming unserviceable.

The proportion referred to may be worked by arithmetic, but the use of "Proportional Logarithms" is simpler and more general. These are found in Norie, Table XXXIV.

The proportional logarithm of a given quantity is the logarithm of the ratio of some constant greater than that quantity to the given quantity. In the N.A. the constant is 3 hours, or 10,800 seconds, and the prop. log (P.L.) for any other time, for instance, $1^h\ 42^m\ 11^s$, is $= \log \frac{3^h}{1^h\ 42^m\ 11^s}$

$$= \log 10800 - \log 6131$$
$$= 4{\cdot}033424 - 3{\cdot}787531$$
$$= 0{\cdot}245893$$

which is entered in tables as 2459, the nearest fourth place only being used. This table facilitates the finding of a time corresponding to a true distance, as by it only two references are necessary, whereas by ordinary logs four entries are made before obtaining the result.

[1] Discontinued since 1908.

Suppose x and y the distances in the N.A. between which the true distance d lies, and t the number of hours elapsed since the time for which x is tabulated.

Then—

$y - x$ is change in distance in three hours,
$d - x$ „ „ „ the required time t.

Hence— $$y - x : d - x :: 3 : t$$

$$\text{or } \frac{3}{t} = \frac{y - x}{d - x}$$

$$= \frac{3}{d - x} \div \frac{3}{y - x}$$

which is P.L. t = P.L. $(d - x)$ − P.L. $(y - x)$

The quantity P.L. $(y - x)$ is tabulated in N.A. opposite the distance x.

In using these distances the instructions in the N.A. are—

"Take the difference between the reduced distance and the *nearest* distance *preceding* it, in order of time, in the ephemeris; from the P.L. of this difference subtract the P.L. in the ephemeris; the remainder will be the P.L. of a portion of time to be added to the hour answering to the *nearest* preceding distance, to obtain the approximate Greenwich mean time corresponding to the given distance."

When selecting a suitable object for observation, it must be remembered that those objects are most favourably situated from which the moon is receding, or to which the moon is approaching most rapidly, and this is shown by the smallness of the P.L. opposite to it; attention should also be paid to the object having as small a difference in declination from the moon as possible compared with their difference in right ascension.

Examples.—

(1) The true distance between the Moon and α Arietis on August 13th is 37° 51′ 32″. Find G.M.T.

Tr. dist.	37° 51′ 32″				
N.A. „	37 20 18	P.L. 3493	...	Aug. 13^d	12^h 0^m 0^s
	31 14	P.L. 7607			
		P.L. 4114	... ...		1 9 48
P.L. 3527					
„ 3463			Approx. M.T.G.	13^d	13 9 48
2)64		With diff. − 32 of P.L.'s and Int. 1^h 10^m cor. for 2nd diffs. =			+ 9
Mean diff. 32			M.T.G.	13^d	13 9 57

(2) The true distance between the Moon and Spica on August 26th is 43° 1′ 29″. Find G.M.T.

Tr. dist.	43° 1′ 29″							
N.A. ,,	42 11 22	P.L. 2357	...	Aug. 26^d	15^h 0^m	0^s		
	50 7	P.L. 5553						
P.L. 2362		P.L. 3196	...	...		1	26	14
,, 2352								
			Approx. M.T.G.		26^d	16	26	14
2)10	With diff. −5 of P.L.'s and							
	Int. 2^h 16^m cor. for 2nd diff. =							+1·5
Mean diff. 5								
			M.T.G.		26^d	16	26	15·5

EXERCISES.

1. The true distance between the Moon and Aldebaran on August 6th is 80° 29′ 7″. Find G.M.T.
2. The true distance between the Moon and Jupiter on August 7th is 89° 21′ 12″. Find G.M.T.
3. The true distance between the Moon and α Pegasi on August 12th is 69° 40′ 15.″ Find G.M.T.
4. The true distance between the Sun and Moon on August 22nd is 42° 35′ 17″. Find G.M.T.
5. The true distance between the Moon and Spica on August 28th is 65° 32′ 11.″ Find G.M.T.

ART. 111. **The Lunar.**—Let ZX and ZY be two vertical circles passing through the apparent positions M′ and S′ of the Moon and another celestial object (Sun, Star, or Planet); the true positions M and S, as seen from the Earth's centre, will be on the same vertical circles, and differ from the apparent positions by the effects of refraction and parallax. The Moon's parallax exceeds the refraction, but the parallax of any other body is less than the refraction. Hence the Moon's apparent place is lower, and that of the other body higher, than the true positions.

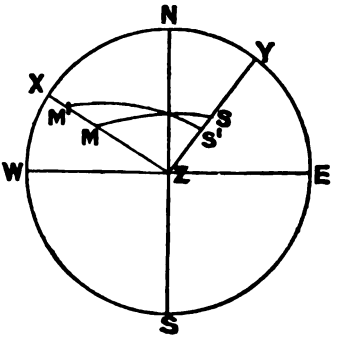


FIG. 171.

M′S′ and MS are arcs of two great circles, and represent the apparent and true distances.

XM′ and XM are the apparent and true altitudes of the Moon, and YS′ and YS the apparent and true altitudes of the other body.

The angle Z is common to the two triangles ZM′S′ and ZMS.

The problem is to find the true distance MS, the apparent distance M′S′, and the true and apparent altitudes being given.

In the spherical triangle ZM′S′ are given the three sides,

ZM', ZS', and M'S', to find the angle Z. Then in the triangle ZMS, the two sides ZM and ZS and the included single Z being known, the third side MS or true distance can be found.

The problem can therefore be solved by simply applying the formulæ of Cases I. and II. of oblique-angled spherical trigonometry, but as this involves two separate computations, several methods have been devised for *clearing the distance* by a single logarithmic computation. The method most generally used is known as *Borda's*, which requires only one table of logarithms. It is obtained as follows:—

Applying the general formula, $\cos A = \dfrac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$

In the triangle ZM'S', $\cos Z = \dfrac{\cos M'S' - \cos ZM' \,.\, \cos ZS'}{\sin ZM' \,.\, \sin ZS'}$. (1)

In the triangle ZMS, $\cos Z = \dfrac{\cos MS - \cos ZM \,.\, \cos ZS}{\sin ZM \,.\, \sin ZS}$. (2)

Let m and m' represent the Moon's true and apparent altitudes;
s and s' the true and apparent altitudes of the other body;
d and d' the true and apparent distances;

Then equation (1) becomes—

$$\cos Z = \frac{\cos d' - \sin m' \,.\, \sin s'}{\cos m' \,.\, \cos s'}$$

and equation (2) becomes—

$$\cos Z = \frac{\cos d - \sin m \,.\, \sin s}{\cos m \,.\, \cos s}$$

$$\therefore \frac{\cos d - \sin m \,.\, \sin s}{\cos m \,.\, \cos s} = \frac{\cos d' - \sin m' \,.\, \sin s'}{\cos m' \,.\, \cos s'}$$

} by putting altitudes for zenith distances

Adding 1 to each side—

$$1 + \frac{\cos d - \sin m \,.\, \sin s}{\cos m \,.\, \cos s} = 1 + \frac{\cos d' - \sin m \,.\, \sin s'}{\cos m' \,.\, \cos s'}$$

$$\therefore \frac{\cos d + \cos m \,.\, \cos s - \sin m \,.\, \sin s}{\cos m \,.\, \cos s} = \frac{\cos d' + \cos m' \,.\, \cos s' - \sin m' \,.\, \sin s'}{\cos m' \,.\, \cos s'}$$

that is—

$$\frac{\cos d + \cos (m + s)}{\cos m \,.\, \cos s} = \frac{\cos d' + \cos (m' + s')}{\cos m' \,.\, \cos s'}$$

$$\therefore \cos d + \cos (m + s) = \frac{\cos m \,.\, \cos s}{\cos m' \,.\, \cos s'} \,.\, \{\cos d' + \cos (m' + s')\}$$

But $\cos d = 1 - 2\sin^2\frac{d}{2}$, and $\cos(m+s) = 2\cos^2\frac{m+s}{2} - 1$

$$\therefore 1 - 2\sin^2\frac{d}{2} + 2\cos^2\frac{m+s}{2} - 1 = -2\sin^2\frac{d}{2} + 2\cos^2\frac{m+s}{2}$$

$$= \frac{\cos m \cdot \cos s}{\cos m' \cdot \cos s'} \cdot \{\cos d' + \cos(m'+s')\}$$

$$= \frac{\cos m \cdot \cos s}{\cos m' \cdot \cos s'} \cdot 2\cos\frac{m'+s'+d'}{2} \cdot \cos\frac{m'+s'-d'}{2}$$

Dividing both sides by 2, and putting $\sin^2\theta$ for

$$\frac{\cos m \cdot \cos s}{\cos m' \cdot \cos s'} \cdot \cos\frac{m'+s'+d'}{2} \cdot \cos\frac{m'+s'-d'}{2}$$

we have—

$$-\sin^2\frac{d}{2} + \cos^2\frac{m+s}{2} = \sin^2\theta$$

$$\therefore \sin^2\frac{d}{2} = \cos^2\frac{m+s}{2} - \sin^2\theta$$

$$= \tfrac{1}{2}\{1 + \cos(m+s)\} - \tfrac{1}{2}(1 - \cos 2\theta)$$
$$= \tfrac{1}{2}\{\cos(m+s) + \cos 2\theta\}$$
$$= \tfrac{1}{2}\left\{2\cos\left(\frac{m+s}{2} + \theta\right) \cdot \cos\left(\frac{m+s}{2} - \theta\right)\right\}$$

$$\therefore \sin\frac{d}{2} = \sqrt{\cos\left(\frac{m+s}{2} + \theta\right) \cdot \cos\left(\frac{m+s}{2} - \theta\right)}$$

Form for Computation.

		° ′ ″		
	App. dist. (d')	— — —		
Moon's	,, alt. (m')	— — —	...	sec
Sun's or Star's	,, ,, (s')	— — —	...	sec
	2)	— — —		
½ sum = $\left(\frac{m'+s'+d'}{2}\right)$		— — —	...	cos
½ sum − d' = $\left(\frac{m'+s'-d'}{2}\right)$		— — —	...	cos
Moon's	true alt. (m)	— — —	...	cos
Sun's or Star's	,, (s)	— — —	...	cos
	$(m+s)$ = 2)	— — —		
	$\left(\frac{m+s}{2}\right)$	— — —		

$$
\begin{array}{lll}
 & & 2)\text{---}\ \text{---} \\
(\theta)\ \text{---}\ \text{---}\ \text{---} & \ldots & \sin \\
\text{sum}\left(\frac{m+s}{2}+\theta\right)\ \text{---}\ \text{---}\ \text{---} & \ldots & \cos \\
\text{diff.}\left(\frac{m+s}{2}-\theta\right)\ \text{---}\ \text{---}\ \text{---} & \ldots & \cos \\
 & & 2)\text{---}\ \text{---} \\
\frac{d}{2}\ \text{---}\ \text{---}\ \text{---} & \ldots & \sin \\
2 & & \\
\hline & & \\
d = & &
\end{array}
$$

Remark.—It will be noticed that the first four logs are the same as in the alt.-azimuth problems, and that all other logs taken out are cosines, whilst the angles sought correspond to sines.

ART. 112. **Rule for clearing the Distance by Borda.**—Add together apparent distance, app. alt. of moon, and app. alt. of body observed for distance; find half sum, and the remainder, which is the difference between the app. dist. and half sum. Then add following logs: Sec app. alt. of moon, sec app. alt. of body, cos half sum, cos remainder, cos true alt. of moon, and cos true alt. of body; reject 40 from index, and divide by 2, and look back sine of angle, which call B. Now find half sum of two true alts., which call A. Add together the cosines of the sum and difference of A and B; divide sum by 2, and look out sine of angle, which is half true distance.

Example.—

The apparent distance between the Sun and Moon's centre is 105° 56′ 46″; Moon's app. alt., 26° 7′ 19″, true alt., 26° 55′ 26″; Sun's app. alt., 14° 24′ 27″, true alt., 14° 20′ 56″. Find true distance.

App. dist.	105°	56′	46″		
Moon's A.A.	26	7	19	sec	10·046792
Sun's A.A.	14	24	27	sec	10·013878
	2)146	28	32		
	73	14	16	cos	9·459997
	32	42	30	cos	9·925019
Moon's T.A.	26	55	26	cos	9·950174
Sun's T.A.	14	20	56	cos	9·986236
	2)41	16	22	2)	19·382096
A	20	38	11		
B	29	24	14	sin	9·691048
A + B	50	2	25	cos	9·807703
B − A	8	46	3	cos	9·994895
				2)	19·802598
	52	49	0	sin	9·901299
			2		
True Dist.	105	38	0		

From the figure.

In the figure AX and AN are the true and apparent alts. of the Sun. BY and BM are the true and app. alts. of the Moon. MN is the app. dist., and XY the true dist.

In the triangle ZMN, given the three sides, ZM, Moon's a.z.d.: ZN, Sun's a.z.d. and MN, to find Z.

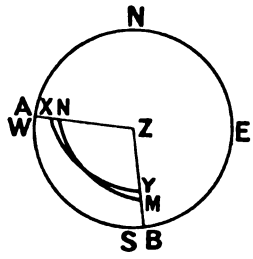


FIG. 172.

$$\text{Cos}\ \frac{Z}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - MN)}{\sin ZM \,.\, \sin ZN}}$$

where $s = \frac{1}{2}(ZM + ZN + MN)$.

MN =	105° 56′ 46″	
ZM =	63 52 41	cosec 10·046792
ZN =	75 35 33	cosec 10·013877
	2)245 25 0	
s =	122 42 30	sin 9·925019
s − MN =	16 45 44	sin 9·459995
		2)19·445683
$\frac{Z}{2}$ =	58 6 45	cos 9·722841

Now, in triangle ZXY, given two sides, ZY, Moon's t.z.d.; ZX, Sun's t.z.d., and included angle Z (just found), find XY, true dist.

$$\text{Tan}\ \tfrac{1}{2}(Y + X) = \frac{\cos \frac{1}{2}(ZX - ZY)}{\cos \frac{1}{2}(ZX + ZY)} \,.\, \cot \frac{Z}{2}$$

$$\text{Cos}\ \frac{XY}{2} = \frac{\cos \frac{1}{2}(ZX + ZY)}{\cos \frac{1}{2}(Y + X)} \,.\, \sin \frac{Z}{2}$$

ZX =	75° 39′ 4″					
ZY =	63 4 34	$\frac{Z}{2}$ =	58° 6′ 45″	cot	9·793890	sin 9·928953
ZX + ZY =	138 43 38	$\frac{1}{2}$(ZX + ZY) =	69 21 49	sec	10·452920	cos 9·547080
ZX − ZY =	12 34 30	$\frac{1}{2}$(ZX − ZY) =	6 17 15	cos	9·997379	
		$\frac{1}{2}$(Y + X) =	60 19 13·5	tan	10·244189	sec 10·305264
						52° 49′ 1″ cos 9·781297
						2
						True dist. 105 38 2

EXERCISES.

1. Given app. dist., 96° 52′; Sun's app. alt., 36° 18′ 7″, true alt., 36° 16′ 57″; Moon's app. alt., 24° 17′ 14″, true alt., 25° 5′ 27″. Find true dist.
2. Given app. dist., 107° 35′ 18″; Saturn's app. alt., 9° 51′ 21″, true alt., 9° 46′ 2″; Moon's app. alt., 38° 43′, true alt., 39° 27′ 20″. Find true dist.
3. Given app. dist., 101° 0′ 4″; Mars' app. alt., 14° 36′ 7″, true alt., 14° 32′ 35″; Moon's app. alt., 48° 10′ 5″, true alt., 48° 49′ 29″. Find true dist.
4. Given app. dist., 56° 14′ 32″; Sun's app. alt., 65° 15′ 36″, true alt., 65° 15′ 13″; Moon's app. alt., 40° 32′ 16″, true alt., 41° 15′ 48″. Find true dist.

ART. 113.—Rules for Lunar Problem.

(1) Find the G.M.T.
(2) Correct the R.A.M.S., the Moon's R.A., Dec., S.D., and H.P. Correct the R.A., and Dec., of object observed for distance.
(3) Correct the obs. alt., finding app. and true alts., and calculate H.A. of object; if both alts. are given, either may be chosen.
(4) With H.A. just found, calculate R.A.M. and H.A. of the other object, and thence its alts. true and app. (unless given).
(5) Correct observed distance for ind. cor. and S.D., adding for Near limbs, and subtracting for Far or remote limbs. In the case of a Sun Lunar, the enlightened limbs must be the near limbs.
(6) Clear the distance.
(7) Find corresponding G.M.T. and error of chronometer.
(8) From R.A.M. in (4) find M.T.S. and longitude.

Example.—

August 16th, P.M. at ship in lat. 20° 29′ S., when the time by a chron. was $16^d\ 2^h\ 14^m\ 50^s$, which was estimated $2^m\ 56^s$ fast of M.T.G.; the obs. alt. of the Sun's L.L. was 48° 1′ 20″; ind. cor., + 2′ 12″; height of eye, 27 ft.; observed distance between the enlightened limbs of Sun and Moon, 41° 3′ 10″; ind. cor., − 2′ 4″. Find error of chron. for G.M.T. and longitude.

Time by chron.	Aug. $16^d\ 2^h\ 14^m\ 50^s$			R.A.M.S.	$9^h\ 35^m\ 52^s{\cdot}5$		
Fast	$-\ 2\ 56$			Accel.		$21^s{\cdot}7$	
M.T.G.	$16^d\ 2\ 11\ 54$				$9\ 36\ 14{\cdot}2$		
S.D.	15′ 1″·4	4″		H.P.	55′ 2″·4	14″·6	
Cor.	+ ·7	2·2		Cor.	+ 2·7	2·2	
	15 2·1	12)8·8			55 5·1	12)32·12	
Aug.	+ 3·2	—		Red.	− 1·3	—	
		·7				2·7	
	15 5·3				55 3·8		
Moon's Dec.	20° 17′ 41″·5 N.	Var. ·25		R.A.	$6^h\ 51^m\ 0^s{\cdot}6$	Var. 2·18	
Cor.	− 3	11·9		Cor.	+ 25·9	11·9	
	20 17 38 N.	2·97			6 51 26·5	25·942	
Sun's Dec.	13° 57′ 31″·4 N.	H.V. 47·03		R.A.	$9^h\ 40^m\ 9^s{\cdot}1$	H.V. 9·37	
Cor.	− 1 43·4	2·2		Cor.	20·6	2·2	
	13 55 48 N.	103·46			9 40 29·7	20·592	

O.A. ⊙	48° 1′ 20″
I.C.	+ 2 12
	48 3 32
Dip	− 5 5
	47 58 27
S.D.	+ 15 49
⊙'s A.A.	48 14 16
R. & P.	− 45
⊙'s T.A.	48 13 31

⊙'s T.A.	48	13	31		
Lat.	20	29	0	sec	10·028365
P.D.	103	55	48	cosec	10·012964
	2)172	38	19		
	86	19	10	cos	8·807492
	38	5	39	sin	9·790254
⊙'s W.H.A.	1h	36m	22s·5	xxxi. =	8·639075
⊙'s R.A.	9	40	29·7		
R.A.M.	11	16	52·2		
☽'s R.A.	6	51	26·5		
☽'s W.H.A.	4	25	25·7	log rising	5·777405
Lat.	...	20°	29′ 0″	S. cos	9·971635
Dec.	...	20	17 38	N. cos	9·972169
				526271 log	5·721209
Z.D.	40°	46′	38″	Nat. cos 757255	
☽'s T.A.	13	21	18	N. sin 230984	
Cor.	−	49	33		
☽'s A.A.	12	31	45		
Obs. dist.	41°	3′	10″		
I.C.	−	2	4		
	41	1	6		
⊙'s S.D.		15	49		
☽'s S.D.		15	5		
App. dist.	41	32	0		
☽'s A.A.	12	31	45	sec	10·010468
⊙'s A.A.	48	14	16	sec	10·176498
	2)102	18	1		
	51	9	1	cos	9·797461
	9	37	1	cos	9·993854
☽'s T.A.	13	21	18	cos	9·988094
⊙'s T.A.	48	13	31	cos	9·823607
	2)61	34	49		
A.	30	47	24·5	2)	19·789982
B.	51	44	27·5	sin	9·894991
A + B	82	31	52	cos	9·113903
B − A	20	57	3	cos	9·970295
				2)	19·084198
	20	23	26	sin	9·542099
			2		
True dist.	40	46	52		

True dist.	40°	46′	52′					
N.A. ,,	41	49	2	P.L. 3267	...	...	Aug. 16^d 0^h 0^m	0^s
	1	2	10	P.L. 4617				
				P.L. 1350	...	...	...	2 11 54·5
							Approx. M.T.G. 16^d	2 11 54·5
Cor. for, − 12, Diff. P.L.'s, and Int. 2^h 10^m from N.A.								+ 3
							M.T.G. 16^d	2 11 57·5
							Time by chron. 16^d	2 14 50
							Fast	2 52·5

R.A.M.	11^h	16^m	52^s·2
R.A.M.S.	9	36	14·2
M.T.S. 16^d	1	40	38
M.T.G. 16^d	2	11	57·5
		31	19·5

Long. 7° 49′ 52″ W.

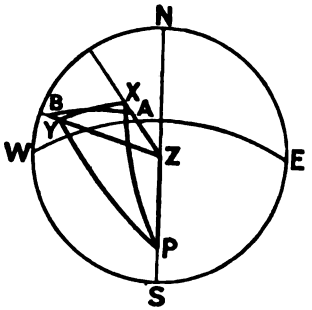


FIG. 173.

From the figure.—

In the figure X and A are true and app. positions of the Sun; Y and B are true and app. positions of the Moon.

First, to find Sun's H.A.—In triangle PZX, given co-lat. = PZ, Pol. dist. = PX, and zen. dist. = ZX, to find P.

$$\text{Cos}\,\frac{P}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - ZX)}{\sin PX \,.\, \sin PZ}}$$

where $s = \frac{1}{2}(ZX + PX + PZ)$.

ZX =	41° 46′ 29″	
PX =	103 55 48	cosec 10·012964
PZ =	69 31 0	cosec 10·028365
	2)215 13 17	
s =	107 36 39	sin 9·979154
s − ZX =	65 50 10	sin 9·960174
		2)19·980657
	0^h 48^m 11^s·5	cos 9·990328
⊙'s W.H.A.	1 36 23	
⊙'s R.A.	9 40 29·7	
R.A.M.	11 16 52·7	
☽'s R.A.	6 51 26·5	
☽'s H.A.	4 25 26·2	

Second, to find Moon's True and Apparent Altitudes.

In triangle PYZ, given PZ = co-lat., PY = ☽'s Pol. dist., P = Hour angle, to find ZY = ☽'s true Z.D.

$$\text{Tan } \tfrac{1}{2}(Z+Y) = \frac{\cos \frac{1}{2}(PY-PZ)}{\cos \frac{1}{2}(PY+PZ)} \cdot \cot \frac{P}{2}$$

$$\text{Cos } \frac{ZY}{2} = \frac{\cos \frac{1}{2}(PY+PZ)}{\cos \frac{1}{2}(Z+Y)} \cdot \sin \frac{P}{2}$$

PY = 110° 17′ 38″

PZ = 69 31 0

$\frac{P}{2} = 2^h\ 12^m\ 43^s{\cdot}1$ cot 10·184507 sin 9·738198

PY + PZ = 179 48 38 $\frac{1}{2}$(PY + PZ) = 89° 54′ 19″ sec 12·781671 cos 7·218329

PY − PZ = 40 46 38 $\frac{1}{2}$(PY − PZ) = 20 23 19 cos 9·971902

89 56 2·2 tan 12·938080 sec 12·938080

38° 19′ 23″·5 cos 9·894607

2

☽'s true Z.D. = 76 38 47

☽'s T.A. 13 21 13

Cor. − 49 33

☽'s A.A. 12 31 40

Third, to clear the Distance.

In triangle ZAB, given AB = app. dist., ZA = ⊙'s A.Z.D., ZB = ☽'s A.Z.D. to find angle Z.

$$\text{Cos } \frac{Z}{2} = \sqrt{\frac{\sin s \cdot \sin (s - AB)}{\sin ZA \cdot \sin ZB}}$$

where $s = \frac{1}{2}(AB + ZA + ZB)$.

AB = 41° 32′ 0″

ZA = 41 45 44 cosec 10·176499

ZB = 77 28 20 cosec 10·010466

2)160 46 4

s = 80 23 2 sin 9·993855

s − AB = 38 51 2 sin 9·797469

2)19·978289

$\frac{Z}{2}$ = 12 45 27·5 cos 9·989144

Finally, in triangle ZXY, given ZX = ⊙'s true Z.D., ZY = ☽'s true Z.D, and angle Z, to find XY = true dist.

$$\text{Tan } \tfrac{1}{2}(X+Y) = \frac{\cos \frac{1}{2}(ZY-ZX)}{\cos \frac{1}{2}(ZY+ZX)} \cdot \cot \frac{Z}{2}$$

$$\text{Cos } \frac{XY}{2} = \frac{\cos \frac{1}{2}(ZY+ZX)}{\cos \frac{1}{2}(X+Y)} \cdot \sin \frac{Z}{2}$$

ZY =	76° 38′ 47″				
ZX =	41 46 29	$\frac{Z}{2}$ = 12° 45′ 27·5″	cot 10·645090	sin 9·344053	
ZY+ZX =	118 25 16	$\frac{1}{2}$(ZY+ZX) = 59 12 38	sec 10·290828	cos 9·709172	
ZY−ZX =	34 52 18	$\frac{1}{2}$(ZY−ZX) = 17 26 9	cos 9·979573		
		83 4 25	tan 10·915491	sec 10·918672	
			20° 23 26″	cos 9·971897	
			2		
		True dist. 40 46 52			

Finish as before.

Remark.—Questions are sometimes set where it is not stated whether the object whose altitude is given is E. or W. of the meridian, but this may be determined by noticing in the "Nautical Almanac" whether the other object is E. or W. of the Moon, and, taking the H.A. in conjunction with the distance, the wrong supposition would put one of the objects too near the horizon, or even below it.

Exercises.

1. August 13th, about 10ʰ 30ᵐ A.M. at ship, in lat. 35° 46′ S., long. D.R. 93° E.; when a chron. estimated 6ᵐ fast of G.M.T. showed 4ʰ 20ᵐ 15ˢ, the obs. alt. of the Sun's L.L. was 34° 13′ 30″; ind. cor., − 1′ 44″; height of eye, 30 ft.; obs. dist. between near limbs of Sun and Moon, 78° 22′ 50″; no ind. cor. Find long. and error of chron. on G.M.T.

2. August 28th, about 3ʰ 35ᵐ P.M. at ship, in lat. 24° 14′ N., long. by acct., 163° 40′ W.; when a chron. whose supposed error on G.M.T. was 9ᵐ 18ˢ slow showed 2ʰ 20ᵐ 18ˢ, the obs. alt. of the Moon's U.L. was 14° 27′ 40″; ind. cor., + 1′ 11″; eye, 35 ft.; obs. dist. between enlightened limbs of Sun and Moon, 117° 32′ 40″; ind. cor., + 2′ 12″. Find the error of chron. on G.M.T. and longitude.

3. August 10th, A.M. at ship, in lat. 37° 50′ N.; when the M.T.G. by a chron. supposed correct was 9ᵈ 3ʰ 49ᵐ 18ˢ, the obs. alt. of the Moon's L.L. E. of meridian was 53° 11′ 20″; ind. cor., − 3′ 21″; eye, 31 ft.; obs. dist. between Aldebaran and the Moon's near limb, 49° 17′ 50″; ind. cor., + 2′ 13″. Find long. and error of chron. on G.M.T.

4. August 22nd, about 6ʰ 30ᵐ P.M. at ship, in lat. 29° 45′ S., long. D.R. 34° E.; when a chron. estimated 10ᵐ 18ˢ fast of G.M.T. showed 4ʰ 20ᵐ 30ˢ, the obs. alt. of α Aquilæ (Altair) was 28° 21′ 50″; ind. cor., + 2′ 16″; eye, 40 ft.; obs. dist. between Altair and the Moon's remote limb, 113° 44′ 10″; ind. cor., − 3′ 11″. Find long. and error of chron. on G.M.T.

5. August 13th, about 2ʰ A.M. at ship, in lat. 30° 27′ N., long. by acct., 41° W.; when a chron. supposed correct for G.M.T. showed 4ʰ 38ᵐ 12ˢ, the obs. alt. of the Moon's L.L. was 23° 12′ 20″; no ind. cor.; eye, 35 ft.; obs. dist. between Saturn's centre and the Moon's further limb., 78° 43′ 10″; ind. cor., −1′ 18″. Find long. and error of chron. on G.M.T.

6. August 24th, at 11ʰ 23ᵐ A.M. at ship, in lat. 53° 34′ N., long. 4° 25′ E.; when a chron. which had run down and been set going indicated 11ʰ 20ᵐ, the following observations were taken for determining long. and error of chron. on G.M.T.; height of eye, 28 ft. :—

Obs. alt. Sun's L.L.	Obs. alt. Moon's U.L.	Obs. dist. Sun and Moon.
46° 53′ 40″	10° 15′ 40″	56° 24′ 10″
I.C. + 1 18	I.C. − 2 18	I.C. − 2 9

7. August 29th, at 8^h P.M. at ship, in lat. 48° 43′ S., long. 30° 49′ 45″ W.; when a chron. which had run down and been set going showed $10^h\ 15^m$, the obs. alt. of Antares was 56° 40′ 20″; ind. cor., + 1′ 57″; eye, 35 ft.; obs. dist. between Star and Moon's near limb was 35° 57′ 20″; ind. cor., − 3′ 33″. Find the long. and error of chron. on G.M.T.

8. August 9th, about 5^h A.M. at ship, in lat. 5° 14′ N., long. by acct., 20° 40′ W.; when a chron. estimated correct for G.M.T. showed $6^h\ 25^m\ 48^s$, the obs. alt. of Jupiter's centre was 30° 30′ 20″; ind. cor., − 2′ 16″; obs. alt. of Moon's L.L. was 70° 48′; ind. cor., + 2′ 9″; eye, 32 ft.; obs. dist. between Jupiter's centre and Moon's near limb, 78° 3′ 50″; ind. cor., − 2′ 15″. Find long. and error of chron. on M.T.G.

9. September 1st, about $1^h\ 30^m$ A.M. at ship, in lat. equator, long. by acct., 43° E.; when a chron. supposed to be $12^m\ 10^s$ slow of G.M.T. indicated $10^h\ 25^m\ 32^s$, the obs. alt. of α Pegasi (Marcab) was 67° 52′ 50″; obs. dist. between Marcab and the Moon's remote limb, 44° 55′ 20″; ind. cor. for alt., + 1′ 31″; for dist., − 2′ 17″; height of eye, 31 ft. Find long. and error of chron. on M.T.G.

PART II.

CHAPTER XXVI.

CONSTRUCTION OF CHARTS.

ART. 114.—Before proceeding to the actual drawing of a chart one or two matters must be considered. In the first place there are two geometrical problems which are continually in use: (1) To erect a perpendicular to a given straight line at a given point in the same. (2) To divide a given straight line into a required number of equal parts. There are several ways for performing each operation. Consider (1) when the given point is the end of the line. Let AB be line, through B draw a line perpendicular to AB. Select any point C above AB, with C as centre and CB as radius describe an arc of a circle cutting AB in D. A line through DC cuts the arc again in E, which is perpendicularly above B, since angle DBE is angle in a semicircle, and therefore a right angle.

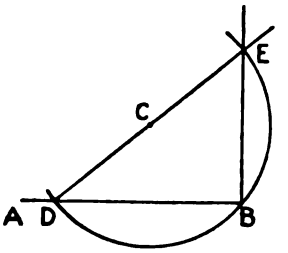


FIG. 174.

For (2) the trial method will give as good results as any to any one unaccustomed to use drawing instruments. Good practice in the above may be obtained by the construction of Diagonal Scales.

Example.—
Construct a diagonal scale of 1¼ inch to 1° long., to measure miles.

Draw a horizontal line, AB, near the bottom of the drawing-paper, and supposing the scale to be 1¼ inch to 1° long., along it measure off distances A 1, 1 2, 2 3, each = 1¼ inch; at A and 3 erect perpendiculars AD, 3F, and measure along these perpendiculars any ten equal parts—2, 4, 6, 8, show the even points; join these points by ten lines, which will be all parallel to AB. On the upper one mark off points 1, 2 to correspond with the points 1, 2 on AB, and join 1 to 1 and 2 to 2. Through D draw a line DO, making any angle with DF, and on DO measure any six equal parts at *a, b, c, d, e, f;* lines

drawn through *a*, *b*, *c*, *d*, and *e* parallel to *f*1 will divide D1 into six equal parts, and since D1 represents 1°, each part represents 10′ of long.; with these distances divide A1 into six equal parts, and join up diagonally as shown in figure, by lines similar to 1*g*. Considering the triangles 1*hm* and 1 1*g*, these

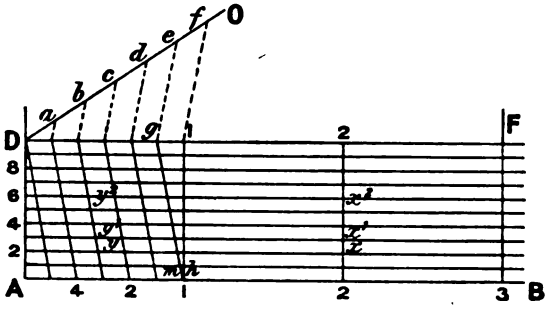


FIG. 175.

are seen to be similar, and since 1 1 is ten times 1*h*, therefore 1*g* is ten times *hm*; but 1*g* is 10′ of long., hence *hm* is 1′ of long.; that is, the portions of the lines intercepted between 1 1 and 1*g* represent the single miles from 1 to 10.

Angles may be set off (1) directly from a protractor; (2) by using the scale of chords, marked Cho. On this scale the radius for describing the arc is the length to 60°, and the length of the chord subtending any required angle at the centre is the length of the scale to the angle; (3) by using the formula for finding the length of chord (*l*) for any convenient radius (*r*) to subtend angle (A) viz., $l = 2r \times \sin \frac{A}{2}$.

Thus for an angle of 37° with radius 1·5 inch—

$$l = 2 \times 1{\cdot}5 \times \sin 18\tfrac{1}{2}° \text{ inch}$$
$$= 3 \times {\cdot}3173 \text{ inch} = {\cdot}952 \text{ inch.}$$

On a plan is sometimes found a fraction in the title space thus $\frac{1}{10944}$, called the *Natural Scale*, which shows the ratio that the length of a certain unit on the plan bears to the real length of that unit on the earth's surface. Hence 1 inch represents 10944 inches.

" $\frac{10944}{12}$ feet
" 912 "
" 1·5 cables.

Again, if 2 inches represent a nautical mile, the natural scale is—

$$\frac{2}{6080 \times 12} = \frac{1}{36480}.$$

When the scale for a minute of longitude is given the scale for a mile of latitude may be found, and *vice versâ*. On a Mercator Chart

Mile of latitude = minute of long. × sec. lat.
or min. of long. = mile of lat. × cos. lat.

and so may be calculated, or found in traverse table.

ART. 115.—Any straight line drawn in a circle and terminated at the circumference is called a chord, and the portions into which it divides the circle are called segments. When the line passes through the centre, the chord becomes a diameter, and the segments semicircles. The angle subtended by the chord at any point in the circumference of the circle is called the angle in the segment, and all angles in the same segment of a circle are equal to one another. When the segment is equal to a semicircle this angle is a right angle; when the segment is greater than a semicircle this angle is less than a right angle; when the segment is less than a semicircle this angle is obtuse. A segment of a circle may always be described on a given chord which contains an angle of a given size.

When the given angle is *acute*, the segment is greater than a semicircle, hence the centre for describing the arc and required arc are on the *same* side of the chord.

When the given angle is *obtuse*, the segment is less than a semicircle, hence the centre for describing the arc and required arc are on *different* sides of the chord.

To find the centre set off the complement of the given angle at each end of the chord, these lines will intersect in the centre.

Let A, B, and C be three conspicuous objects, A on the extreme left of the observer, C on his right, B the central object, the horizontal angle ($a°$) between A and B may be observed and the horizontal angle ($b°$) between B and C. If now segments of circles be described on AB and BC as chords to contain angles of $a°$ and $b°$ respectively, the intersection of these arcs will give the position of the observer. Care must be taken in the selection of the objects, or the cut of the two arcs is uncertain as to position. When the two arcs coincide it shows that the observer and the three points are on one circle, and there is no fix; to avoid this the central object should be chosen as nearly in a line as possible with the other two, or should be nearer the observer than the others. The station pointer performs the operation practically and the fix is tabulated thus: A.$a°$ B.$b°$ C.

ART. 116.—**To construct a Mercator's Chart.**—The advantages of such a chart are that all parallels of latitude are projected as parallel straight lines, and all meridians are projected as parallel straight lines cutting the parallels of latitude at right angles; a rhumb line projects as a straight line, since it cuts all meridians at the same angle, and all places of small area retain their proper form on the chart, but with increased area as the latitude increases.

Example 1 (Fig. 176).

Construct a Mercator's chart on a scale of 1·9 inch to 1° long., to extend from lat. 53° N. to 54° 30′ N., and from long. 3° W. to 7° W. Insert a

magnetic compass. Variation 17° 20′ W., and the following positions. Chicken Rock 54° 2′ N., 4° 50′ W., Langness 54° 3′ N., 4° 37′ W., N.W. Lt. V. 53° 31′ N., 3° 31° W.

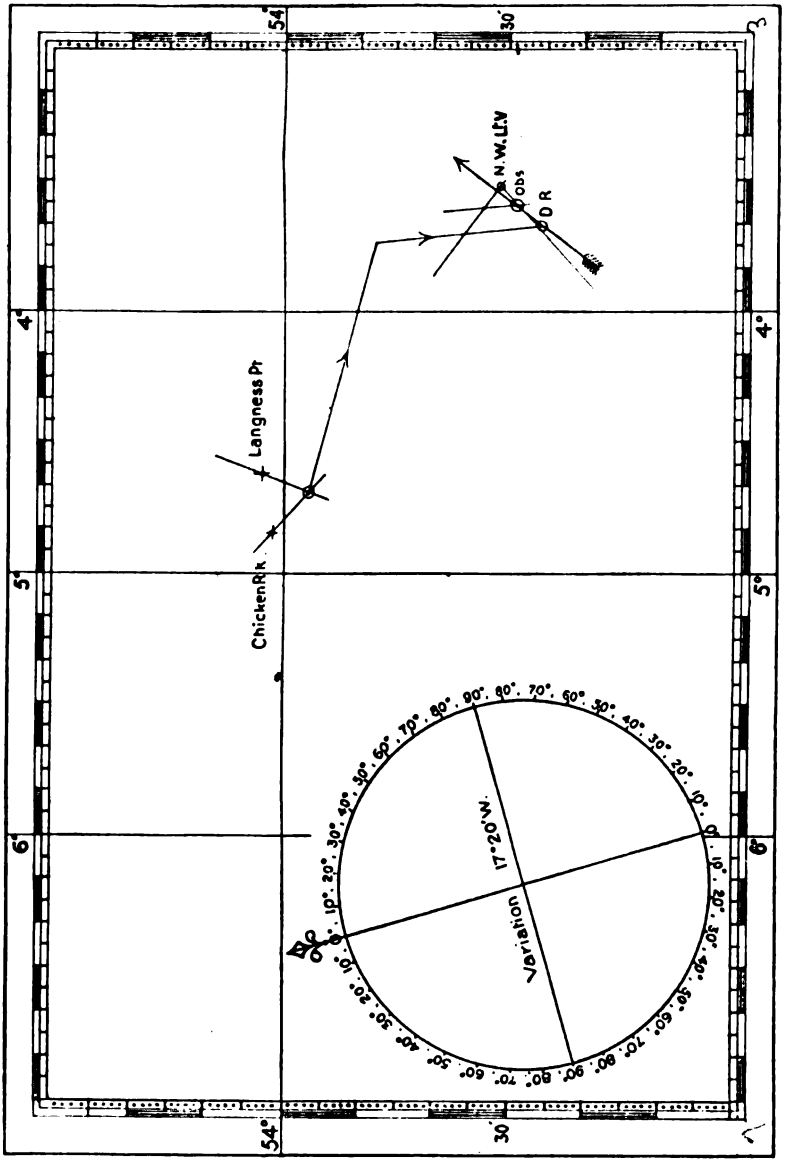


FIG. 176.

At 2ʰ 20ᵐ p.m. Chicken Rk. bore N. 32° W., and Langness N. 39° E. Shaped course S. 59° E., 15 knots, and at 4ʰ 40ᵐ p.m. altered course to S.

9° W., at 5ʰ 50ᵐ p.m. N.W. Lt. V. bore S. 37° E. ; and at 6ʰ 10ᵐ p.m. it bore N. 64° E.; place courses on chart, find true and D. R. positions of ship at last observation, set and drift of current. Courses and bearings magnetic.

1st. To find the size of the chart.

As there are four degrees of long., 3° to 7° W., the breadth of the chart is 4 × 1·9 inches = 7·6 inches.

Take out mer. pts.	53°	= 3763·76	M.D.L. = 50·14
,, ,,	53°½	= 3813·90	,, = 50·74
,, ,,	54	= 3864·64	,, = 51·35
,, ,,	54½	= 3915·99	
		Total	,, 152·23

60′ = 1·9 inch. Hence 152′·23 = 4·82 inches.

The height of the chart is 4·82 inches.

2nd. To draw the frame, meridians, and parallels.

Near the bottom of the paper draw a horizontal line 7·6 ins. long and divide into four equal parts, at each end set up perpendiculars as explained (Fig. 174). The lower line represents the parallel of 53° N. Find the distances on the chart corresponding to M.D.L.'s, viz : 50·14 = 1·59 inch, 50·74 = 1·61 inch, 51·35 = 1·62 inch. Now set these distances up on each perpendicular, and measure the distance between the perpendicular at the upper divisions to see if it corresponds with the base line 7·6 inches : if so join them and the frame is complete. Measure off the whole degrees on the top line, draw the parallel of 54°, and the meridian of 4°, 5°, and 6°.

Draw around the frame two parallel lines for graduation purposes, graduate the degrees, and enclose in a thicker outside line.

3rd. To draw in the compass.

Select a portion of the chart clear away from the problem, and draw a circle of suitable radius. A diameter through the centre gives the true north and south line, measure the amount of the variation 17° 20′ W. to the left of this diameter for magnetic north, draw the diameter and top it with a half fleur-de-lis. At right angles draw another diameter for the east and west line and graduate.

4. To complete the problem.

Insert the three given points from their lats. and longs., and from the compass set off the given bearings through Chicken Rk. and Langness, their intersection gives the point of departure. From this point set off the course S. 59° E. for 35 miles, the distance run between 2ʰ 20ᵐ p.m. and 4ʰ 40ᵐ p.m. ; here the course is altered to S. 9° W., which lay off for 22·5 miles, the distance run between 4ʰ 40ᵐ p.m. and 6ʰ 10ᵐ p.m. This gives the D.R. position which can be taken off. From the N.W. Lt. V. set off the two bearings observed, and fit in exactly the distance run by the ship between taking the observations (that is 5 miles run between 5ʰ 50ᵐ p.m. and 6ʰ 10ᵐ p.m.) on a line parallel to run made S. 9° W., the point of cut on the 2nd bearing is the *true* position of the ship which can now be taken off.

Set and drift of current is the course and distance from D. R. to true position.

ANSWER.—True pos. lat. 53° 29′ N.
long. 3° 35′ W.
D.R. pos. lat. 53° 26′ N.
long. 3° 40′ W.
Set N. 54° E., 4½ miles.

Example 2 (Fig. 177).

Construct a plan 7″ by 5″. Scale 5″ = 1 nautical mile : also scale of cables and 10ths. Insert compass showing 15° E., variation. △ to be 1″ from bottom and left-hand corner. From △, A bore N. 8° W., 4·2 cables ; B bore N. 51° E., 6·2 cables ; C bore N. 61° E., 10 cables ; all magnetic.

At buoy "*a*" A 103° B 43° C.
„ "*b*" A 47° B 108° C.
„ "*c*" A 45° B 74° C.

Plot buoys and give their bearings and distances from △.

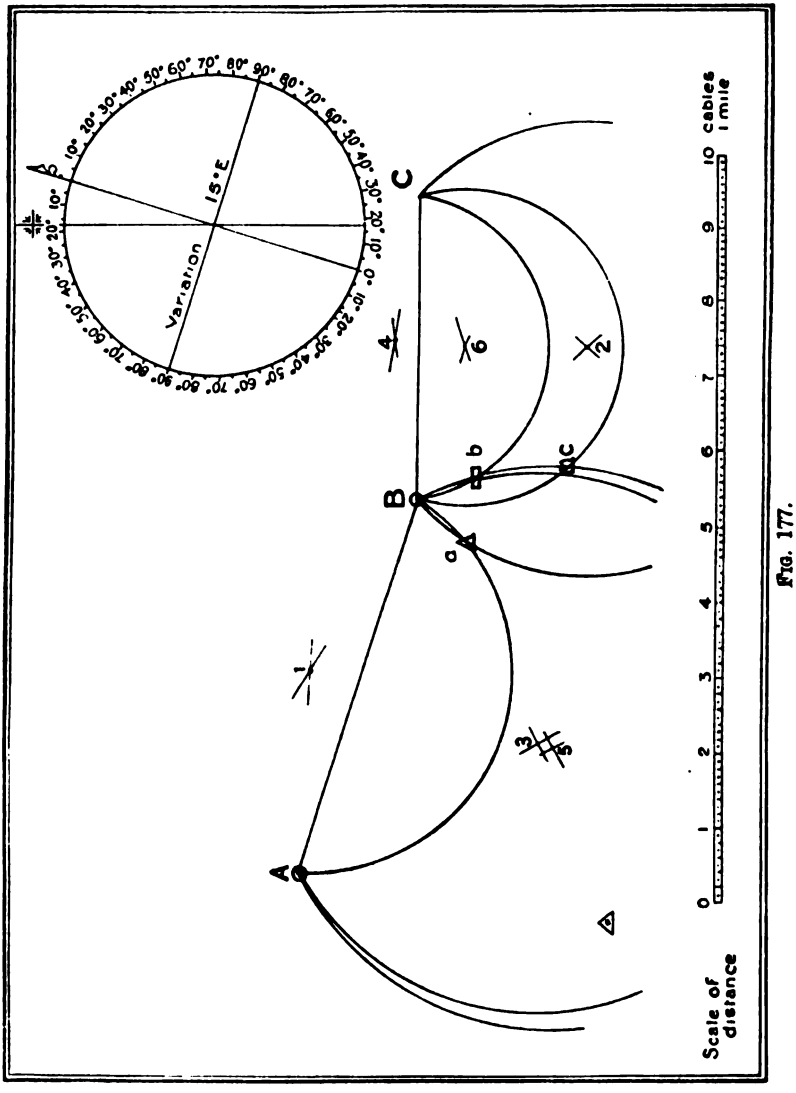


FIG. 177.

Draw a horizontal line 7″ long at bottom of paper, and at each end erect perpendiculars as in Art. 114, and make them 5″ long ; measure to see if their extreme ends are 7″ apart, if so join up and frame is complete. Inside the frame near the bottom draw scale of cables and 10ths as in

diagram, and put in a compass with magnetic north and south inclined 15° to sides of frame. Graduate compass.

Plot in △ 1″ from bottom and left-hand side; and from it set off bearings and distances from your scale and compass to plot positions of A, B, and C. Join AB and BC.

For buoy "*a*" it is noticed that A is on the left, as seen from the buoy, and the angle between A and B is 103°, the arc is less than a semicircle, and the complement of 103°, that is 13°, is laid off at each end of A and B, on the side away from where the fix is to be, the centre "1" in Fig. is the intersection of these lines, and the arc is drawn. Again the angle between B and C is 43°, so its complement 47° is laid off at each end of BC, on the same side as the fix is to be, and intersect in the centre "2"; where the former arc cuts this is buoy "*a*."

At buoy "*b*" A is again on the left, since the angle between A and B is 47°, set off lines making 43° with AB at each end to intersect in centre "3" on same side as fix, describe arc; then set off lines making 18° with BC at each end to intersect in centre "4," on opposite side to fix, since angle between B and C is 108°; describe arc: these arcs cut at position of buoy "*b*."

For buoy "*c*" it will be noticed that both angles being less than 90°, the centres "5" and "6," determined as before, are both on the same side as the buoy.

Now take off the bearings and measure distances.

From △ bearing of	"*a*"	N. 53° E.:	5·4	cables.
,, ,,	"*b*"	N. 58° E.:	6·2	,,
,, ,,	"*c*"	N. 68° E.:	6	,,

Exercises in Construction of Mercator's Charts.

1. Construct a Mercator's chart on a scale 2″·4 to a degree of longitude, extending from lat. 56° N. to 57° 30′ N., and from long. 25° W. to 28° W. A ship from lat. 57° 8′ N., long. 25° 10′ W., sailed as follows by compass: S.W. by S. 50 miles, dev. 6° 10′ W.; W.N.W. 53 miles, dev. 6° 50′ W.; N.E. 40 miles, dev. 10° E.; N. by W. 52 miles, dev. 1° 10′ E. The variation is two points W. Find latitude and longitude in.

2. Construct a Mercator's chart, scale 0·8″ to 1° long., extending from 63° 30′ S. to 65° 30′ S., and from 120° to 123° E., putting in parallels at each ½°. A ship leaves A, lat. 65° 7′ S., long. 120° 52′ E., and sails the following true courses and distances: N.E. 47 miles; N.N.W. 63 miles; S.W. by S. 42 miles; E. 27 miles. Find latitude and longitude in. The variation being 8° 30′ W., what is the magnetic bearing of A from ship?

3. Construct a Mercator's chart to extend from lat. 54° 30′ N. to 56° N., and long. 30° W. to 33° W., scale 1″·3 to 1° long. A point of land in lat. 54° 32′ N., long. 32° 56′ W., bears S.W. from the ship by compass, distant 7 miles; ship's head, E. ¾ N.; dev. 9° 55′ E.; variation, 18° W. Ship sailed as follows by compass: E. ¾ N. 90 miles, dev. 9° 55′ E.; N.N.W. ½ W. 40 miles, dev. 2° 20′ W.; W.S.W. 45 miles, dev. 7° 50′ W.; N.E. ¾ E. 75 miles, dev. 10° 35′ E. Find latitude and longitude in, true course, and distance made good.

4. Construct a Mercator's chart to extend from lat. 54° N. to 55° 30′ N., and long. 3° to 6° W., scale 1″·75 = 1° long. Insert following positions: Abbey Head, lat. 54° 47′ N., long. 3° 58′ W.; Little Ross, 54° 46′ N., 4° 5′ W.; Burial Island, 54° 29′ N., 5° 25′ W.; South Rock, 54° 22′ N., 5° 26′ W. At the ship, Abbey Head bore N. 46° E., and Little Ross, N. 37° W.; she then steamed S. 63° W. 12 knots for 4 hours, when her position was fixed by cross-bearings of Burial Island W. 8° N., and South Rock S. 23° W. Find true position of ship, set and drift of current.

5. Construct a Mercator's chart, scale 2½″ = 1° long., extending from

lat. 68° to 71° N., and 6° to 10° E. long. A ship leaves 69° 38′ N., 8° 20′ E., and sails as follows by compass: N.E. ¾ E., 30′, dev. 12° E.; N.W. by W. ½ W., 55′, dev. 8° W.; South, 80′, dev. 40° W.; variation, 20° W. Find her latitude and longitude in, course, and distance made good.

6. Construct a Mercator's chart on a scale of 1·3 inch to 1° long., extending from lat. 48° 45′ S. to 52° S., and long. 20° E. to 26° E. Position of lighthouse, lat. 51° S., long. 25° E. A peak lies 15 miles true North from Lt. Ho. Variation 20° E. A ship steamed from 50° S., 21° E., as follows:—

Comp. Co.	Dev.	Dist.	Soundings.	Bottom.
S. 32° E.	2° E.	120 miles	100 ftms.	coral.
N. 6° E.	4° E.	180 ,,	20 ,,	no bottom.
S. 8° E.	2° W.	120 ,,		

When the Lt. Ho. bore S. 45° E. true with an angle of 75° between it and the peak, give the lat. and long. of the position, amount of current experienced, and assuming the same rate of speed to have been maintained, plot the soundings in their correct positions.

7. Draw a plan on a scale 0·85 inch = 1 mile; and insert following: B, C, D are three objects, the true bearings and distances of which from A are: B, S. 81° E., 3½ miles; C, S. 85° E., 6½ miles; D, S.S.E. ¼ E., 3 miles. Fix the following soundings:—

A ϕ D 65° 20′ C (5 feet, mud).
A 41° B 65° C (3¼ fathoms, broken coral).
A 116° B 123° D (7 fathoms, sand and shells.

8. Construct a plan on a scale 0·8 inch to 1 mile lat., to extend from 52° 3′ N. to 52° 18′ N., 6° to 6° 25′ W. Insert a compass showing 21° W. variation and graduate.

A ship steering east magnetic, observed a lighthouse lat. 52° 12′ N., long. 6° 7′ W., in transit with a buoy when the horizontal angle between light vessel, lat. 52° 7′ N., long. 6° 24′ W., and Lt. Ho. was 81°. After steering 1·5 miles on above course, the horizontal angle was observed to be 50° between Lt. V. and Lt. Ho., and 22° between buoy and Lt. Ho. Find position of buoy with reference to its charted position, which was at first S. 31° W. of Lt. Ho. and 1·6 miles from it.

9. On a scale of 1·4 inch to 10 miles long, construct a Mercator's chart to extend from 55° 15′ N., to 55° 45′ N., and from 6° to 7° 30′ W., insert a magnetic compass variation 19° 15′ W., graduate margins.

At $7^h\ 30^m$ a.m. a point (P) bore 130° from a ship proceeding on a magnetic course S. 80° E., 10 knots, in a current setting N. 34° E. magnetic. At $9^h\ 45^m$ a.m. it bore 206°, and at $10^h\ 30^m$ a.m. it bore 225°. P. lat. 55° 14′ N., long. 6° 39½′ W.

Find the ship's position and distance from P when the last bearing was taken, average drift of the current, course and distance made good.

CHAPTER XXVII.

LAWS OF STORMS.

ART. 117.—The facts gathered from observations, and known as the *Laws of Storms*, apply more particularly to the violent Hurricanes, Cyclones, and Typhoons (which may be all classed under the name "Cyclone") met with between the parallels of 10° and 35° of latitude; but also in a modified degree to the strong winds, gales, and storms of higher latitudes.

These facts may be summarized as follows:—

1. The tropical Cyclones are never met with in the belt between 10° N. and 10° S. lat., but outside of 10° they occur in the following regions:—

N. Atlantic: The western part near the West Indies.

N. Indian Ocean: The Bay of Bengal and Arabian Sea.

N. Pacific: The China and Java Seas.

S. Indian Ocean: The Western part, passing near Mauritius, and hence called "Mauritius Hurricanes."

S. Pacific: Eastern part.

2. Cyclones (as the name implies) have a rotatory motion round a centre, and a progressive motion, varying from three or four miles an hour to twelve or more.

Near their origin they cover only a small area, but afterwards expand to as much as a thousand or more miles in diameter.

3. The barometer stands lowest at the centre (often 28 inches or less), and gradually increases in height towards the circumference, whilst the wind force is generally greater the nearer the centre.

4. The bearing of the centre from the ship may be inferred approximately by counting from the wind direction 8 or 10 points to the *Right* in the N. Hemisphere, and the same number of points to the *Left* in the S. Hemisphere.

5. The path of the centre, or *Line of Progression*, divides the Storm Area into two semicircles, and by observing the change of wind it may be determined whether the ship is on the right- or left-hand side of the Line of Progression—a matter of great importance in the handling of the ship. Thus, if the wind changes to the *Right*, the ship is on the Right-hand side, but if to the *left*, she is in the Left-hand semicircle.

If the wind is "steady," *i.e.* changes very slightly in direction, it indicates that the Ship is *on* the Line of Progression.

6. The Direction of Rotation in the N. Hemisphere is to the Left, and in the S. Hemisphere to the Right.

Fig. 178.

7. The approach of a Cyclone is often indicated by well-marked signs, such as—

(*a*) An oppressive stillness, with threatening sky and much lightning.

(*b*) A long "ground swell."

(*c*) A dense cloud bank on the horizon.

(*d*) A rapid motion of the upper clouds.

8. When within the Storm Area, the *approach* of the centre is shown by—

(*a*) A rapidly falling barometer.

(*b*) Increase of wind, with heavy squalls, much lightning and rain, and general murkiness.

(*c*) Heavy and confused sea.

(*d*) Continuous "veering" or "backing" of the wind, except when *on* the Line of Progression.

The *receding* of the centre would be indicated by—

(*a*) Rising barometer.

(*b*) Wind becoming more "steady" and decreasing in force.

(*c*) Weather clearing, but the sea still confused and dangerous.

9. Vessels, especially steamships, sometimes *overtake* Hurricanes, because their speed is greater than the rate of progression of the Storm Centre. In this case, if the wind changes to the *Left*, the ship is on the Right-hand side of the storm, but if to the *Right*, she is in the Left-hand semicircle; contrary to par. 5.

10. In entering the Central Area, which may be up to a hundred miles in diameter, the wind suddenly ceases, with glimpses of a clear sky and a general calm of perhaps hours' duration, interrupted by puffy squalls, whilst the sea is particularly confused and dangerous. After passing through the centre, the wind strikes the ship from the opposite point of the compass with renewed hurricane force.

11. The paths of Storm Centres follow the same general directions for the same regions of the Globe. The following are the usual tracks:—

(*a*) N. Atlantic: Having their origin near the Windward Islands, they first travel about W.N.W., then more northerly, and about 25° N. to 30° N. they recurve to the N.E. towards Mid-Atlantic.

(*b*) N. Pacific: The same general direction as in the N. Atlantic.

(*c*) N. Indian Ocean, including Bay of Bengal and Arabian Seas: A general N. and W. direction.

(*d*) China Seas: Between N.W. and W.S.W.
(*e*) Java Sea: Between N. and W.
(*f*) S. Indian Ocean: Starting from about 12° South, they travel in a S.W. direction towards Mauritius, then more southerly, and recurve to the S.E. in 25° to 30° S. lat.

12. **Seasons of Greatest Frequency of Cyclones.**—The following Table of recorded Cyclones from Scott's Meteorology shows that Cyclones are most frequent in the N. Indian Ocean at the Change of Monsoons, and in the other Cyclone Regions during the hottest months:—

Cyclone region.	Recorded cyclones for each month.												Totals of recorded observations.	Average no. per annum.
	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.		
West Indies (in 300 years)	5	7	11	6	5	10	42	96	80	69	17	7	355	1·18
S. Indian Ocean (39 years)	9	13	10	8	4	—	—	—	1	1	4	3	53	1·36
Bombay (25 years)	1	1	1	5	9	2	4	5	8	12	9	5	62	2·5
Bay of Bengal (139 years)	2	—	2	9	21	10	3	4	6	31	18	9	115	0·8
China Seas (85 years) ...	5	1	5	5	11	10	22	40	58	35	16	6	214	2·5

The Table also shows that the *most* cyclone-infested regions of the globe are the W. Indies, Arabian Sea, and China Seas, and that the worst cyclone months in the various regions are—

(*a*) Bay of Bengal and Arabian Sea: May, June, October, and November.
(*b*) N. Atlantic: July to November.
(*c*) China and Java Seas: July to October.
(*d*) S. Indian Ocean: January to March.

The same rule would give for—

(*e*) N. Pacific: July to November.
(*f*) S. Pacific: January to March.

13. The Rules of Action to be taken when a ship is caught in a Cyclone can be best understood by reference to the diagrams on p. 276.

These Rules may be summed up as follows:—

(1) If *on* the Line of Progression of an approaching Storm Centre, run or steam before the wind until clear of the track of the centre; then, in the N. Hemisphere, heave to on the Port Tack, but in the S. Hemisphere on the Starboard Tack. By so doing, the ship would always "come up" to the sea when hove to.

(2) In the Right-hand semicircle—

(*a*) Northern Hemisphere: Head reach or heave to on the Starboard Tack.
(*b*) Southern Hemisphere: Run with wind on Port Quarter, or heave to on Starboard Tack.

(3) In the Left-hand Semicircle—

(*a*) Northern Hemisphere: Run, having the wind on Starboard Quarter, or heave to on the Port Tack.

(*b*) Southern Hemisphere: Head reach or heave to on the Port Tack.

(4) When overtaking a Cyclone, it is obvious that the ship should be stopped or her course altered so that she may not approach the centre.

14. **General Remarks.**—The most dangerous position for a ship is the track of the Centre of a Cyclone, which should be avoided if possible. This can generally be done, if there is sea room, by attention to the foregoing Rules.

Observations tend to show that a Storm Area is more circular in front than in the rear; so that the Rule for finding the bearing of the centre is most reliable in the most dangerous situation.

The same Rules may be applied when Heavy Gales are encountered in higher latitudes.

It is common to give the name of "Dangerous Semicircle" to the Right-hand Semicircle in N. Lat., and the Left-hand Semicircle in S. Lat., because the direction of the air-currents is towards the Line of Progression in front of the Storm Centre.

If it is suspected that the ship is overtaking a storm, the engines should be stopped, or the vessel "hove to," and the barometer carefully watched. Should the barometer begin to rise, it is a sure sign of *overtaking;* but if it continues to fall (although less rapidly), it is a case of *meeting* a storm.

As these storms have their origin in the hottest parts of the earth, their cause has been thought due to the rushing in of winds from the equatorial and polar regions to fill the partial vacuum caused by a vertical sun in rarefying the air and vaporizing the water. The path of a revolving storm is at first to the westward, but in consequence of the resistance due to meeting air carried round in an opposite direction by the rotation of the earth on its axis from W. to E., the storm trends to a place where there is less resistance, that is, in both hemispheres towards the poles. This is called the "Recurving" of the storm, and takes place in about 30° latitude.

ART. 118.—

LAW OF STORMS.

Questions to be answered by candidates for Mates' and Masters' Certificates.

1. Suppose the wind in a cyclone to blow from , state what is the probable bearing of the centre in the hemisphere.

2. If the wind in the same cyclone changes to , on what side of the line of progression is the ship, and what action would you take?

3. Under what circumstances would the change in the direction of the wind be the reverse of the above?

4. What are the usual indications that a ship is on the line of progression of the centre of a cyclone?

5. What are the usual indications that a ship is approaching the centre of a cyclone or receding from it?

6. State what are the usual tracks taken by cyclones in , and at what season of the year do they most frequently occur in those regions?

LAW OF STORMS.

Answers.

1. In the Northern Hemisphere the centre bears about eight to ten points to the right, and in the Southern Hemisphere eight to ten points to the left, from the given direction of the wind.

2. If the wind changes to the right the ship is on the right-hand side of the line of progression, if to the left she is on the left-hand side. These rules apply to both hemispheres.

The action to take is as follows:—

Northern Hemisphere—

RIGHT-HAND SIDE.—If possible, sail out on the starboard tack to a safer distance from the centre, or heave to on the starboard tack.

LEFT-HAND SIDE.—If possible, run with the wind on starboard quarter until in a safer position, or heave to on the port tack.

Southern Hemisphere—

RIGHT-HAND SIDE.—If possible, run with the wind on the port quarter until in a safer position, or heave to on the starboard tack.

LEFT-HAND SIDE.—If possible, sail out on the port tack to a safer position, or heave to on the port tack.

3. The wind would change the reverse way if the ship were on the other side of the line of progression, also if she were overtaking the storm on the same side as before.

4. In advance of the centre the barometer falls more and more rapidly as the centre approaches, the wind continues to blow from the same quarter, but increases in violence, with frequent heavy squalls, a confused angry sea, a dense cloud bank, and general threatening appearances of weather.

In the central area the barometer is lowest, with a confused and dangerous sea, and calms varied by strong gusts.

In the rear of the centre the barometer rises, and the wind blows at first with great violence from the opposite quarter, afterwards moderating and the squalls becoming less frequent, the sea being heavy and confused, but the weather gradually clearing.

5. (*a*) When approaching the centre, not on the line of progression, there are the same barometer and weather indications as stated in the preceding answer, but the wind changes in direction as it increases in force, with heavy cross-seas.

(*b*) When receding from the centre the barometer rises, the wind becomes more steady, with less frequent squalls and less threatening appearance of weather, though the sea is still cross and dangerous.

6. **North Atlantic (Tropical).**—The cyclones, in this region, are supposed to take their rise near the Windward or Leeward Islands, and first travel about W.N.W. towards Florida, then more northerly, and on reaching the parallel of 30° often recurve to the N.E. They occur most frequently from July to November; worst months, August and September.

South Atlantic (Tropical).—They travel first about W.S.W., then

more southerly, and sometimes recurve to the S.E. They occur most frequently in the hottest months—December, January, and February.

South Indian Ocean (Tropical).—They are supposed to have their origin in the belt of calms, south of the Equator, often near Java, and travel first W.S.W., then more southerly towards the Mauritius, and sometimes recurve to the S.E. They occur from November to May, but most frequently in February and March.

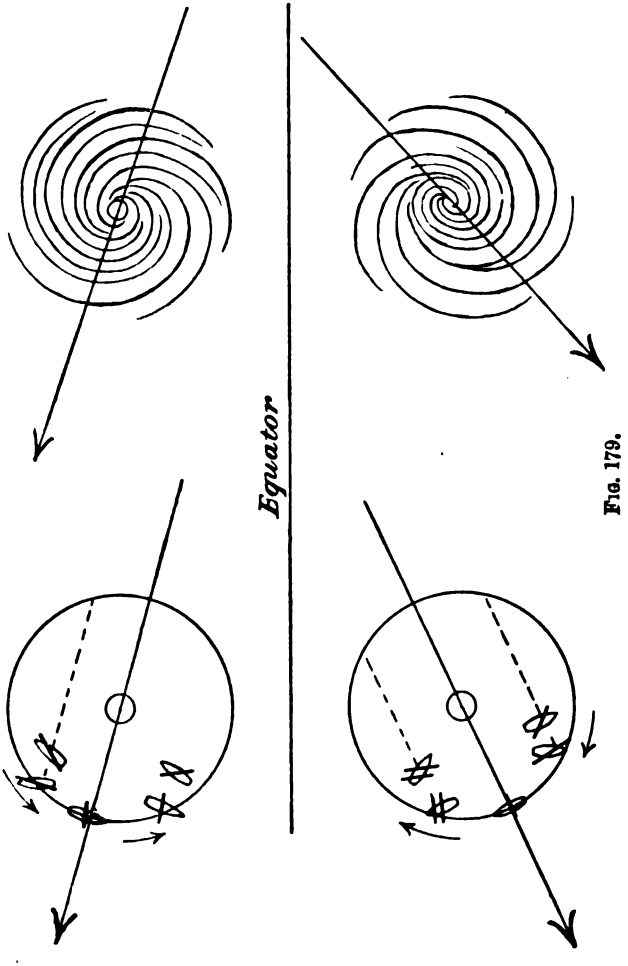


Fig. 179.

China Seas.—The storms take their origin near the Philippine Islands, and travel generally between W.N.W. and W.S.W. They occur most frequently from July to November; worst months, August and September.

Bay of Bengal.—The storms in this region begin in the neighbourhood of the Andaman and Nicobar Islands, and travel between W. and N.N.W. They occur most frequently in April to June, and September to November; worst months, May, October, and November.

Arabian Sea.—Between N. and W., and most frequent in May and October.

North Pacific (Tropical).—The cyclones of this region begin in about 10° N., and travel first W.N.W., then more northerly, and finally recurve to the N.E. July, August, and September are the worst months.

South Pacific (Tropical).—The cyclones of this region first travel about W.S.W., then more southerly, and finally to the S.E. They occur most frequently from November to April.

Outside of lat. 30°, storms in the North Atlantic and North Pacific travel to N.E., and in the South Indian Ocean and South Pacific to the S.E.

Mauritius.—Storms occur from December to April, worst months being February and March; they travel from S.S.W., recurving to S.S.E.

CHAPTER XXVIII.

MAGNETISM AND DEVIATION OF THE COMPASS.

ART. 119.—For a full and scientific discussion of the deviation of the compasses in iron ships, the student is referred to the "Admiralty Manual," or to the "Elementary Manual," edited and revised by Captain Creak, R.N. For general students Merrifield's "Magnetism and Deviation of the Compass" is recommended.

The purpose of the following remarks is to enable students to clearly understand the principles involved in the Board of Trade Syllabus of Questions dealing with the practical application of the laws of magnetism to the case of iron ships and their compasses.

Definition.—Magnetism is a property which may be acquired by masses of iron and steel, by virtue of which they attract or repel each other under certain circumstances.

Kinds of Magnets.—There are three kinds of magnets, viz. natural, artificial, and electro-magnets. The natural magnets are the Earth and pieces of magnetic iron ore or "lodestone." Artificial magnets are bars or "needles" of hard-tempered steel, which have been magnetized by rubbing contact with other magnets, or by the influence of an electric current. Electro-magnets are elongated or horseshoe-shaped masses of soft iron wound round with copper wire, through which an electric current is passing. The strongest magnets are made in this way; but the magnetic properties disappear when the current ceases.

Degrees of Magnetism.—Magnetism may exist in three different states or degrees in steel or iron, viz. PERMANENT, SUB-PERMANENT, and TRANSIENT INDUCED.

A bar of hard-tempered steel receives magnetism slowly, but when once magnetized it retains its magnetic strength for an indefinite length of time, and is said to have *permanent magnetism.*

Iron or steel of a less degree of hardness is more easily magnetized, but is liable to lose a considerable portion of its magnetism with lapse of time, and is therefore said to have *sub-permanent* magnetism.

Soft or malleable iron becomes instantly magnetized by induction from any magnetic source, but loses it when the

inducing cause is removed. This temporary magnetic condition is known as TRANSIENT INDUCED magnetism.

Saturation.—There is a limit to the magnetic strength which

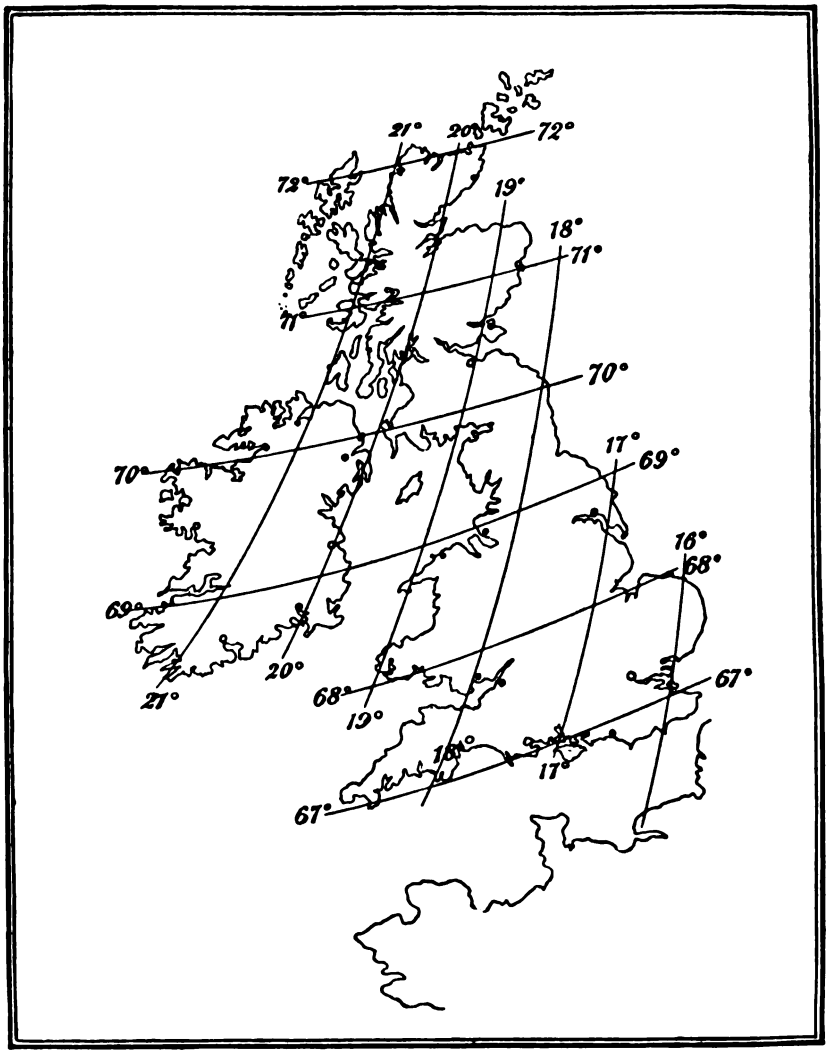


FIG. 180.—Variation chart of British Islands.

a piece of steel or iron can acquire, and when this limit is reached it is said to be *saturated*.

Magnetic Poles.—Every magnet has two POLES, or points

where the greatest energy is concentrated, distinguished as RED and BLUE poles, because it is common to paint artificial magnets in these colours. The red pole of a permanent magnet is the one which would point north when the magnet is suspended at its centre and free to turn round. This end is also called the N. or *marked* end. Therefore the end of a compass-needle which points N. is the red pole.

ART. 120. **First Law of Magnetism.**—It is a well-known law of magnetism that *like poles repel each other, and unlike poles attract each other.* This can be proved on a compass-needle, for if the red pole of a magnet be brought near to the N. or marked end, it will repel it, whilst the blue pole will attract it. Similarly, the S. end will be repelled by the blue pole, and attracted by the red. It follows from this experiment that the Earth's NORTH magnetic pole is BLUE, because the red pole of the needle is attracted towards it. The needle, if floating on water and free to move, would not, however, travel towards the N. pole, because its length is as nothing compared with the distance from the Earth's pole, its blue end being as strongly repelled as the red end is attracted. It is the same with the action of the south magnetic pole. The only result is, therefore, to give the needle a north and south *direction.*

If a needle hundreds of miles in length could be supposed to be afloat in the North Atlantic, with its red pole towards north, no doubt it would travel bodily towards the N. pole of the Earth, because its red pole would then be attracted more strongly than the blue would be repelled.

Second Law.—It is another law of magnetism that the effect of a magnetic force (such as the pole of a magnet or an induced pole in soft iron) on a small magnet varies nearly in the inverse ratio of the cube of the distance; that is, at double the distance it is one-eighth ($2 \times 2 \times 2 = 8$); at treble the distance it is only one twenty-seventh ($3 \times 3 \times 3 = 27$), etc. Also—

Third Law.—A magnet placed *end on* to a pivoted needle causes twice the deflection that it would do if *broadside on* at the same distance between the centres of the magnet and needle.

These facts have a very important bearing on the effect of the magnetic forces in a ship on her compasses, and show the advantage of placing the compasses as far as practicable from masses of iron having magnetic properties, especially when end on.

ART. 121. **Transient Induced Magnetism.**—When either pole of a magnet is brought near to one end of an iron rod, it develops two magnetic poles in the iron, the near end being of the opposite name to that of the pole applied. The process may be repeated through a series of rods end on, so that the N. point of a compass-needle would be deflected by the last one in the same manner as by the magnet itself.

The same fact may be illustrated by the familiar experiment of causing a number of soft iron bars or nails to hang on to the

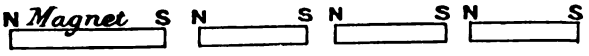


FIG. 181.

end of a strong magnet; but whether in contact or not, the effect is the same.

The above are examples of *magnetic induction* and the acquiring of *transient induced magnetism.*

Lines of Force.—The mysterious action of one magnet on another at a distance, and the magnetization of iron with or without contact, are explained by the existence of *lines of force*, which issue from the red pole and return to the blue pole. These lines of force pass without interruption through all intervening non-magnetic substances, and it is the passing of lines of force into iron which gives it magnetic properties. Their existence and direction can be shown by experiment (Fig. 182). If a sheet of white paper be laid on a magnet and iron filings sprinkled on the paper, the filings will arrange themselves in lines corresponding to the direction of the lines of force from one pole of the magnet to the other.

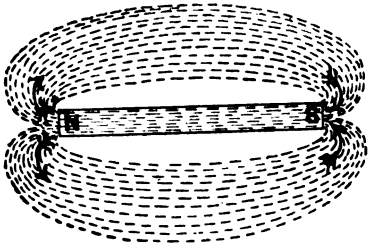


FIG. 182.

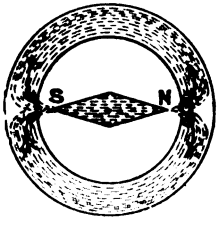


FIG. 183.

If a suspended magnetic needle were enclosed in a thick iron shell, the lines of force would follow round through the iron by preference to reach the blue pole, and the consequence would be that the needle would lose its directive power. This is the reason why compass-bowls are made of copper, and not of iron. Another reason is that poles would be induced in an iron bowl, which would oppose the Earth's directive action on the needle. Therefore a ship's compass which is closely surrounded by iron would have a weakened directive power.

The Earth's Magnetism.—The Earth itself acts as a magnet, having a blue pole towards the north and a red pole towards the south. Its action on a compass needle is merely to give a *directive* tendency, as before explained, but it is also a powerful *inducing* source. No iron can escape from its influence. Hence fireirons, knife-blades, needles, etc., are nearly always magnets with two fixed poles, as may be tested on a compass-needle,

whilst bars or rods of soft iron have also induced poles from the same cause, which are not, however, fixed. A fairly strong magnet can be made by holding a bar of hardened steel in a N.-and-S. direction and tapping it with a hammer; the vibration quickens the process of magnetization by the Earth's induction.

A Ship's Induced Magnetism.—Even a ship becomes a huge magnet from the Earth's induction in the process of building, whilst the separate parts, such as beams, stanchions, davits, rudder and stern-posts, etc., become so many temporary magnets, liable to act separately and collectively in disturbing the compasses.

ART. 122. **Variation.**—The magnetic poles of the Earth do not coincide with the true geographical poles, the N. or blue pole being in about lat. 70° N., long. 90° W., and the S. or red pole was found by Lieut. Shackleton to be in lat. 72½° S., long. 154° E. (from which it is seen they are not exactly opposite). This is the cause of the variation of the compass, which is the difference in direction between true N. and magnetic N. The amount of variation is stated on ordinary charts, and for the entire globe on special variation charts. The variation is not constant at any place, but changes at a known rate (in the United Kingdom now about 8′ or 9′ annually).

Previous to the year 1657 the variation was E. in this country. In that year there was no variation at London, the compass pointing true N. Then it became westerly, and attained its maximum W. variation in 1816. Since then it has constantly decreased, and will again become 0 in 1977; then, becoming E. again, it will reach its maximum in the year 2290. The explanation of this progressive change of variation is that the N. magnetic pole describes a complete circle round the true N. pole in a period of about 640 years. Twice in the course of this period the compass would point true N. at Greenwich, *i.e.* when the magnetic pole is on the meridian of Greenwich, and on the opposite meridian (Fig. 184). The annual change would evidently be greatest when the magnetic pole is at or near to the points 0 and 0_1 in the figure, and slowest when near the points W and E, when the direction of the needle is a tangent to the small circle. At present the magnetic poles are coming towards the meridian of Greenwich. The same remarks apply to other places. A line in the direction magnetic N. and S. at any place is called the magnetic meridian; hence variation may be defined as the

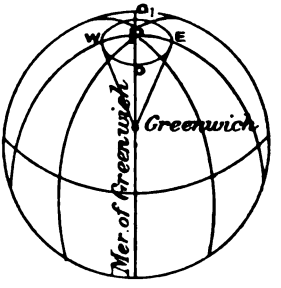


FIG. 184.

angle between the true and magnetic meridians. An irregular line round the Earth, about midway between the poles, is the magnetic equator, which cuts the true geographical equator in two nearly opposite points about longs. 170° E. and 12° W.

Magnetic Dip.—A magnetized needle, poised at its centre so as to be free to move in a vertical plane, and provided with a scale of degrees, is called a dipping-needle (Fig. 185).

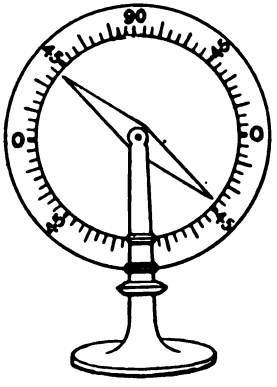


FIG. 185.

At the magnetic equator, and placed in the plane of the magnetic meridian (*i.e.* pointing N. and S. magnetic), such a needle would lie horizontal; but if carried northwards the red pole would dip more and more, until the magnetic pole is reached, where it would stand upright, red pole downwards. South of the magnetic equator the blue pole would dip, and the needle would again assume an upright position at the S. magnetic pole, blue pole downwards. At Greenwich the dip at the present time is 68°, but in the West Atlantic and Gulf of St. Lawrence it is much greater. Like the variation, the dip at Greenwich is decreasing annually, and will be least in the year 1977, when the variation will be 0.

These are the results of observation. The explanation is that in N. magnetic latitude the red pole of the needle points *direct* to the N. magnetic pole, and in S. magnetic latitude the blue pole to the S. magnetic pole of the Earth, notwithstanding the mass of the Earth intervening, whilst the horizontal plane at any place is a tangent to the Earth's surface (see Fig. 186).

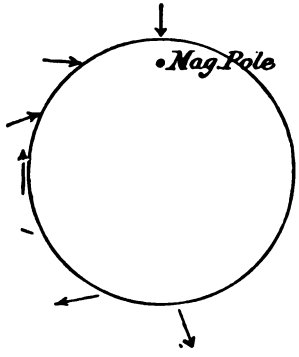


FIG. 186.

The dip at any place decreases when the magnetic pole, in making its circuit round the true pole, is approaching the meridian of the place, and increases when it is receding. Thus in the Fig. 184, the dip at Greenwich decreases when the magnetic pole is moving from 0_1 to 0, and increases from 0 to 0_1, being greatest when the pole is at 0_1, and least when at 0. At present the dip at Greenwich is decreasing.

The position assumed by the dipping-needle may be con-

sidered the *natural* one for any magnetized needle which is free to move, but the horizontal position of a compass needle is a *forced* one. The reason why the compass card lies horizontal under the circumstances is that the point of support is above the centre of gravity of the card and needles, and the dragging down force is not sufficient to overcome their weight. It is evident from the above that an ordinary compass would be of no use for steering purposes near the magnetic poles, and is at its best on the magnetic equator.

ART. 123. **Parallelogram of Forces.**—Before considering the magnetism of iron ships and its effects on their compasses, it is necessary to understand the principle of the parallelogram of forces.

It is a law in mechanics that if two adjacent sides of a parallelogram represent two given forces in direction and amount, then the diagonal from the point of meeting of these two sides will represent the resultant (or equivalent force) both

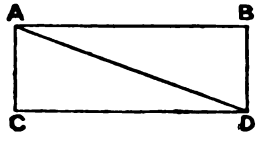


FIG. 187.

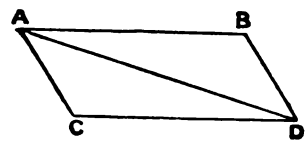


FIG. 188.

in direction and amount. Thus, if AB and AC in the parallelogram ABCD represent two given forces, then the diagonal AD will represent the joint effect or resultant. The sides of the triangle ABD represent equally well the forces and resultant.

A good illustration of this principle may be taken from the case of sailing in a current. For example, let the direction from A to B (Fig. 188) be the course steered, and the length of AB the rate of sailing per hour; also let AC represent the "set" of the current and "drift" in one hour.

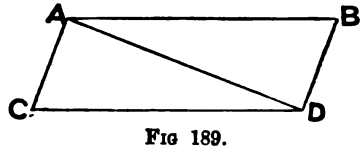


FIG 189.

Draw BD parallel to AC, and CD parallel to AB, and join AD; then AD will be the course and distance made good, and represents the resultant or joint effect of wind or steam and the current acting for the same length of time.

It is the same with any other kind of force, and it will be seen how the principle applies in dealing with the *magnetic* forces in a ship.

AB and AC are called components, and these can be found from the resultant, as well as the resultant from the components. The finding of the components is called "resolution," and of the resultant "composition." Thus, a force represented by AD is

"resolved" into components represented by AB and AC. This expression will be frequently used hereafter.

ART. 124. **Magnetic Condition of Iron Ships.**—(*a*) *Sub-permanent Magnetism.*—It has been stated that the hull of an iron ship becomes a magnet from the Earth's induction. The effect of lying for a considerable time in one direction, and the hammering to which the iron is submitted, is to fix the acquired magnetism so that it becomes sub-permanent, *i.e.* less than permanent. After launching there is some reduction of the strength of this sub-permanent magnetism, but it can often be traced throughout the whole existence of the ship, showing that a portion of it is almost of a permanent character.

If the hull be supposed to be divided into halves as she lies on the stocks by a plane at right angles to the line of dip (*q.v.*), the half towards the N. will have red polarity, and the other half blue.

Thus, if built in the United Kingdom, with head towards N., the line of dip being DP (Fig. 190) and EQ at right angles,

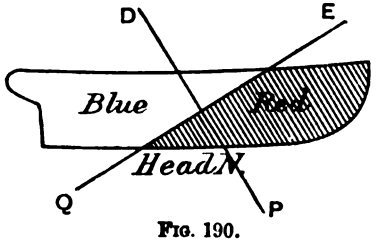


FIG. 190.

the ship would be an irregularly shaped magnet, red towards the bow, and blue towards the stern.

If built heading S., the red polarity would be at the stern, and blue at the bow.

If built heading E., the port side would be red, and the

FIG. 191.

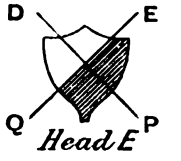


FIG. 192.

starboard side blue (Fig. 192); but if built head W., the starboard side would be red, and the port side blue (Fig. 191).

The magnetic poles or points of greatest concentration of magnetic strength might be supposed to be about the centre of the two halves. So that in all these cases the line joining the poles would be inclined, one pole being nearer the keel than the other (unless built on the magnetic equator).

But it is only the horizontal component of the total force which causes deviation when the ship is upright. (The effect

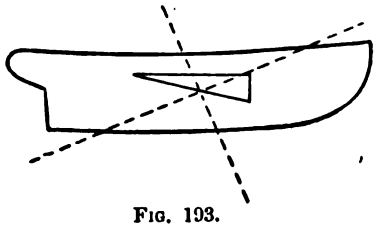


Fig. 193.

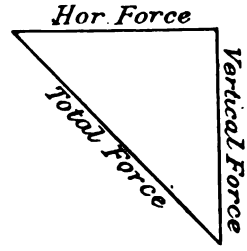


Fig. 194.

of the vertical component will be considered later.) The parallelogram (or triangle) of forces will show the relation of the horizontal to the total force (Fig. 194).

It is clear that the greater the dip the less will be the horizontal component of the ship's magnetic force, and *vice versâ*. Hence, other things being equal, a ship built in a high latitude

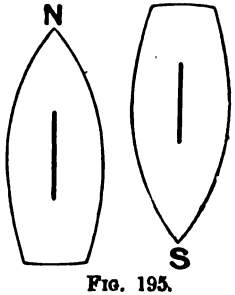


Fig. 195.

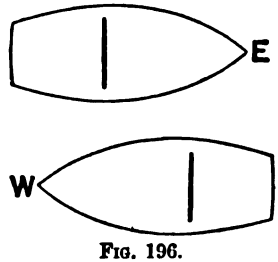


Fig. 196.

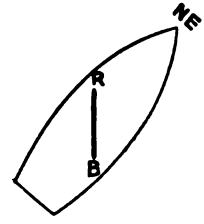


Fig. 197.

should cause less deviation from sub-permanent magnetism than one built in a low latitude, the ship being upright.

If a ship were built heading magnetic N. or S., the line representing the horizontal force of sub-permanent magnetism would be parallel to the keel (Fig. 195); if E. or W., it would be at right angles to the keel (Fig. 196); but if in any other direction it would lie obliquely (Fig. 197). In the latter case, the parallelogram of forces would again resolve the total horizontal force into two components—one represented by a line parallel to the keel, and the other at right angles to it.

Induction in Horizontal and Vertical Iron.—The effect of the Earth's induction on the horizontal and vertical iron of a ship may now be considered.

A bar of soft iron has the greatest magnetic strength from the Earth's induction when it lies in the line of dip, but is neutral when at right angles to it.

Thus, in N. magnetic latitude the end B of the horizontal bar AB and the lower end of the vertical bar CD are red poles, because towards the N. dipping-pole (Fig. 198).

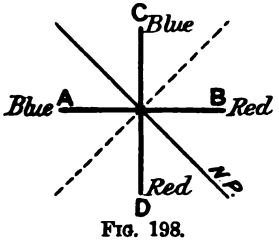


FIG. 198.

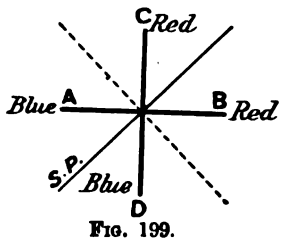


FIG. 199.

In S. magnetic latitude, the end A of the horizontal bar and the lower end of the vertical bar are blue, because towards the S. magnetic pole (Fig. 199).

A bar lying at right angles to the line of dip would be neutral.

This explains why the stern post, rudder post, and other vertical iron *below* the compass attract the N. point of the compass when north of the magnetic equator and repel it when south, whilst they have no effect when the ship is *on* the magnetic equator. Also, why the ends of horizonal beams attract and repel alternately as the ship's head turns from one quadrant to another, but are neutral when her head is N. or S. magnetic.

Another inference is that the magnetic strength of vertical iron in a ship *decreases* on going towards the magnetic equator, and *increases* towards the poles, whilst that of a horizontal beam lying N. and S. *increases* towards the equator, and *decreases* towards the poles.

ART. 125. **Coefficients.**—The term COEFFICIENTS is used to represent the components of magnetic forces in a ship. It may be assumed that these forces have a mechanical value, although a very small one, and could be measured by the same standard as other mechanical forces; but in reference to a ship's compass they are estimated by the amount of deviation they produce when acting most effectively. This deviation itself is often called the coefficient.

The letters A, B, C, D, E name the coefficients, of which B, C, and D are the most important.

Coefficient B represents a magnetic force whose poles lie horizontally in the fore-and-aft midship line, and is a combination of the fore-and-aft component of sub-permanent magnetism and the induced magnetism of vertical iron before or abaft the compass. It is named +B if the combined effect is to attract the N. point of the compass towards the bow, and −B if towards the stern.

Coefficient C represents a force whose poles lie horizontally at right angles to the fore-and-aft midship line, and is the athwartship component of the sub-permanent magnetism only, for vertical iron is supposed to be evenly balanced towards the sides. It is named +C when the N. point of the compass is attracted to the starboard side, and −C when to the port side.

The horizontal force of sub-permanent magnetism is therefore represented by coefficients B and C, whose signs and relation to each other depend on the direction of ship's head during her construction, as illustrated in the following figures:—

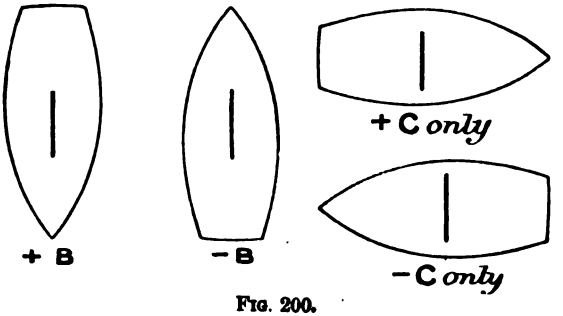


FIG. 200.

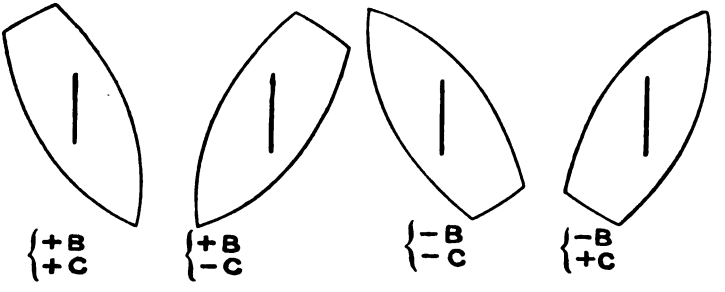


FIG. 201.

Semicircular Deviation.—The deviation due to coefficient B or coefficient C is called *semicircular*, because it is east on courses over one half of the compass, and west in the other half.

Coefficient +B (blue pole forward) causes E. deviation when the ship's head is to the eastward, and W. deviation when to the westward; −B (blue pole aft) gives deviations of opposite name to the above. The maximum amount in both cases is found when the ship's head is E. or W. *by compass.*

Coefficient +C gives E. deviation when the ship's head is to the northward, and W. deviation when to the southward; −C gives the opposite, the maximum amount being found when heading N. or S. *by compass.* B causes *no* deviation when ship's

head is N. or S. by compass, and C causes *no* deviation when her head is E. or W., because in these cases the forces act in a line with the compass needle.

The kind of deviation due to coefficients B and C is seen at a glance in the following figures, where + stands for E. deviation, and − for W. deviation:—

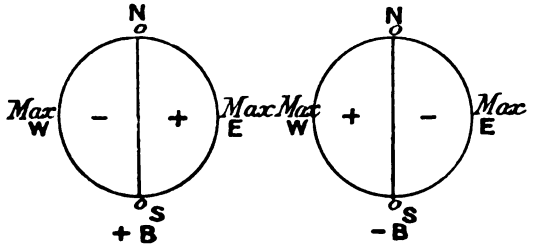


Fig. 202.

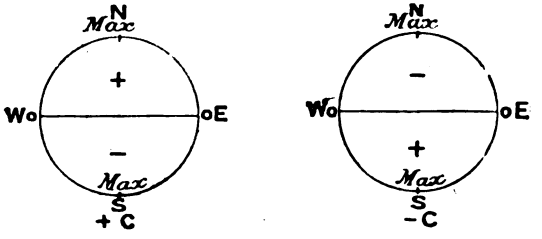


Fig. 203.

Transient Induced Magnetism in Ships.—The *transient induced* magnetism of horizontal iron is represented by coefficients D and E. Beams from side to side, and elongated iron lying fore-and-aft *end on* to the compass, give coefficient +D; athwartship beams *end on* and fore-and-aft iron extending beyond the compass both ways, gives −D (Fig. 204).

Horizontal stringers from starboard quarter towards port bow give +E, and when extending from port quarter towards starboard bow they give −E. When end on to the compass, they give a coefficient of the opposite sign (Fig. 205).

Quadrantal Deviation: Coefficients D and E.—The deviations caused by coefficients D and E are called *quadrantal*, because they change their signs with change of course from quadrant to quadrant.

Thus, +D gives E. deviation between N. and E. and between S. and W., but W. deviation between E. and S. and between W. and N. Also, + E gives E. deviation between N.W. and N.E. and between S.E. and S.W., but W. deviation between N.E. and S.E.

and between S.W. and N.W. −D and −E give deviations of opposite sign.

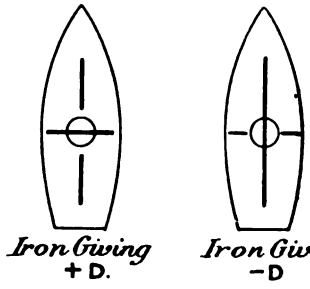


FIG. 204.

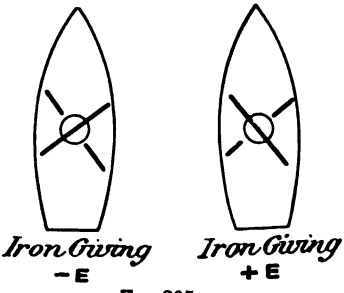


FIG. 205.

It is an instructive exercise to prove the foregoing results by tracing the influence of one of these masses of horizontal iron on the compass as the ship's head is swung round.

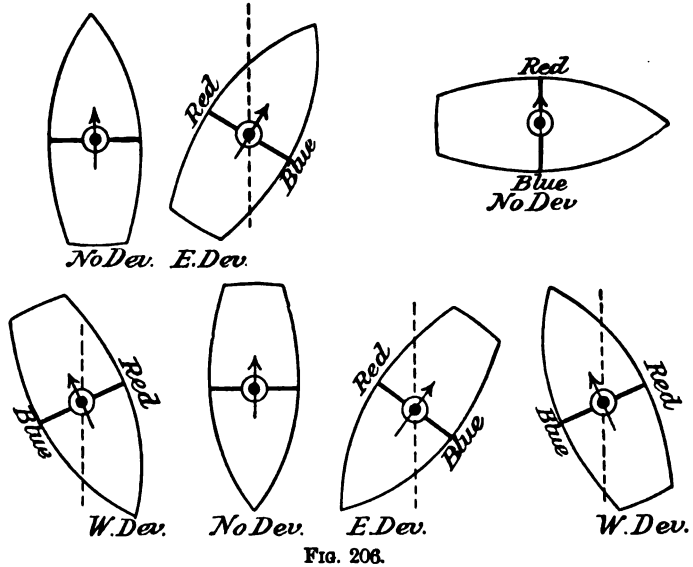


FIG. 206.

These figures show the magnetic polarity of a beam from side to side (giving coefficient + D) and the deviation produced in each quadrant. The zero points (*i.e.* points on which there is no deviation) are N., E., S., and W., and the maximum deviation is on courses halfway between these points, namely, at N.E., S.E., S.W., and N.W.

Similarly, it can be shown that the zero points for coefficient

E are N.E., S.E., S.W., and N.W., and the courses of maximum deviation N., E., S., and W.

The student should reason out for himself the effect of the various masses of horizontal iron as the ship is swung round the compass.

It is conceivable that the iron producing +D or +E might be exactly counterbalanced by iron causing −D or −E, but if not, then the balance on the side of the greater is the coefficient.

The following figures show the quadrantal deviations due to coefficients D and E, and the zero and maximum points. The sign + stands for E. deviation, and − for W. deviation:—

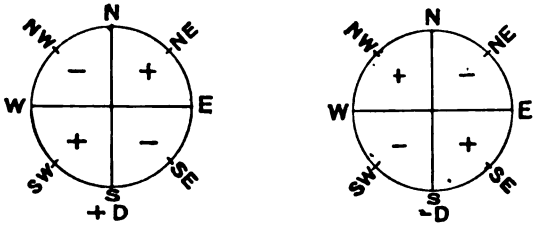


FIG. 207.

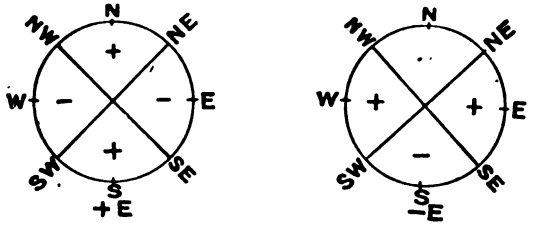


FIG. 208.

Coefficient A and its Causes.—Coefficient A is of the nature of an index error, being of the same sign or name on all courses, and is often not due to magnetism at all. It may arise from any of the following causes:—

(*a*) The misplacing of the lubber line.

(*b*) The magnetic axis of the needles not coinciding in direction with the line through the N. and S. points of the card.

(*c*) An error in the correct magnetic bearing used in finding the deviations.

(*d*) Unsymmetrical arrangement of iron with respect to the compass.

(*e*) Retained magnetism during the process of "swinging" the ship.

The first three causes, if detected, could be removed, but the

last two can scarcely be remedied. In any case the deviation from these causes is small, and is liable to be confounded with the deviation due to coefficient E, which is also very small.

The existence of a coefficient A is made evident in a table of deviations when the sum of the E. deviations is not equal to the sum of the W. deviations. When the former is greater, A is + ; when the latter, it is −.

ART. 126. **Analysis and Compensation.**—The importance of making an analysis of the causes of deviation and of knowing how much is due to each, is that suitable methods of compensation may be employed, namely, to counterbalance the permanent magnetism by means of magnets, and the transient induced magnetism by iron correctors which themselves have the same kind of magnetism from the same source (the Earth), but act on the compass in the opposite sense.

Separation of Coefficients.—It is a fortunate circumstance that the effects of the various coefficients of forces can be sorted out from the observed deviations, as explained in the answer to Question 62, and a table can be made of the deviations due to each coefficient for every point of the compass (see answer to Question 62), which, being summed up algebraically, give the total deviation for each course.

The practical method of compensation is explained in answers to Question 40.

Compensation of Coefficient B.—The most troublesome coefficient to deal with is B, because it is a combination of sub-permanent and transient induced magnetism, as before explained. When on the magnetic equator, it would be easy to compensate the former with a magnet, and then, on proceeding north or south, the amount of change in the deviation when the ship's head is E. or W. could be compensated by a "Flinders bar."

But since B and C of sub-permanent magnetism stand to each other in the relationship of difference of Latitude and Departure, the ship's head whilst building being the course, it is possible to find this B if that direction is known. For this purpose the ship's head should be steadied on N. or S. by compass, and the deviation from coefficient C observed. Then, with the direction of ship's head whilst being built as a course, and C in departure column (in the Traverse Table), the required B will be found in the latitude column. This is the amount to be compensated by the fore-and-aft magnet, and the balance of observed B with the Flinders bar, either before or abaft the compass as required.

Effect of Magnetic Forces on Different Compasses.—It may be remarked that although the same coefficients of magnetic forces in a ship disturb both steering and standard compasses, yet the *measure* of the coefficients as shown by the deviations will not be the same for both, because the forces act at different distances.

Consequently, observations and corrections must be made for each compass separately.

It is explained in the answer to Question 63 how this can be done for several compasses simultaneously.

Compensation good for all Latitudes.—If the compensations are properly made when a ship is ready for sea, they should stand good for all latitudes, provided the sub-permanent magnetism does not change, for on this supposition the strength of the magnets and the coefficients of sub-permanent magnetism would always be in the same relation to each other, and no deviation would occur, although the directive power of the needle varies with change of latitude, whilst the horizontal force of transient induced magnetism and of the correctors would also remain in the same relation to each other.

Changes in Deviation.—But however carefully a compass may be compensated, changes may be expected with the lapse of time, and as these changes will be chiefly in the deviation due to sub-permanent magnetism, a readjustment of the magnets will be necessary from time to time. The effect of the vibration caused by the propeller, the buffeting of heavy seas, collisions, or stranding is to considerably modify the magnetic condition of the ship when she left the builder's hands.

Retained Magnetism.—Another troublesome cause of change of deviation is what is called *retained magnetism;* that is, if a ship remains for a considerable time in one position in dock or on the same course at sea for several days, the induced magnetism from the earth is retained for a certain length of time, and causes a deviation when the course is altered. In sailor's language, the ship tends to follow her last course. For example, a steamer in crossing the Atlantic, bound for Liverpool, lays on an easterly course for several days together, and when on rounding Tuskar her course is changed to the northward, the retained red polarity on the port side (and blue on the starboard side) causes an E. deviation, which tends to set her to the eastward of her proper course. To this cause is owing, no doubt, the fact that steamers from the westward, coming up Channel in thick weather, are apt to be found too near the Welsh coast, and steamers from the south, bound up the English Channel, too near the English coast.

ART. 127. **Uncompensated Compasses.**—An uncompensated compass having large deviations on some courses and very small deviations on others is very unsatisfactory for steering purposes, for two reasons:—

(1) The deviation will be subject to greater changes, because when a ship approaches the magnetic equator the directive force of the needle increases, and therefore the coefficients of sub-permanent magnetism, which are assumed to remain nearly

constant, will cause less deviation. On receding from the magnetic equator, the deviation would increase for the same reasons.

(2) The directive tendency of the needle would be much increased when a strong blue magnetism in the ship is towards the north from the compass, and correspondingly decreased when red is towards the north. The consequence would be that in some directions a considerable change of course would show only a small change in the compass course, and in other directions a small change of course would appear a large one by compass.

Horizontal Iron and Uncompensated Compass.—The deviation caused by horizontal iron would not be subject to these changes, because the horizontal component of transient induced force would vary in the same ratio as the directive force of the needle.

Diminution of Directive Force.—In an ordinary iron ship, the directive force is rarely more than six-sevenths of what it would be in a wooden ship or on shore, and in armoured war-ships not more than three-fourths, according to Captain Creak, R.N., who has had very large experience of such matters. The chief cause of this is the horizontal iron, for in most cases it has a red polarity to the north of the compass. Sub-permanent magnetism weakens the directive force on *some* courses, but strengthens it on others, according as a red pole is north or south of the compass, and so the mean effect all round is *nil*. These remarks apply to uncompensated compasses.

The consequence of a diminution of directive force is *greater deviations*, for the forces which cause deviation are acting on a weakened compass.

The object aimed at by compensating a compass is to secure a uniform directive force on all courses, so that if any deviations remain they will be small in amount and regular in their changes.

ART. 128. **Heeling Error.**—So far it has been assumed that the ship is upright, but when she has a list from the pressure of the wind, or shifting of cargo, or unequal trimming of bunker coals, further deviations—sometimes very large, and known as heeling error—occur on northerly and southerly courses, but disappear on east and west courses.

The Cause of Heeling Error.—(*a*) It has been already explained that there is a *vertical* component of sub-permanent magnetism as well as a horizontal. The upper pole of this force, nearest to the steering compass aft, is blue if the ship is built head north (Fig. 209), and red if built head south (Fig. 210), and so long as the ship is upright, it only tends to *attract* the N. point of the needle *downwards* or *repel* it *upwards*, according as the upper pole is *blue* or *red*, without causing any deviation. The case is different when the ship heels, for then this upper pole is no longer in the vertical plane passing through the centre of the compass, but

towards the higher side, deviating the N. point of the compass towards the higher or lower side. This effect is evidently

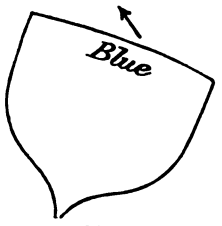


FIG. 209.

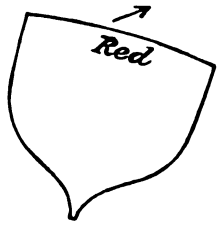


FIG. 210.

greatest on N. and S. courses, and *nil* on E. and W. courses, because in the former case the force acts at right angles to the needle, and in the latter case in the plane of the magnetic meridian.

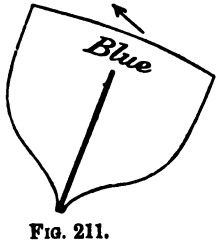


FIG. 211.

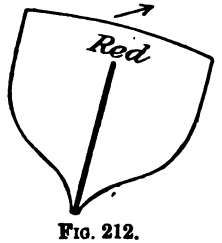


FIG. 212.

(*b*) The upper pole of vertical iron below the compass affects the compass in exactly the same way as the sub-permanent pole when the ship heels, only it must be remembered that vertical iron has the blue pole uppermost only in N. magnetic latitude, and red pole uppermost in S. magnetic latitude, being neutral on the magnetic equator. Therefore the heeling error from this cause decreases on sailing towards the magnetic equator, and *vice versâ*.

(*c*) The athwartship iron, which gives coefficient D when the ship is upright, acquires a vertical component when she heels, which is of the same nature as the magnetism of vertical iron, and is subject to the same changes—the higher end being *blue* in N. magnetic latitude, and red in S. magnetic latitude.

It is evident that in some cases these three causes of heeling error would act together, and the heeling error would be large. In other cases the first would be opposed to the other two, and they would tend to neutralize each other, and give a small heeling error.

Their combined forces might be represented by a single force

acting vertically under the centre of the compass, which would therefore be the resultant of—

(1) The vertical component of sub-permanent magnetism.
(2) The induced magnetism of vertical soft iron.
(3) The vertical component of the induced magnetism of transverse beams.

This resultant force is called + if the balance on the side of the greater is *blue uppermost*, and − if *red*. The total effect may be summed up by saying that a + vertical force causes a heeling error towards the *higher* side, and a − force a heeling error towards the *lower* side (Figs. 213 and 214).

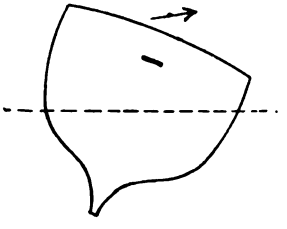


FIG. 213.

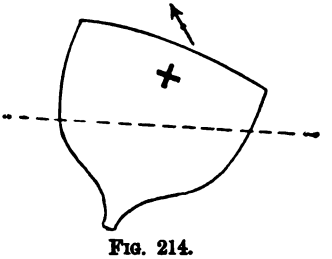


FIG. 214.

Experience shows that in the majority of sailing ships the force below the compass is + in N. latitude and − in S. latitude, with the result that N. of the magnetic equator the ship is drawn to windward on northerly courses, and to leeward on southerly courses. South of the magnetic equator the rule would generally be reversed. The same rules apply to steam ships, if "higher side" is substituted for "windward," etc.

Navigators make a practical application of these facts by keeping "away" on northerly courses, and "closer" on southerly courses, when north of the magnetic equator, and the reverse when south of it.

Since the vertical force draws the N. point of the compass to one side or the other when the ship heels, it is called a coefficient C, although it would be more logical to distinguish it in some way from the coefficient C already discussed, due to horizontal force, say by using C_1 to represent it.[1]

Like coefficient C of horizontal sub-permanent force, the new coefficient C_1 causes no deviation (heeling error) when the ship's head is E. or W. (magnetic), and the greatest deviation on N. and S. courses, because in the former case it acts in the plane of the magnetic meridian, and in the latter at right angles to the needle.

ART. 129. **Compensation.**—The heeling error is usually

[1] For a similar reason, the part of coefficient B due to vertical iron before or abaft the compass might be distinguished as B_1.

compensated by means of one or more magnets placed vertically under the compass, and capable of being moved up or down. There are two modes of procedure:—

(1) The ship's head being at N. or S. magnetic, she is caused to heel, say about 10°, and the error observed. Then the magnet is placed in position, and moved up or down, until the compass points correct magnetic N.

(2) The card having been removed, and the ship upright, with her head E. or W. magnetic, a dipping needle, weighted so as to lie horizontal when in the plane of the magnetic meridian on shore, is placed on the pillar in the centre of the bowl, and lying N. and S. Should the *red* end dip, showing a *blue* force below the compass, the *red* pole of the vertical magnet should be uppermost, and *vice versâ*, being moved up or down until the needle is again horizontal.

In either of these methods, what has been done is to counteract the force below the compass by a force of opposite name; by observation of the heeling error in the first case, and in the second case by observing the vertical action of the force below the compass on the dipping needle.

In a compensated compass the vertical force of the athwartship iron (producing coefficient D) is counterbalanced for moderate angles of heeling by a vertical force developed in the correctors; which leaves only the effects of vertical iron and the sub-permanent magnetism to be dealt with.

The Flinders bar increases the heeling error, for its upper pole is of the same name as that of the vertical iron it is intended to counteract when the ship is upright. Therefore the compensating magnet has to neutralize the combined vertical forces of sub-permanent magnetism, vertical iron, and Flinders bar. But as the two latter change with change of latitude, it is evident that this mode of compensation is effective only so long as the ship remains in the same latitude, and constant shifting of the magnet would be necessary as the ship proceeds north or south, and probably a reversal of the poles on the other side of the magnetic equator.

The only efficient compensation would be to use the magnet for the part due to sub-permanent magnetism only, and a vertical soft iron bar, having its lower end *above* the compass, to counteract the vertical force of soft iron *below* the compass. Some arrangement might possibly be made for the purpose in the construction of the binnacle.

It will now be understood that the principle of compensation is to counteract the existing magnetic forces in a ship by employing counter equal forces of the same kind, to act on the compass needles.

The order of procedure recommended in the "Elementary

Manual" for ships of the Royal Navy, where the conditions are pretty constant, is—

1st. Coefficient D.

2nd. Heeling error.

3rd. Coefficients B and C.

But in the Merchant Service, where cargoes change every trip, and where the compensation is often hurriedly made when ready for sea, the "Tentative Method" of adjustment is usually carried out in the following order:—

1. Coefficient C (athwartship magnet).
2. Coefficient B. (1) Flinders bar, and (2) fore-and-aft magnet.
3. Coefficient D (soft-iron correctors).
4. Heeling error (vertical magnet).

Coefficient E is rarely compensated, although a counter E could be introduced by moving one of the D correctors forward, and the other a little aft. For +E the starboard corrector should be moved aft, and the port one as much forward; for −E, the reverse.

A table of the remaining deviations is obtained by "swinging" the ship in dock or other convenient place.

Yet, however accurately the compensation may be made, changes in the deviation, from causes already explained, will occur during a voyage, and no careful navigator will place much reliance on "deviation cards." In fact, nothing can relieve him from the obligation to find the deviation from frequent observations, which is now happily made easy by Azimuth Tables.

CHAPTER XXIX.

SYLLABUS.

Questions on Deviation of the Compass for Ordinary Masters.

The applicant is to answer correctly at least 12 of such of the following questions as are marked with a cross by the examiner.

The examiner's attention is specially called to the importance of questions 11, 12, 13, 14, and 39, which must be marked in all cases.

1. State briefly the essentials of an efficient compass.
2. State briefly the chief points to be considered when selecting a position for your compass on board ship, and what should be particularly guarded against.
3. What do you mean by deviation of the compass; and how is it caused?
4. Describe how you would determine the deviation of your compass: (1) by reciprocal bearings; (2) by figures on the dock walls; (3) by bearings of a distant object; (4) by the bearings of the sun or other celestial body.
5. Having determined the deviation with the ship's head on the various points of the compass, how do you know when it is easterly and when westerly?
6. Why is it necessary, in order to ascertain the deviations, to bring the ship's head in more than one direction?
7. For accuracy, what is the least number of points to which the ship's head should be brought for constructing a curve or table of deviations?
8. How would you find the deviation when sailing along a well-known coast?
9. Name some suitable objects by which you could readily obtain the deviation of the compass when sailing along the coasts of the Channel you have been accustomed to use.
10. Supposing you have no means of ascertaining the magnetic bearing of a distant object when swinging your ship for deviations, how could you find it approximately from equidistant compass bearings; and at what distance, as a rule, should the object be from the ship?
11. Having taken the following compass bearings of a distant object, find the object's magnetic bearing, and thence the deviation.

Magnetic bearing required :—

Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation required.
N.	S. 4° E.		S.	S. 13° E.	
N.E.	S.		S.W.	S. 23° E.	
E.	S. 4° W.		W.	S. 21° E.	
S.E.	S. 1° W.		N.W.	S. 11° E.	

12. With the deviation as above, construct a curve of deviations on a Napier's diagram, and give the courses you would steer by the standard compass to make the following courses, correct magnetic :—

Magnetic courses : S.S.W. W.N.W. N.N.E. E.N.E. S.S.E.
Compass courses required.

13. Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above curve of deviations :—

Compass courses : W.S.W. N.N.W. E.N.E. S.S.E.
Magnetic courses required.

14. You have taken the following bearings of two distant objects by your standard compass as above; with the ship's head at W. ½ S., find the bearings, correct magnetic :—

Compass bearings : W. by S. and N. ¾ W.
Magnetic bearings required.

15. Do you expect the deviation to change? If so, state under what circumstances.

16. How often is it desirable to test the accuracy of your table of deviations?

17. What is meant by variation of the compass? What is it caused by? And where can you find the variation for any given position?

18. The earth being regarded as a magnet, which is usually termed the blue, and which the red magnetic pole?

19. Which end of a magnet (or compass needle) is usually termed the red or "marked" end, and which the blue?

20. What effect has the pole of one magnet of either name on the poles of another magnet?

21. What is meant by transient induced magnetism?

22. Which is the red and which is the blue pole of a mass of soft vertical iron, by induction, and what effect would the upper and lower ends of it have on the compass needle (*a*) in the northern hemisphere, (*b*) in the southern hemisphere, (*c*) on the magnetic equator?

23. Describe what is usually termed the sub-permanent[1] magnetism of an iron ship, and state when and how it is acquired, and which is the red and which is the blue pole, and why it is called sub-permanent magnetism.

24. Describe the meaning of the expression "coefficient A."

25. Describe the meaning of the expression "coefficient B," its signs and effects.

26. Describe the meaning of the expression "coefficient C," its signs and effects.

27. Describe the meaning of the expression "coefficient D," its signs and effects.

28. Describe the meaning of the expression "coefficient E," its signs and effects.

29. Would you expect any change to be caused in the error of your compass by the ship heeling over either from the effect of the wind or the cargo, etc.?

30. The compasses of iron ships being more or less affected by what is termed the heeling error, on what courses is this error usually at its minimum, and on what courses at its maximum?

---

[1] The term sub-permanent magnetism in these questions is used in the original sense as proposed by the late Sir G. B. Airy, to denote the character of the permanent magnetism of an iron ship as distinguished from the permanent magnetism of a magnetized steel bar. The terms "sub-permanent" and "permanent" throughout these questions may, therefore, be considered as synonymous.

31. Describe clearly the three principal causes of the heeling error on board ship.

32. State to which side of the ship in the majority of cases is the north point of the compass drawn when the ship heels over in the northern hemisphere.

33. Under what conditions (that is, as regards position of ship whilst building, and the arrangement of iron in the ship) is the north point of the compass needle usually drawn to windward or the high side of the ship in the northern hemisphere; and if not allowed for, what effect has it on the assumed position of the ship when she is steering on northerly, also on southerly courses, in the northern hemisphere?

34. Under what conditions (as in Question 33) is the north point of the compass needle usually drawn to leeward or the low side of the ship in the northern hemisphere; and, if not allowed for, what effect would it have on the assumed position of the ship, when she is steering on northerly, also on southerly courses, in the northern hemisphere?

35. The effects being as you state, on what courses would you keep away, and on what courses would you keep closer to the wind in the northern hemisphere in order to make good a given compass course (*a*) when north point of compass is drawn to windward or the high side of ship; and (*b*) when drawn to leeward or the low side?

36. Does the same rule hold good in both hemispheres with regard to the heeling error?

37. State clearly how that part of the heeling error due to the permanent part of the magnetism of the ship varies as the ship changes her position on the globe; and what is the reason of this.

38. State clearly how that part of the heeling error due to the induction in transverse iron (which was horizontal when ship was upright) and iron vertical to the ship's deck, varies as the ship changes her position on the globe.

39. Your steering compass having a large error, show by "Beall's Compass Deviascope" how you would correct it by compensating magnets and soft iron (as usually practised by compass adjusters in the Mercantile Marine) in order to reduce the error within manageable limits. Show also how the heeling error can be compensated.

40. As the coefficient B (capable of being corrected) usually consists of two parts, one due to the permanent magnetism of the ship, and the other to vertical induction in soft iron, how should each of the two parts, strictly speaking, be corrected when compensating the compass?

41. If the whole of coefficient B be corrected by a permanent magnet, as is usually done, what is likely to ensue as the ship changes her magnetic latitude?

42. Provided the needles of your compass are not so long and powerful, and so near, as to cause the soft-iron correctors to become magnetized by induction, would the coefficient D, if properly compensated, be likely to remain so in all magnetic latitudes and both hemispheres? If so, state the reason why.

43. State at what distance, as a general rule, the magnets and soft-iron correctors should be placed from the compass needles, and what will be the consequence if they are placed too near the needles.

44. Is it necessary that the magnets used for compensating coefficients B and C should be placed on the deck? If not, state where they may also be placed, and the rules to be observed in placing them into position.

45. Can the compensation of the heeling error be depended upon when the ship changes her latitude? If not, state the reason.

Answers to the Questions on Deviation of the Compass.

1. The card should be light, accurately centred and divided, and the needles well magnetized, the magnetic axis coinciding with the N. and S. points of the card. The cap should be sound, and the pivot sharp and smooth. The point of the pivot should be level with the gimbals. The bowl should be of copper, and the mounting should not allow of any lateral play.

2. The standard compass should be in the midship line, and should afford a clear view all round for taking bearings, and not less than five feet from iron of any kind, especially vertical iron and any iron which is likely to change its position, such as derricks, cowls, davits, etc. Also as far as possible from a dynamo, if electric lighting is used. Steering compasses should be so placed that the card can be well seen by the steersman, and should be as far removed as possible from the influence of iron.

3. Deviation is the amount of deflection of the N. point of a compass from the correct magnetic N. caused by the magnetic influence of iron in the ship, whether in hull, equipment, or cargo.

4. (1) By reciprocal bearings: An observer is stationed at a compass on shore in a position free from local attraction, and another observer at the standard compass on board. Then, as the ship is swung round, bearings are taken of each compass by the other observer at the same time, whilst the ship's head is steadied on the required points. The difference between the bearing observed on board and the reversed shore bearing is the deviation. (2) By figures on the dock wall: These figures at Liverpool show the true bearings of the Vauxhall Chimney from the river, so that the figure in a line with the chimney as seen from the ship shows its true bearing from the observer's position. The difference between this and the bearing by the standard compass is the total error, from which the variation may be separated, and the result is the deviation. (3) By bearings of a distant object: When the correct magnetic bearing of a distant object is known, the difference between the bearing of it by compass on board, and the correct magnetic is the deviation. (4) By the bearings of a celestial object: The true azimuth of the object being found for a given moment, either by calculation or by tables, the bearing of the object at the same time is taken by compass; the difference shows the error of compass for the actual position of ship's head, and the variation being known, the deviation may be found.

5. If the correct magnetic bearing reads to the *right* of the compass bearing, the deviation is *east*; if to the *left*, it is *west*.

6. Because the deviation changes with every change of course.

7. Eight equidistant points, viz. N., N.E., E., S.E., S., S.W., W., N.W.

8. By comparing the bearing of two known objects when in one (or in line) with the known correct magnetic direction of one from the other obtained from a chart.

9. (*a*) English Channel: Bolt Head and Start Lighthouse, Prawle Point and Start Point, etc. (*b*) Irish Channel: Skerries and South Stack, Smalls and Bishop Lights, Wolf and Longship. (*c*) North Sea: South Foreland Lights, Whitby Lights, and Tyne Harbour Lights.

10. The ship's head should be steadied on eight equidistant points in succession, and the bearing of a distant object observed when on each point. The mean of these may be taken as the correct magnetic. The object should be seven or eight miles distant, so that its bearing would not be sensibly affected by the swinging of the ship.

11, 12, 13, and 14 are practical.

15. Yes, the deviation may be expected to change: (*a*) From lapse of time; (*b*) By keeping the ship's head for a considerable time in an opposite

direction to that in which she was built; (*c*) By change of latitude; (*d*) By shocks or strains from heavy seas; (*e*) By being kept for a long time on one course, magnetism is induced which is retained on a change of course.

16. The deviation table should be tested at every opportunity.

17. Variation is the difference in direction between the magnetic north and true north, and is due to the fact that the earth's magnetic poles do not coincide with its geographical poles. The amount, together with the annual change, may be found on a magnetic chart.

18. The earth's north magnetic pole is the blue pole, and the south the red.

19. The end of a magnetized needle, freely suspended, which points north is the red pole, and the other the blue pole.

20. Poles of the same name repel each other, and poles of different names attract each other.

21. Transient induced magnetism is the magnetic condition of soft iron caused by the influence of a magnetic force in its neighbourhood or that of the earth itself.

22. North of the magnetic equator the upper end is the blue pole and the lower the red, but south of the magnetic equator this rule is reversed. (*a*) In the northern hemisphere the upper end would attract the north end of the compass needle, and the lower end repel it. (*b*) In the southern hemisphere the upper end would repel and the lower attract the north end of the needle. (*c*) On the magnetic equator the effect would be *nil*.

23. The sub-permanent magnetism of an iron ship is the magnetic condition of a more or less enduring character which she has when she is launched. It is acquired in the course of building, from the earth's inductive influence, aided by the vibration due to hammering, riveting, etc. The red pole is that which was towards the north, and the blue towards the south when the ship was on the stocks. It is called sub-permanent magnetism to distinguish it from the magnetism of a tempered steel bar, and because it is liable to be weakened or modified by circumstances and by lapse of time.

24. Coefficient A is a deviation of the same amount and sign on all courses, usually due to a faulty or badly placed compass.

25. Coefficient B represents a horizontal magnetic force, whose poles are in the fore-and-aft midship line. It is called +B when the blue pole is towards the bow, and −B when towards the stern. +B causes E. deviation on easterly courses, and W. deviation on westerly courses. −B causes W. deviation on easterly courses, and E. deviation on westerly courses. The effect is *nil* on N. and S. courses, and greatest on E. and W. courses.

26. Coefficient C represents a horizontal magnetic force, whose poles are in a line at right angles to the fore-and-aft midship line. It is called +C when the blue pole is towards the starboard side, and −C when towards the port side. +C causes E. deviation on northerly courses, and W. deviation on southerly courses. −C causes W. deviation on northerly courses, and E. deviation on southerly courses. The effect is *nil* on E. and W. courses, and greatest on N. and S. courses.

27. Coefficient D represents the effect of transient induced magnetism in horizontal iron in fore-and-aft and athwartship directions. The deviation due to this cause changes its name in successive quadrants, and is thence called quadrantal. +D causes E. deviation from N. to E. and S. to W. courses, and W. deviation from S. to E. and N. to W. −D gives W. deviation from N. to E. and S. to W. courses, and E. deviation from S. to E. and N. to W. The deviation due to this cause is *nil* on the cardinal points and greatest on N.E., S.E., S.W., and N.W.

28. Coefficient E represents the effect on the compasses of induced magnetism in horizontal iron lying in a direction oblique to the fore-and-aft line. The deviation due to this cause is quadrantal, and is *nil* on N.E., S.E., S.W., and N.W., and greatest on the cardinal points. +E gives E. deviation

from N.W. to N.E. and S.E. to S.W., and W. deviation from N.E. to S.E. and S.W. to N.W. −E gives W. deviation from N.W. to N.E. and S.E. to S.W., and E. deviation from N.E. to S.E. and S.W. to N.W.

29. Yes, because when the ship heels, horizontal athwartship iron acquires a vertical component, and besides, the upper poles of vertical iron and the poles of sub-permanent magnetism will then not be in the same vertical plane with the centre of the compass.

30. The heeling error is usually greatest on N. and S. courses, and least on E. and W. courses, because in the former case the force causing it acts at right angles to the compass needle, and in the latter case coincides with the direction of the needle.

31. The three principal causes of heeling error are: (1) Traverse iron acquires a vertical component. (2) The upper induced poles of vertical iron will not be in the same vertical plane with the centre of the compass. (3) The poles of the fore-and-aft component of the ship's sub-permanent magnetism fall to the right or left of the vertical plane, through the centre of the compass.

32. In the N. hemisphere the N. point is usually drawn towards the higher side.

33. In ships built with head northerly, compass aft, placed over continuous athwartship iron, the N. point of a steering compass will be attracted towards the higher side, and, if not allowed for, will draw the ship to windward on northerly courses, and to leeward on southerly courses.

34. In ships built with head towards south, with compass aft, placed over divided athwartship iron, the effect of the sub-permanent magnetism and soft iron would be to draw the N. point of the compass to leeward, and if these exceeded the influence of the vertical soft iron, the N. point of the compass would be drawn towards the lower side. If not allowed for, it would take the ship to leeward on northerly courses, and to windward on southerly courses.

35. (*a*) When the N. point is drawn to windward, I would keep away on northerly courses, and closer to the wind on southerly courses. (*b*) When the N. point is drawn to leeward, I would keep closer to the wind on northerly courses, and keep away on southerly courses.

36. The rule for the northern hemisphere is generally reversed in the southern, especially in ships built heading south, for the poles of vertical induction are reversed, but in ships built heading north, the same rule may still obtain, that is, when the sub-permanent vertical force exceeds the induced.

37. It *decreases* on approaching the magnetic equator, and *increases* on receding from it, because the directive force of the compass changes as the magnetic latitude changes, being greatest at the magnetic equator and *nil* at the magnetic poles, whilst the sub-permanent magnetism of the ship is nearly constant.

38. The induced vertical force becomes weaker on approaching the magnetic equator, is *nil* at the equator and the poles are reversed on crossing it. Therefore the part of the heeling error due to this cause, drawing the N. point to windward in north mag. lat., disappears at the magnetic equator, and repels the N. point to leeward in S. mag. lat.

39. Practical.

40. The part of coefficient B due to the sub-permanent magnetism should be compensated by means of a magnet, and the part due to vertical induction in soft iron by an upright bar of soft iron before or abaft the compass in the midship line.

41. It will be found to be over-compensated or under-compensated as the ship changes her mag. lat., for the part caused by vertical induction changes with change of latitude.

42. Yes, because the horizontal inductive force of the correctors will vary in the same ratio as that of the horizontal iron which produces coefficient D.

in the same ratio as that of the horizontal iron which produces coefficient D.

43. The magnets used for compensating should not be nearer the centre of compass than twice their own length, and the soft-iron correctors should not be nearer than 1¼ times the length of the needles. If the magnets are too near, the compensation which was made with the ship's head on the cardinal points would not be perfect on other points. If the soft-iron correctors are too near, the poles of the compass needles would have a tendency to induce opposite polarity in the correctors, rendering them useless.

44. The magnets used for compensating B and C need not be on the deck, but may be placed in any convenient position near the compass, in or outside the binnacle, provided the following conditions are complied with: (1) The ship should be upright and on even keel. (2) The fore-and-aft magnets should be parallel to the deck, and bisected by a vertical plane athwartships, passing through the centre of the compass. (3) The athwartship magnets should be parallel to the deck, and placed so as to be bisected by a fore-and-aft vertical plane passing through the centre of the compass.

45. No, because a part of the heeling error is due to vertical induction in soft iron, which varies in force as the latitude changes. This part of the heeling error cannot be compensated by means of a permanent magnet fixed in position.

Questions on Deviation of the Compass for Masters (Extra).

1. Describe an artificial magnet, and how a steel bar or needle is usually magnetized.
2. Which end of the compass needle, or a magnet, is commonly termed the red, and which the blue pole?
3. Which is the red magnetic pole of the earth, and which the blue? and give their geographical positions.
4. What effect has the pole of one magnet of either name on the pole of the same name of another magnet? and what would be the consequence of the pole of one magnet of either name being brought near enough to affect the pole of contrary name, if in these cases both magnets were freely suspended?
5. By applying this law to all magnets, natural as well as artificial, describe what would be the result on a magnetic bar or needle, freely suspended, but by weight or by the nature of its mounting constrained to preserve a horizontal position; and what would be the result, if so mounted, but free to move in every direction, the earth being regarded as a natural magnet.
6. What is the cause of the variation of the compass?
7. What is meant by the deviation of the compass?
8. What is meant by the term "local attraction"? under what circumstances have ships' compasses, from recent careful investigation, been found to be affected by it? and name some of the localities in different parts of the world where this disturbance is to be found, and consequently where increased vigilance is necessary (see page 22, "Creak's Elementary Manual," 1908).
9. What do you understand by the term "soft" iron; and what are its properties as regards acquiring and retaining magnetism?
10. What do you understand by the term "hard" iron; and what are its properties as regards acquiring and retaining magnetism?
11. Describe the meaning of the term "horizontal force" of the earth; where is it the greatest, and where the least, and what effect has it in respect to the increase or decrease of the directive force of the compass needle?
12. Does the magnetic equator coincide with the geographical equator? if not, state clearly how it is situated.

13. Where can the values of the magnetic dip, the earth's horizontal force, and the variation, be found?

14. State in what parts of the globe lying in the usual tracks of navigation the variation changes very rapidly, and what special precautions should be observed when navigating these localities; also why a "variation" chart is then very useful.

15. Why is a knowledge of the magnetic dip and the earth's horizontal force important in dealing with compass deviations?

16. Describe the meaning of the term "vertical force" of the earth; where is it the greatest, and where the least?

17. Would you expect a compass to be the more seriously affected by any given disturbing force when near the magnetic equator, or near the poles? and state the reason.

18. State briefly (*a*) the essentials of an efficient compass; and (*b*) what you would consider a good arrangement of the needles (that is, whether long or short, single or double, etc.) with the view to good compensation.

19. In stowing away spare compass cards or magnets, how would you place them with regard to each other, or what might be the probable consequence?

20. State briefly the chief points to be considered when selecting a position for your compass on board ship, and what should be particularly guarded against.

21. What is meant by transient induced magnetism?

22. Which is the *red* and which the *blue* pole of a mass of soft vertical iron (or indeed of any soft iron not in a horizontal position) by induction; and what effect would the upper and lower ends of it have on a compass needle in the *Northern* hemisphere?

23. Which is the *red* and which the *blue* pole of a mass of soft vertical iron by induction; and what effect would the upper and lower ends of it have on the compass needle in the *Southern* hemisphere?

24. What effect would a bar of soft vertical iron have on the compass needle on the magnetic equator?

25. Describe what is usually termed the sub-permanent[1] magnetism of an iron ship, and state when and how it is acquired, and which is the sub-permanent red and which is the blue pole, and why it is called sub-permanent magnetism.

26. What is meant by "the composition of forces" and "the parallelogram of forces"? and show how the knowledge of these is valuable in ascertaining and compensating the sub-permanent magnetism of an iron ship.

27. Describe the nature of the coefficients B and C, plus (+) and minus (−), and the different magnetic forces they represent; also why they are said to produce semicircular deviations.

28. Can semicircular deviations be produced by any other force than the sub-permanent magnetism of the ship? If so, by what?

29. On what points, by compass bearings of the ship's head, does + B give westerly deviation, and on what points does it give easterly; also on what point does − B give westerly, and on what points easterly?

30. On what points does + C give westerly deviation, and on what points easterly; also on what points does − C give westerly, and on what points easterly deviation?

31. The value of either coefficient B or C being given, also the magnetic direction of the ship's head while she was being built, determine by the Traverse Tables the approximate value of the other coefficient C or B; and,

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[1] See footnote, p. 300.

the value of both these coefficients being given, determine approximately the direction by compass of the ship's head whilst being built, assuming, of course, that these coefficients resulted altogether from sub-permanent magnetism.

32. Would you expect the greatest disturbance of the needle from the effects of sub-permanent magnetism alone to take place when ship's head is in same direction as when building, or when her head is at right angles to that direction, and in what direction of the ship's head would you expect to find the least disturbance?

33. Describe quadrantal deviation, and state what coefficients represent it; also on what points of the ship's head, by compass, each of these coefficients gives the greatest amount of deviation, and why it is called quadrantal deviation.

34. On what points of the compass will each of the coefficients D and E, + and −, give easterly, and on what points westerly, deviation?

35. What conditions of the iron of a ship will produce +D, and what −D?

36. State clearly, then, which end of the horizontal iron running athwartship (such as beams, etc.), and of horizontal iron running fore and aft of a ship, acquires *red* and which *blue* polarity, by induction, when ship's head is at N.E., S.E., S.W., and N.W. respectively.

37. Describe the nature of the deviation represented by coefficients +A and −A, and describe the errors in the construction of the compass, and other causes, that frequently produce it.

38. What is the object of compensating the compass by magnets, etc.; and what are the general advantages of a compensated compass over an uncompensated one?

39. Before adjusting the compass of an iron ship, what is it desirable to do with the view to eliminating, as far as possible, what may be termed the unstable part of the magnetism of the ship?

40. Describe clearly the tentative method of compass adjustment (that is, the compensation of coefficients B, C, and D, with ship upright) as generally practised by compass adjusters in ships of the Mercantile Marine.

41. State at what distance, as a general rule, the magnets and soft-iron correctors should be placed from the compass needles, and what will be the consequence if they are placed too near the needles.

42. Is it necessary that the magnets used for compensating coefficients B and C should be placed on the deck? If not, state where they may also be placed, and the rules to be observed in placing them into position.

43. Does the B found on board ship usually arise altogether from sub-permanent magnetism, or does part of it usually arise from some other cause or causes?

44. If the part of B due to induced magnetism in vertical soft iron, as well as the part due to sub-permanent magnetism, are corrected by a magnet alone, as is generally the case, what is frequently the consequence on the ship changing her magnetic latitude and hemisphere?

45. How should each of these two parts of B then, strictly speaking, be compensated?

46. Assuming, for the sake of clearness, that your steering compass is unavoidably placed very near to the head of the stern-post (and other vertical iron at the stern), thereby causing a very large −B from induced magnetism, describe briefly any method by which the approximate position for the compensating vertical iron bar (Flinders' or Rundell's) could be estimated in order to reduce the error; describe also how you would proceed, in order to improve, if not to perfect, its position after observations have been made on the magnetic equator.

47. State if standard compasses, as well as steering compasses, are

generally subject to this disturbance from induced magnetism in vertical iron; also whether the attraction in all cases is found to be towards the stern; and if not, state the conditions under which it might be toward the bow, and how the compensating soft-iron bar should then be placed.

48. Generally speaking, does the magnetism induced in vertical iron usually have any effect in producing the coefficient C (ship upright), or is it generally produced by sub-permanent magnetism alone? State also your reasons for saying so.

49. Provided the needles of your compass are not so long and powerful, and so near, as to cause the soft-iron correctors to become magnetized by induction, would the coefficient D, if properly compensated as you have described (Ans. 40), be likely to remain so in all latitudes and both hemispheres? If so, state the reason why.

50. Under what circumstances does the character of A and E so change as to render it desirable that these coefficients should be disregarded or modified?

51. Supposing your compasses were allowed to remain uncompensated, explain clearly what would be the probable changes (ship upright) in the deviations produced separately by (1) the sub-permanent magnetism of the ship alone, (2) by the induced magnetism in vertical soft iron, (*a*) on reaching the equator, (*b*) in the southern hemisphere.

52. Assuming you were able to arrive at the proper proportions to be corrected, and were then to exactly compensate the sub-permanent magnetism of the ship by means of a permanent magnet, and the induced magnetism in vertical iron by a soft-iron bar, would you expect any deviation to take place in your compass as the ship changed her latitude and hemisphere? And state your reasons for saying so.

53. Supposing the coefficient D from horizontal soft iron were allowed to remain uncompensated, would you, or would you not, expect the D to differ in name or amount on the ship changing her magnetic latitude, and hemisphere? And state the reason.

54. Describe how you would determine the deviation of your compass, (1) by reciprocal bearings, (2) by figures on the dock walls, (3) by bearings of a distant object.

55. Describe, in detail, how you would determine the deviation of your compass by the bearings of the Sun. Also by a star or planet.

56. Describe the uses to which the Napier's diagram can be applied, and its special advantages.

57. Describe clearly how the Napier's diagram is constructed.

58. For accuracy, what is the least number of points to which the ship's head should be brought for constructing a complete curve of deviations, or a complete table of deviations?

59. Nearing land, and being anxious to check your deviations on a few courses you may probably require to steer, what is the least number of points it would be necessary to steady the ship's head upon, if making use of a Napier's diagram, in order to ascertain the deviation on each of the points, say, in a quadrant of the compass? and describe clearly how you would do this at sea.

60. Supposing you have no means of ascertaining the magnetic bearing of the distant object when swinging your ship for deviations, how could you find it, approximately, from equidistant compass bearings; and how far, as a rule, should the object be from the ship when swinging, or steaming round?

61. Having taken the following equidistant compass bearings of a distant object, find the object's magnetic bearings, and thence the deviations:—

(*a*) Magnetic bearing required—

Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation.	Ship's head by standard compass.	Bearing of distant object by standard compass.	Deviation.
N.	S. 75° W.		S.	S. 34° W.	
N.E.	S. 64° W.		S.W.	S. 31° W.	
E.	S. 56° W.		W.	S. 49° W.	
S.E.	S. 50° W.		N.W.	S. 71° W.	

(*b*) Construct a curve of deviations on a Napier's diagram, with the deviations as above, and give the courses you would steer by the standard compass to make the following courses, correct magnetic:—

Magnetic courses: N.N.W. S.S.E. W.N.W. E.S.E.
Compass courses required.

(*c*) Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above curve of deviations:—

Compass courses: N.N.E. E.N.E. S.S.W. W.S.W.
Magnetic courses required.

(*d*) You have taken the following bearings of two distant objects by your standard compass as above; with the ship's head at N.E. ½ E., find the bearings, correct magnetic:—

Compass bearings: S.E. by S. and N.N.W.
Magnetic bearings required.

62. Assuming the deviations observed with the ship's head by compass to be as follows (or as in Question 61, whichever may be given), determine the value of the coefficients A, B, C, D, and E, and from them construct a complete table of deviations (or for as many points as the Examiner may direct).

Deviation at N.	(21° W.)	S.	(20° E.)
N.E.	(10° W.)	S.W.	(23° E.)
E.	(2° W.)	W.	(5° E.)
S.E.	(4° E.)	N.W.	(17° W.)

63. When swinging your ship, if it be required to construct Deviation Tables for two or more compasses situated in different parts of the vessel, describe the process, and how you would employ the Napier's diagram for this purpose.

64. State your rule for determining whether deviation is easterly or westerly.

65. Is a knowledge of the value of the various coefficients of any advantage? If so, state why?

66. Describe (*a*) what is commonly known by the term "retentive" or "retained" magnetism, and how the ship acquires it when in port and at sea; (*b*) its effect on the compass needle whilst ship's head continues in the same direction; (*c*) the immediate consequence when the direction of the ship's head is altered; and (*d*) the special precautions to be invariably observed at sea on the alteration of the ship's course.

67. Describe a "Dumb-card" or "Pelorus," and its use (*a*) in compensating a compass, (*b*) in determining the deviation.

68. If you determine the deviation by an azimuth or an amplitude of a heavenly body, it is then combined with variation, which together is sometimes called the *correction* for the compass. State when the deviation is the difference between the variation and the *correction*, and when the sum; and when it is of the same name as that of the *correction*, and when of the contrary name.

69. In observing azimuths of heavenly bodies, the best method is by "time azimuths," since these can be observed without an altitude when the ship is in port, or when the horizon cannot be defined from any cause. Given the Sun's declination, the hour of the day, and the latitude, to find the true bearing of the Sun.[1]

70. By night, if it be desirable to observe the *correction* of the compass. Given the day of the year, and time at ship, also the latitude of the place, to determine what stars will be in good position for this purpose.

71. If your correcting magnets are so mounted that their positions can be altered, describe the process by which, on open sea, you can place the ship's head correct magnetic N. (or S.), and correct magnetic E. (or W.), and can make the correction perfect.

72. Given the name of a star, the time, the place of ship, the variation of the compass, and the bearing of the star by compass, determine the deviation, and name it east or west.

73. Would you expect any change to be caused in the error of your compass by the ship heeling over, either from the effect of the wind or the cargo?

74. Describe clearly the three principal causes of the heeling error on board an iron ship?

75. Towards which side of the ship would that part of magnetism induced in continuous transverse iron (which was horizontal while ship was upright) help to draw the north point of the needle when ship heels over, (*a*) in the northern hemisphere, (*b*) in the southern hemisphere?

76. Supposing the compass were placed between the two parts of a divided beam or other athwartship iron, towards which side of the ship would iron so situated help to draw the north point of the needle when ship heels over, (*a*) in the northern hemisphere, (*b*) in the southern hemisphere?

77. Would you expect that part of the magnetism induced in iron exactly perpendicular to the ship's deck, such as stanchions, bulkheads, etc., if below the compass, to cause any part of the heeling error when ship heels over, and if so, towards which side of the ship, (*a*) in the northern hemisphere, (*b*) in the southern hemisphere?

78. If an ordinary standard compass placed higher than the iron top sides be compensated whilst the ship is upright, what coefficient will be affected by heeling?

79. Under what conditions (that is, as regards position whilst building and the arrangement of iron in the ship) is the north point of the compass needle usually drawn to windward, or the high side of the ship, in the northern hemisphere?

80. Under what conditions, as a rule, is the north point of the compass needle usually drawn to leeward, or the low side of the ship, in the northern hemisphere?

81. State to which side of the ship, in the majority of cases, is the north point of the compass drawn when ship heels over in the northern hemisphere; and when this is the case, and it is not allowed for, what effect has it on the assumed position of the ship when she is steering on northerly, and also on southerly courses?

82. On what courses would you keep away, and on what courses would you keep closer to the wind in both the northern and southern hemispheres in order to make good a given compass course, (*a*) when north point of compass is drawn to windward, or the high side of ship; and (*b*) when drawn to leeward or the low side?

[1] The process of finding time azimuths by the ordinary formulæ of spherical trigonometry is tedious, and since on board an iron ship these observations should be often repeated, the candidate will be allowed to use any table or graphic or linear method that will solve the problem within a half of a degree, the altitude of the heavenly body not being given.

83. If a ship is beating to windward, when she tacks, under what circumstances will the heeling error retain the same name, and under what circumstances will it take the contrary name?

84. If a ship is placed on the opposite tack by the change of wind, the ship's course being the same by compass, will the heeling error change its name?

85. In which direction of the ship's head does the heeling error attain its maximum value, and in which direction does it generally vanish?

86. Explain clearly how that part of the heeling error due to the permanent part of the magnetism of the ship varies as the ship changes her geographical position, and what is the reason of this?

87. Explain clearly how that part of the heeling error due to the induction in transverse iron (which was horizontal when ship was upright), and iron vertical to the ship's deck, varies as the ship changes her geographical position.

88. What, then, would be the probable nature of the heeling error, that is, whether to high or low side of the ship, and whether the error would be equal to the sum or difference, etc., of the forces—(1) in high north latitudes, (2) on magnetic equator, (3) in high south latitudes? Assuming the polarity of the sub-permanent magnetism of the ship under, and affecting, the compass to be as given below; the vertical induction in soft iron, of course, obeying the ordinary laws in the above geographical positions (1), (2), (3).

(*a*) In cases where the effect of *red* vertical sub-permanent magnetism is equal to that of the vertical induction in the soft iron of the ship.

(*b*) Where the effect of *red* vertical sub-permanent magnetism is greater than that of the vertical induction in the soft iron.

(*c*) Where the effect of *red* vertical sub-permanent magnetism is less than that of the vertical induction in the soft iron.

(*d*) Where the effect of *blue* vertical sub-permanent magnetism is equal to that of the vertical induction in the soft iron.

(*e*) Where the effect of *blue* vertical sub-permanent magnetism is greater than that of the vertical induction in the soft iron.

(*f*) Where the effect of *blue* vertical sub-permanent magnetism is less than that of the vertical induction in the soft iron.

89. Can the heeling error be compensated? If so, state the means to be employed, and how the compensation may be effected.

90. Can the compensation of the heeling error be depended on in every latitude? If not, state the reason.

91. Do the soft-iron correctors used for compensating the coefficient +D have any effect on the compass needle when the ship heels over? and, if so, do they draw the needle towards the low or the high side of the ship, and do they counteract, or otherwise, the effect produced by the vertical induction in the soft iron, (*a*) in the northern hemisphere, (*b*) in the southern hemisphere; and what is the reason of this?

92. Given the heel, the direction of the ship's head by compass, and the heeling error observed, to find the approximate heeling error, with a greater or less given heel, and with the ship's head on some other named point of the compass, the ship's magnetic latitude being in both cases the same.

93. Describe any instrument to show the ship's heel (generally called a clinometer), and state how and where it should be fixed.

94. Should the clinometer be observed when the ship is swung to determine the deviation when the ship is upright? If so, state the reason why.

95. Would you expect the Table of Deviations supplied by the compass adjuster from observations made in swinging the ship to remain good during the voyage, or would you expect the deviations to change? If so, state under what circumstances.

96. Is it desirable that a record of your observations for deviations should

be kept as a guide for any subsequent voyage, in case the ship should be in the same locality, or for further correction of the compass? If so, describe some suitable form for keeping such record.

97. Would you, under any circumstances, consider it a safe and proper procedure to place implicit confidence in your compasses, however skilfully they may have been adjusted? If not, what precautions is it your duty to take at all times?

ANSWERS TO THE QUESTIONS ON COMPASS DEVIATION FOR MASTERS (EXTRA).

1. An artificial magnet is a bar or needle of hard-tempered steel which has been magnetized by artificial means, and so distinguished from a load-stone or natural magnet.

Artificial magnets can be made in several ways:

(*a*) For small needles, it is sufficient to place either pole of a strong magnet on one end of the needle and draw it in contact to the other end, where it is taken off. If the pole used is the red pole, the end of the needle last in contact with it becomes a blue pole, and the other a red pole.

(*b*) The method of "separate touch." The bar to be magnetized is placed with its ends resting on opposite poles of two strong magnets; two other magnets are held, one in each hand, with opposite poles downwards, and brought together at the middle of the bar. The magnets are then inclined so as to make angles of about 30° with the bar, and are moved in contact with it towards the ends, where they are taken off. This process should be repeated several times on both sides of the bar, taking care that a red pole is moved towards the end supported by a red pole, and the blue pole towards the end resting on a blue pole.

(*c*) By using an electro-magnet. One half of the bar is drawn from the middle to the end over one pole of a strong electro-magnet, and the other half over the other pole.

(*d*) By suspending the bar inside a solenoid and passing a strong electric current through the latter.

2. The end which points north when the magnet is freely suspended is named the red pole, and the other end the blue pole.

3. The south magnetic pole of the Earth is the red, and the north the blue pole. The north magnetic pole is situated about 70° N. latitude and 97° W. longitude, and the south magnetic pole about 72½° S. latitude and 154° E. longitude.

4. Poles of the same name repel each other, whilst poles of different names attract each other. If freely suspended in the manner stated, they would assume a position of equilibrium determined by the Earth's directive force acting on the needles and the mutual attraction of their opposite poles.

5. In the first case, the needle would come to rest in the plane of the magnetic meridian, with its red pole pointing north; in the second case, the needle would still coincide with the magnetic meridian, but its red pole would dip in N. latitude, and its blue pole would dip in S. latitude at an angle from the horizontal depending on the magnetic latitude, the dip being 0 on the magnetic equator, and 90° at the magnetic poles.

6. The variation of the compass is due to the fact that the Earth's magnetic poles do not coincide with the geographical poles.

7. Deviation of the compass is the angle of deflection of its north point from magnetic north, caused by the disturbing influence of iron.

8. "Local attraction" is the name given to the magnetic influences (causing a disturbance of the compass) of the locality in which a ship may be found; for example, when in dock near to iron ships, iron pillars, cranes, etc., also in the neighbourhood of some volcanic islands. It is said that the following places have a disturbing effect on ships' compasses: Ascension,

St. Helena, Cape St. Frances (Labrador), Cossack (North Australia), New Ireland, Solomon Islands, Tumbora, Sumbawa, coast of Madagascar, Iceland, Ile de Los (west coast of Africa), and Odessa Bay and Shoal.

9. Malleable iron and cast iron are termed "soft iron." Soft iron has the property of being instantaneously magnetized by induction from the magnetism of the Earth or of a natural or artificial magnet; also by passing an electric current round it. This induced magnetism is only transient in its character, and is lost in great part or entirely when the inducing cause is removed.

10. "Hard iron" is iron combined with a certain percentage of carbon, and is usually termed steel. Tempered hard iron has the property of acquiring slowly and retaining permanently, or nearly so, the magnetic polarity communicated to it.

11. "Horizontal force" is the horizontal component of the Earth's total magnetic force in the line of dip, and varies as the cosine of the magnetic latitude. It is greatest at the magnetic equator, and 0 at the magnetic poles, and therefore gives the greatest directive force to the compass needle in the former case, whilst there is no directive tendency in the latter.

12. The magnetic equator does not coincide with the geographical equator, but intersects it in two nearly opposite points situated in west longitude and east longitude. From the former point in the Pacific Ocean it runs nearly parallel to the Equator to about 100° W. long., then bends to the south on approaching South America, and in crossing Brazil it recedes from the Equator as far as 16° S. lat.; then it bends northwards and crosses the Equator in about 6° W. long., whence it passes across the Gulf of Guinea and the African continent, receding to about 10° N. lat., and then very gradually approaches the Equator to the other point of intersection.

13. The values of the dip, etc., may be found on charts prepared for the purpose.

14. The variation changes rapidly in the English Channel, the western portion of the North Atlantic, the South Atlantic near the coast of Brazil, the South Indian Ocean, the neighbourhood of Japan, and especially in the Gulf of St. Lawrence.

In navigating these localities, if the ship's course is across the magnetic meridians, the precaution should be taken of altering the courses steered by compass in accordance with the change of variation. A variation chart is useful to show what magnetic meridians are crossed and the amount of variation.

15. Because the vertical induced magnetism which causes "heeling error" depends on the dip, and the directive force of the needle depends on the Earth's horizontal force.

16. The "vertical force" is the vertical component of the Earth's total magnetic force in the direction of the dip. It is greatest at the magnetic poles, and least at the magnetic equator.

17. The needle would be least affected near the magnetic equator, because the Earth's horizontal force, and therefore the directive power of the needle, are there at their greatest.

18. (*a*) The card should be accurately centred and graduated, the magnetic axis of the needle (or the common axis, if more than one) coinciding with the north and south points of the card. There should be the least possible friction between the centre cap and the supporting pivot, the needles should have good directive power, the "lubber-point" should be exactly in the fore and aft midship line, and the deviation should be reduced by compensation to a small amount, say under 3°. The gimbals should have little friction, and should not allow any lateral play. When in position in the binnacle, the card and lubber-point should be in full view of the man at the wheel.

(*b*) It is generally agreed that two or more short needles parallel to each

other in the same plane is the best arrangement for securing good compensation.

19. Care should be taken that the magnets are not stowed away with similar poles in contact, because in that position they would tend to weaken or demagnetize each other, or even reverse the poles.

20. A position should be chosen for the steering compass where it would not be near large elongated masses of iron, *e.g.* stern post, rudder post, tiller, stanchions, funnel, etc.

For a standard compass the same precautions are necessary, and the position chosen should be, if possible, symmetrical with regard to the various magnetic forces in the ship.

21. Transient induced magnetism is acquired by soft iron by induction from permanent magnets, from electric currents, or from the Earth. This polarity is retained only so long as the inducing cause is present.

22. The blue pole is uppermost in north magnetic latitudes when the cause of induction is the Earth's magnetism; therefore in the northern hemisphere the north end of the needle would be attracted by the upper end of a soft iron bar, and repelled by the lower end.

23. In south magnetic latitude the upper pole of soft iron by induction is red, and the lower end blue; therefore the north point of a compass needle would be repelled by the upper end, and attracted by the lower.

24. No effect.

25. The sub-permanent magnetism of an iron ship is the magnetic polarity acquired in the course of construction by induction from the Earth and retained after launching. It is liable, however, to a considerable reduction after the ship leaves the stocks, and is therefore termed "sub-permanent" to distinguish it from the fixed magnetism of tempered steel.

The red pole is that which was towards the north when the ship was being built, and the blue towards the south.

26. Composition of forces is the method of finding a single force which is equivalent to two or more forces of given amount and direction.

In the "Parallelogram of Forces," two adjacent sides represent in direction and units of amount two given forces acting at the point of meeting, and a diagonal from this point represents in direction and units of amount the equivalent force, or "resultant" of the composition of the two given forces.

A knowledge of this principle enables a compass adjuster to resolve the sub-permanent magnetic force in a ship into fore and aft, athwartship, and vertical components, and to compensate each independently of the others.

27. Coefficients B and C express the maximum deviations caused by magnetic forces acting horizontally in fore-and-aft and athwartship directions respectively; +B represents a blue pole at the bow, and red at the stern; −B the reverse; +C represents a blue pole at the starboard side, and red at the port side; −C the reverse. They are said to produce semi-circular deviation, because the deviation due to each is easterly on one half of the compass, and westerly on the other half.

28. Yes; it may be caused by vertical soft iron, because the induced poles remain the same whatever the direction of ship's head.

29. +B gives E. deviation on all courses from N. through E. to S., and W. deviation from N. through W. to S.; −B gives the opposite.

30. +C gives E. deviation on all courses from W. through N. to E., and W. deviation from W. through S. to E.; −C gives the opposite.

31. See page 296.

32. The greatest disturbance from sub-permanent magnetism would be found when the ship lay at right angles to the direction in which she was when being built, and least when in the same direction.

33. Quadrantal deviation is the effect of transient induced magnetism in horizontal iron; it is represented by coefficients D and E. D gives the

greatest deviation on N.E., S.E., S.W., and N.W. courses, and E gives the greatest on N., E., S., and W. courses. It is termed quadrantal because it changes its name in successive quadrants.

34. +D gives E. deviation on courses from N. to E. and S. to W., and W. deviation from E. to S. and W. to N.; −D gives the opposite.

+E. gives E. deviation on courses from N.W. to N.E. and S.E. to S.W., and W. deviation from N.E. to S.E. and S.W. to N.W.; −E gives the opposite.

35. Continuous transverse iron produces +D, and continuous fore-and-aft iron −D; but if end on to the compass, or when a beam is divided for a skylight and a compass placed between the parts, then the signs would be reversed.

36. The following table shows the results :—

Ship's head.	Athwartship iron.	Fore-and-aft iron.	Polarity acquired.
N.E.	End to port.	End towards bow.	Red.
	„ starboard.	„ stern.	Blue.
S.E.	„ port.	„ „	Red.
	„ starboard.	„ bow.	Blue.
S.W.	„ „	„ stern.	Red.
	„ port.	„ bow.	Blue.
N.W.	„ starboard.	„ „	Red.
	„ port.	„ stern.	Blue.

37. Coefficient A is a constant deviation of the same amount and sign on all courses, +A being easterly and −A westerly. It may be due to several causes—

(*a*) The magnetic axes of the needles not coinciding in direction with the N. and S. points of the card.
(*b*) The lubber-point not being correctly placed.
(*c*) Unsymmetrical arrangement of iron with respect to the compass.
(*d*) An error in the correct magnetic bearing used for finding the deviation.
(*e*) Retained magnetism when swinging the ship.

38. The object of compensating compasses is to reduce the deviations to the smallest possible amounts. An uncompensated compass having large deviations would be too "sensitive" on some courses, and too "sluggish" on others, so that its indications would not correspond to the actual changes of course; whilst a compass having its deviations reduced to a minimum by compensation would behave equally well on all courses.

39. When being equipped for sea, it is desirable that the ship's head should point the opposite way to the direction when building, the compensations being made just before sailing.

40. A common practice is to draw two lines on the deck where the binnacle stands—one fore and aft amidships, and the other at right angles, both passing through a point vertically under the centre of the compass; then, with the ship's head at correct magnetic N. or S., a bar magnet is laid on the deck at right angles to the fore-and-aft line, with its middle point on it and approached to the compass until the needle points correct magnetic N.

Next, the ship's head is steadied on correct magnetic E. or W., and another bar is laid on the deck at right angles to the athwartship line, with its middle point on that line, and moved to or from the compass until the needle again points correct magnetic N. Both magnets are then fixed in position.

Next, the ship's head is steadied on N.E., S.E., S.W., or N.W. correct magnetic for the purpose of correcting the quadrantal deviation. This is

done by placing two soft iron correctors on the supports attached to the binnacle, and moving them towards or from the compass until the deviation is compensated.

Lastly, the ship's head is brought to N. or S. by compass, and she is heeled over, say 10°, and the heeling error observed. Then a small bar magnet is placed vertically under the centre of the compass, and moved up or down in its groove until the error disappears.

N.B.—The red pole of the magnet must be uppermost when the deviation is towards the higher side, and *vice versâ*.

In Thompson's compass the magnets are inside the binnacle.

41. The magnets should not be nearer to the centre of the card than twice their own length, and the quadrantal correctors not nearer the needles than once and a quarter the length of the needles.

If the magnets are too near, they cause a new deviation called sextantal error, and if the correctors are too near they are liable to receive induced magnetic polarity from the needle, and attract either end indifferently.

42. No. They may be placed either above or below the compass, on a bulkhead, or in the binnacle itself, but parallel to the deck, provided the condition is observed that they are bisected at right angles by transverse and fore-and-aft planes passing through the centre of the compass.

43. Coefficient B is partly due to sub-permanent magnetism and partly to induced magnetism in vertical iron before or abaft the compass.

44. The compass would be found to be over-compensated with respect to B on going to a lower latitude, and on crossing the magnetic equator the part due to vertical induction would be reversed, which would necessitate the shifting or removal, or perhaps reversal, of the magnet.

45. One part should be compensated by a vertical soft iron bar amidships, either before or abaft the compass, as required, and the other by a bar magnet.

46. The ship's head being at E. or W., the deviation should be observed; then the Flinders bar should be moved into position before the compass, to correct, say, one-fourth of this deviation as a first approximation, the rest being compensated by a magnet. When on the magnetic equator the whole of coefficient B is due to sub-permanent magnetism, and should be exactly compensated by the fore-and-aft magnet. Then, as the ship proceeds N. or S., the change observed in the deviation due to coefficient B would be the amount to be compensated by the Flinders bar.

A more correct method would be, when direction of ship's head whilst building is known, to enter the Traverse Table with direction of ship's head when building as a course, and coefficient C in the dep. column, then B will be found in the latitude column. This would be the part of B to be compensated by the magnet, and the remainder by the Flinders bar.

47. Standard compasses *are* liable to be affected by the induced magnetism of vertical iron in the form of stanchions, pillars, funnels, or bulkheads near them. The attraction is not always towards the stern, because the disturbing masses of iron might be forward of the compass, in which case the Flinders bar should be abaft the compass.

48. Vertical iron could not cause any part of coefficient C, unless it was unsymmetrically placed with regard to the compass, therefore the whole of C, when the ship is upright, is usually due to sub-permanent magnetism.

49. Yes; because the horizontal force of the correctors, the disturbing force, and the directive force of the needle would vary in the same ratio.

50. When the joint effect is small, and when the constant deviation is W. when the ship is swung to the right, and E. when swung to the left, and consequently the effect of temporarily retained magnetism, they may be disregarded when compensating the compass.

51. (1) (*a*) It would be found to have decreased.

(*b*) It would increase again and tend to return to its original amount.

(2) (*a*) It would become 0.
(*b*) It would have a different sign.

52. No deviation would occur from the sub-permanent magnetism so long as it remained constant, because it is exactly counterbalanced by the correcting magnet; nor from the vertical induced magnetism, because it varies in the same ratio as that of the Flinders bar.

53. The deviation from this cause would not change its name and amount, because (*a*) the red induced pole is towards the north in all latitudes, and (*b*) the horizontal induced force changes in the same ratio as the directive force of the needle.

54. (1) A compass is set up on shore or in a boat at some distance from the ship, where it is not exposed to local attraction, and an observer stationed by it. The ship is then swung, and as her head comes to the desired points simultaneous bearings are taken by signal of each compass from the other. The bearings by shore compass reversed are correct magnetic, with which the bearings observed on board are compared, and the deviation found.

(2) Figures on dock walls show either the correct magnetic or the true bearings of a prominent distant object from the observer's position. When, therefore, the ship swings with the tide or is specially swung, the difference is noted between the figure on the wall in line with the object and the bearing of the same object by the compass on board, as the ship's head comes to each point on which the deviation is required. These differences are either "compass errors" or deviation, according as the figures on the wall are true or magnetic bearings.

(3) The correct magnetic bearings of the distant object may be found by means of a compass placed in a direct line with the ship and the object in a position free from local attraction. The deviation for the actual direction of the ship's head is the difference between the compass bearing of the object and the correct magnetic as before.

55. (*a*) By the sun: Take its bearing by compass and compute for the same time, or take from the Azimuth Tables its true azimuth. The difference is the compass error, from which the deviation may be found when the variation is known.

(*b*) By a star or planet: Find the hour angle of the body and compute, or take from the Azimuth Tables (when within their limits) its true azimuth, the difference between which and the observed bearing by compass at the time for which the calculation is made will be the compass error, whence the deviation as before.

56. Uses of Napier's diagram:—

(*a*) From known deviations on a few points, either correct magnetic or by compass, a curve showing the deviation on all points may be drawn.

(*b*) The curve so drawn may be used as a complete deviation table, from which may be found mechanically the compass course from the correct magnetic, and *vice versâ*. Its advantages are, that a complete curve of deviations may be drawn from the deviations observed on a few points, whether by compass or correct magnetic, and whether equidistant or not. It also gives a graphic record of the deviations which appeals to the eye, and gives a much better appreciation of the results of compensation than does an ordinary deviation table.

Again, if observations have been obtained of the deviations on two or three points of the quadrant in which the ship's course may lie, a curve will show the deviations on the other neighbouring points.

Further, the deviations obtained by drawing a curve are free from possible errors of observation and calculation.

57. The Napier's diagram consists of a vertical line graduated to points and degrees, with two sets of parallel lines drawn across it at angles of 60°, one set being continuous or *plain*, the other set broken or *dotted*, through

each of the thirty-two marks representing the "points" of the compass. The N. is at the top and bottom, the S. in the middle, and the E. and W. midway between the N. and S. marks, the other points following in order.

58. Eight equidistant points, viz. N., N.E., E., S.E., S., S.W., W., and N.W.

59. Two would be the least number, but three would be better, because a circle could be drawn through the three projected points.

Assuming that deviations had been observed on three compass courses in the quadrant in which my course would lie, I would project them on the dotted lines, and then draw a fair curve through the projected points, extending it to comprise the whole quadrant. The parts intercepted by the curve of the dotted lines through the other courses would give the required deviations, measured on the vertical line.

If only two observations were available, a straight line passing through the points of projection would show approximately the deviations on the intermediate courses.

N.B.—If the observations for deviation had been made on correct magnetic courses, the plain lines would be used instead of the dotted.

60. The mean of the equidistant bearings would be the correct magnetic bearing, nearly.

The object should be so far distant that its bearing by the compass on board would not be sensibly affected by the change of position in swinging the ship.

61. See page 208.

62. See page 322.

63. An observer being stationed at each compass, bearings are taken simultaneously of a distant object as the ship's head is steadied on seven or eight points correct magnetic (by Pelorus) round the compass. Comparing these bearings with the known correct magnetic, the deviations for each compass are found, and are then marked on the plain lines of a Napier's diagram. A curve is then drawn through each set of projected marks, and from these curves a complete table of deviations on compass courses may be constructed by measuring the parts of the dotted lines intercepted by the curve.

64. If the correct magnetic bearing reads to the *right* of the compass bearing, the deviation is easterly; if to the *left*, westerly.

65. A knowledge of the coefficients is of advantage, because—

(1) From the coefficients a complete table of deviations can be constructed.

(2) It shows how much of the deviation is due to each of the disturbing forces in a ship.

(3) It enables a compass adjuster to employ the proper means for compensating.

66. (*a*) "Retentive" or "retained" magnetism is the name given to the temporary magnetic polarity acquired by a ship whilst heading for a considerable time in one direction, and retained for a longer or shorter period after that direction is changed. It is induced by the Earth's magnetism whilst the ship is in dock or on a long-continued course at sea.

(*b*) It has no effect on the compass needle whilst the ship's head continues on the same course.

(*c*) A deviation will appear tending to make the ship follow the last course steered.

(*d*) Allowance should be made for this effect, and an observation for deviation should be made as soon as possible after the course is changed.

67. The "Pelorus" has the general appearance of an azimuth compass, but it has no magnets. The "card" is usually a metal disc, which can be turned about its centre and clamped in any position.

(*a*) Its use in compensating: First clamp the sight vanes to the known

correct magnetic bearing of a distant object; then turn the card round until the desired direction of ship's head is at the lubber-point, and clamp. Next swing the ship until the object is seen through the sight vanes, and place the magnet or corrector in position to effect the compensation.

(*b*) Its use to find the deviation: Clamp the sight vane to the known correct magnetic bearing of a distant object on land, or a celestial object, and turn the card round until the object is seen through the sight vanes. The course by "Pelorus" is the correct magnetic, the difference between which and the course by compass is the deviation for that course.

68. Deviation is the *difference* between the correction and variation when they are of the same name, but the *sum* if of different names. It has the same name or sign as the correction in all cases except when the correction is less than variation of the same name.

69. See page 177.

70. See page 325.

71. When on the magnetic equator, sea smooth, and ship upright, set a watch to show apparent time at ship. Bring the ship's head to near north (or south), and clamp the sight vanes of the Pelorus to the correct magnetic bearing of the Sun—found by taking out the true bearing from the tables for some minutes in advance, and applying the variation; also bring the (N. or S.) point of the Pelorus to the lubber-point. Then steer the ship so that the centre of the Sun's image is on the sight vanes at the moment for which the bearing was computed, and steady her on that course whilst the athwartship magnet is moved into position to make the compass indicate correct magnetic.

In a similar manner place and keep ship's head on correct magnetic east or west, and adjust the position of the fore-and-aft magnet.

72. See page 180.

73. Yes; a deviation appears which is known as the heeling error.

74. (1) When the ship heels, the vertical component of sub-permanent magnetism takes effect in producing deviation, the upper pole not being vertically under the centre of the compass.

(2) Iron which is horizontal when the ship is upright acquires a vertical component of induced magnetism.

(3) The upper ends of rudder post, stern post, or other vertical iron before or abaft the binnacle, are brought out to one side of a vertical plane passing through the centre of the compass.

75. (*a*) Towards the higher side.
(*b*) Towards the lower side.

76. (*a*) Towards the lower side.
(*b*) Towards the higher side.

77. Such iron would cause a heeling error.
(*a*) Towards the higher side.
(*b*) Towards the lower side.

78. Coefficient C.

79. In the northern hemisphere, the N. point of the compass is generally drawn to windward in ships built head north, also in ships whose compasses have +D, and in others which have large masses of vertical iron below the compass.

80. In some ships built head south, and ships having a large −D.

81. In the majority of cases the N. point of the compass is drawn to windward. If not allowed for, the effect would be to draw the ship to windward on northerly courses, and to leeward on southerly.

82. (*a*) In N. latitude keep away by compass on either tack as the ship heels on northerly courses, and keep closer to the wind on either tack on southerly courses.

(*b*) In N. latitude keep closer to the wind by compass on either tack on northerly courses, and keep away on southerly courses.

This holds good in S. latitude.

83. If in tacking the ship's head is northerly on one tack, and southerly on the other, the heeling error retains the same name; but if on both tacks her head is either northerly or southerly, the error changes its name.

84. Yes.

85. The heeling error has its maximum value when heading north or south magnetic, and nearly vanishes at east and west.

86. It diminishes when the ship goes to a lower latitude, and increases when to a higher, because the directive force of the needle increases in the first case, and decreases in the second, whilst the vertical sub-permanent force is assumed to be constant.

87. It decreases on approaching the equator, and *vice versâ*, becomes 0 on the magnetic equator, and changes its sign after crossing; because the vertical force of induced magnetism decreases to 0 at the equator, and increases with increase of latitude, the uppermost poles being blue in north magnetic latitude, and red in south. The directive force of the needle also varies with change of latitude.

88.

(1) High N. latitude.	(2) Magnetic equator.	(3) High S. latitude.
(*a*) Nil.	To low side, due to sub-permanent only.	Sum to low side.
(*b*) Difference to low side.	" "	" "
(*c*) " high side.	" "	" "
(*d*) Sum to "	To high side, due to sub-permanent only.	Nil.
(*e*) " "	" "	Difference to high side.
(*f*) " "	" "	" low "

89. Yes; for a given magnetic latitude, by means of a vertical magnet which can be moved up or down in a groove or tube. There are two ways of proceeding—

(1) Bring the ship's head to N. or S. by compass (assumed to be correct) and give her a list of, say, 10°; then slide the magnet up or down until the error disappears, and fix in position.

(2) Bring the ship's head to E. or W. magnetic, ship upright; remove the card and place on the supporting pillar a small dipping needle, which has been balanced by a sliding weight so as to show no dip when on shore; then place the magnet in its groove and move up or down until the needle is again perfectly level.

90. No; because the heeling error is partly due to vertical induced magnetism, which varies with change of latitude.

91. The soft iron correctors are found to correct the heeling error due to transverse iron for ordinary angles of heeling. (*a*) In the N. hemisphere they deflect the N. point of the needle to the low side; (*b*) in the S. hemisphere, towards the high side. The reason is that they produce a co-efficient C due to an induced vertical force in the correctors, which has a different sign from that of the C caused by transverse iron when the ship heels.

92. See page 326.

93. A clinometer consists of a graduated semicircle, which is fixed on a bulkhead with its diameter horizontal and arc downwards. The zero of the arc should coincide with a plumb-line from the middle of the diameter, so that when the ship heels the plumb-line or weighted index would show on the arc the exact degrees of inclination.

94. Yes; otherwise the deviations would be complicated with the heeling errors.

95. The deviations might be expected to change from several causes—

(1) The place where the ship is swung may have local attraction.

(2) The force of sub-permanent magnetism is liable to change from long continuance on one course, and from concussion and vibration.

(3) The compensation of coefficient B can only be regarded as provisional, there being no certainty that the just proportions were assigned to the magnet and the Flinders bar.

(4) Allowance must be made for a want of scientific knowledge on the part of the adjuster, or a careless performance of his work.

(5) Loss of power in needles or magnets.

96. It is most necessary that such a record should be kept. The following form would serve the purpose :—

Date.	Lat.	Long.	Var.	Ship's head.		Deviation.		Remarks.
				Standard compass.	Steering compass.	Standard compass.	Steering compass.	

97. No, because considerable changes in the deviation may occur. Therefore it is necessary that frequent observations for deviation may be made, especially when the course is altered, or when rapidly changing latitude.

PRACTICAL QUESTIONS IN SYLLABUS OF DEVIATION FOR EXTRA MASTERS.

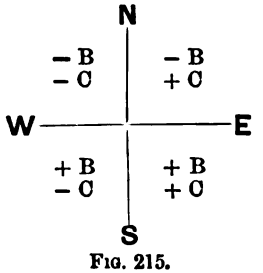


FIG. 215.

31. The values of B and C being given to determine the magnetic direction of ship's head whilst building, supposing coefficients due entirely to sub-permanent magnetism.

According to the direction of ship's head whilst building, the signs of B and C can be easily remembered from the adjoining figure.

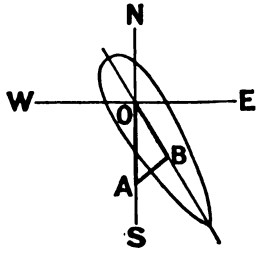


FIG. 216.

Again, let a ship be built with her head in, say, the S.E. quadrant, OB being fore-and-aft line. Then if OA, in magnetic meridian, be taken as representing the horizontal component of the total magnetic force in the ship, it can be resolved into two forces at right angles, OB in a fore-and-aft direction, and BA in an athwartship direction.

OB represents coefficient B.
BA ,, ,, C.
AOB ,, magnetic direction ship's head building.

$$\therefore \cot AOB = \frac{OB}{AB}$$

$$\text{or cot direction} = \frac{B}{C}$$

Hence in the Traverse Table, if the direction of ship's head whilst being built be taken as a *course*, the latitude and departure columns will give the relative values of B and C. Therefore—

(1) With ship's head as a course and B in latitude column, C is found in departure column.

(2) Ship's head as course and C as departure give B in latitude column.

(3) B in latitude and C in departure columns give the direction of ship's head from N. or S.

Examples.—

1. Given the direction of ship's head whilst building N.E. by N., and coefficient B = −16°·5. Find coefficient C.

Here the course 3 points and lat. 16·5, give dep. 11·1. Coefficient C is therefore +11°·1, because the ship's head was easterly.

2. Given ship's head S.S.W., and coefficient C = −3°·6. Required coefficient B.

Course 2 points and dep. 3·6 give lat. 8·7; therefore coefficient B = +8°·7, because the ship's head was southerly.

3. Given coefficient B = +7°·3, and coefficient C = −10°·7. Required the direction of the ship's head when building.

Lat. 7°·3 and dep. 10·7 give the course 56°, which is S. 56° W., because B is + and C is −.

Exercises.—

1. Given coefficient B = −11° and C = −8°·5. Find direction of ship's head whilst being built.

2. Given coefficient +C = 12°·5, and −B = 6°·8. Required the direction of ship's head.

3. Given +B = 14°·7 and +C = 8°·7. Required ship's head.

4. Given ship's head N. 65° W., and −B = 5°·4. Required coefficient C.

5. Given ship's head S. 14° W., and −C = 3°·5. Required coefficient B.

6. Given ship's head S.E. ¼ E., and coefficient C = +7° 5. Required coefficient B.

62. **Example.**—Assuming the deviations observed with the ship's head by compass to be as follows, determine the value of the coefficients A, B, C, D, and E, and from them construct a table of deviations.

Deviation at N.	(21° W.)	S.	(20° E.)
N.E.	(10 W.)	S.W.	(23 E.)
E.	(2 W.)	W.	(5 E.)
S.E.	(4 E.)	N.W.	(17 W.)

NOTE.—In computing the coefficients, E. deviation is marked +, and W. deviation −.

To find coefficient A.

Add (algebraically) the deviations on N., E., S., and W., and divide by 4.

Deviation on N.	−21°
E.	− 2
S.	+20
W.	+ 5
	4)+ 2
	+ ½° = A, *i.e.* +A = ½°

To find coefficient B.

Add (algebraically) the deviation on E. and W., after reversing the sign of the latter, and divide by 2.

Deviation on E. $-2°$
W. -5 (reversed)

$2)-7$

$-3\frac{1}{2}°$ *i.e.* $-B = 3\frac{1}{2}°$

To find coefficient C.

Add (algebraically) the deviations on N. and S., after reversing the sign of the latter.

Deviation on N. $-21°$
S. -20 (reversed)

$2)-41$

$-20\frac{1}{2}°$ *i.e.* $-C = 20\frac{1}{2}°$

To find coefficient D.

Add (algebraically) the deviations on N.E., S.W., S.E., and N.W., after reversing the signs of the two latter, and divide by 4.

Deviation on N.E. $-10°$
S.W. $+23$
S.E. -4 } (reversed)
N.W. $+17$ }

$4)+26$

$+6° 30'$ *i.e.* $+D = 6\frac{1}{2}°$

To find coefficient E.

Add (algebraically) the deviations on N., S., E., and W., after reversing the signs of the two latter, and divide by 4.

Deviation on N. $-21°$
S. $+20$
E. $+2$ } (reversed)
W. -5 }

$4)-4$

$-1°$ *i.e.* $-E = 1°$

The formula for obtaining the deviation for any point from the above coefficients is, where d = deviation, and Z = azimuth or direction of ship's head by compass—

$$d = A + B \cdot \sin Z + C \cdot \cos Z + D \cdot \sin 2Z + E \cdot \cos 2Z$$

Here A represents an amount constant in value and sign.

Here $B \cdot \sin Z + C \cdot \cos Z$ represents an amount depending on the azimuth for value and sign, and giving semicircular deviation.

Here $D \cdot \sin 2Z + E \cdot \cos 2Z$ represents an amount depending on twice the azimuth for value and sign, and giving quadrantal deviation.

In the following table the first column represents all the points from N. to N.

Second column. Coefficient A, and is the same for every point.

Third column. All the values of $B \cdot \sin Z$ that can be obtained either by multiplying the value of coefficient B by the natural sine of azimuth; *or* from

the Traverse Table, by taking the azimuth as "course," coefficient B as "dist.," and finding corresponding values in the "dep." column.

Fourth column. All the values of C . cos Z that can be obtained either by multiplying coefficient C by the natural cosine of azimuth; *or* from the Traverse Table, by taking azimuth as "course," coefficient C as "dist.," and finding corresponding values in "d. lat." column.

Fifth column. All the values of D . sin 2Z that can be obtained either by multiplying coefficient D by the natural sine of twice the azimuth; *or* from Traverse Table, by taking twice the azimuth as "course," coefficient D in "dist." column, and finding corresponding values in "dep." column.

Sixth column. All the values of E . cos 2Z that can be obtained either by multiplying coefficient E by the natural cosine of twice the azimuth; *or* from Traverse Table, by taking twice the azimuth as "course," coefficient E in "dist." column, and finding corresponding values in "d. lat." column.

Seventh column contains the amount obtained by adding across columns 2 to 6 algebraically.

In forming the table, use the coefficients expressed as degrees and decimals of a degree, and all entries to be made in the same manner from the Traverse Table. It is only necessary to convert the sums found in the seventh column to degrees and miles. Begin by inserting in each column the maximum and minimum values against the proper directions of the ship's head; when a deviation passes through "0" it changes in name.

Ship's head by compass.	Coefficient A = +0°·5.	Coefficient B = −3°·5.	Coefficient C = −20°·5.	Coefficient D = +6°·5.	Coefficient E = −1°.	Total deviation.
	°	°	°	°	°	° ′
North	+ ·5	0	− 20·50	0	− 1	− 21 0
N. by E.	+ ·5	− 0·68	− 20·11	+ 2·49	− 0·92	− 18 43
N.N.E.	+ ·5	− 1·34	− 18·94	+ 4·60	− 0·71	− 15 53
N.E. by N.	+ ·5	− 1·94	− 17·04	+ 6·01	− 0·38	− 12 51
N.E.	+ ·5	− 2·47	− 14·50	+ 6·50	0	− 9 58
N.E. by E.	+ ·5	− 2·91	− 11·39	+ 6·01	+ 0·38	− 7 25
E.N.E.	+ ·5	− 3·23	− 7·85	+ 4·60	+ 0·71	− 5 16
E. by N.	+ ·5	− 3·43	− 4·00	+ 2·49	+ 0·92	− 3 31
East	+ ·5	− 3·50	0	0	+ 1	− 2 0
E. by S.	+ ·5	− 3·43	+ 4·00	− 2·49	+ 0·92	− 0 30
E.S.E.	+ ·5	− 3·23	+ 7·85	− 4·60	+ 0 71	+ 1 14
S.E. by E.	+ ·5	− 2·91	+ 11·39	− 6·01	+ 0·38	+ 3 21
S.E.	+ ·5	− 2·47	+ 14·50	− 6·50	0	+ 6 2
S.E. by S.	+ ·5	− 1·94	+ 17·04	− 6·01	− 0·38	+ 9 13
S.S.E.	+ ·5	− 1·34	+ 18·94	− 4·60	− 0·71	+ 12 47
S. by E.	+ ·5	− 0·68	+ 20·11	− 2·49	− 0·92	+ 16 31
South	+ ·5	0	+ 20·50	0	− 1	+ 20 0
S. by W.	+ ·5	+ 0·68	+ 20·11	+ 2·49	− 0·92	+ 22 52
S.S.W.	+ ·5	+ 1·34	+ 18·94	+ 4·60	− 0·71	+ 24 40
S.W. by S.	+ ·5	+ 1·94	+ 17·04	+ 6·01	− 0·38	+ 25 7
S.W.	+ ·5	+ 2·47	+ 14·50	+ 6·50	0	+ 23 58
S.W. by W.	+ ·5	+ 2·91	+ 11·39	+ 6·01	+ 0·38	+ 21 11
W.S.W.	+ ·5	+ 3·23	+ 7·85	+ 4·60	+ 0·71	+ 16 53
W. by S.	+ ·5	+ 3·43	+ 4·00	+ 2·49	+ 0·92	+ 11 20
West	+ ·5	+ 3·50	0	0	+ 1	+ 5 0
W. by N.	+ ·5	+ 3·43	− 4·00	− 2·49	+ 0·92	− 1 38
W.N.W.	+ ·5	+ 3·23	− 7·85	− 4·60	+ 0·71	− 8 1
N.W. by W.	+ ·5	+ 2·91	− 11·39	− 6·01	+ 0·38	− 13 37
N.W.	+ ·5	+ 2·47	− 14·50	− 6·50	0	− 18 2
N.W. by N.	+ ·5	+ 1·94	− 17·04	− 6·01	− 0·38	− 21 0
N.N.W.	+ ·5	+ 1·34	− 18·94	− 4·60	− 0·71	− 22 25
N. by W.	+ ·5	+ 0·68	− 20·11	− 2·49	− 0·92	− 22 20

Exercises.—

1. From the following deviations on ship's head by compass, find the coefficients A, B, C, D, E, and construct a table of deviations from W. by N. to N.E. by E. :—

Deviation at N.	(29½° W.)	S. (27½° E.)
N.E.	(26½ W.)	S.W. (17½ E.)
E.	(6½ E.)	W. (4½ W.)
S.E.	(30½ E.)	N.W. (21½ W.)

2. From deviations found on ship's head by compass as follows, find the coefficients A, B, C, D, E, and construct a table of deviations from E. to W. through S. :—

Deviation at N.	(22° E.)	S. (25° W.)
N.E.	(15 E.)	S.W. (18 W.)
E.	(0)	W. (0)
S.E.	(15 W.)	N.W. (20 E.)

3. From the following deviations on ship's head by compass, find the coefficients A, B, C, D, E, and make a table of deviations for all points from E.N.E. to S. :—

Deviation at N.	(26° E.)	S. (26° W.)
N.E.	(14 E.)	S.W. (15 W.)
E.	(16 W.)	W. (9 E.)
S.E.	(23 W.)	N.W. (25 E.)

4. Given the following deviations on ship's head by compass, required the coefficients A, B, C, D, E, and make a table of deviations for all points from N. through W. to S. :—

Deviation at N.	(13° W.)	S. (18° E.)
N.E.	(18 W.)	S.W. (15 E.)
E.	(5 W.)	W. (4 E.)
S.E.	(5 E.)	N.W. (8 W.)

5. Given the following deviations on ship's head correct magnetic, construct a Napier's curve, find the coefficients B, C, D, and make a table of deviations for all points from N.N.W. to S.W. by S. :—

Deviation at N. by E. (mag.)	(6° E.)	S.W. by S.	(15° W.)
E.N.E.	(15 E.)	W. by S.	(22 W.)
S.E. by E.	(20 E.)	N.W. by W.	(16 W.)
S.S.E.	(14 E.)	N.N.W.	(5 W.)

70. Remark.—In the short time available at an examination, it would be troublesome to find, by direct calculation, what stars would satisfy the conditions as to altitude and hour angle to render them suitable for azimuths. Therefore it is better to make use of tables, where the work is already done.

Table IV. in "Towson's Deviation" will answer the purpose, the sidereal time and latitude being known.

Example.—

August 10th, at $11^h\ 30^m$ P.M. mean time at ship, in lat. 50° 50′ N., long. 62° W., what stars would be in a good position for determining the correction of the compass?

To find the sidereal time—

M.T.S. Aug. 10^d	$11^h\ 30^m$	Long. 62° W. =	$4^h\ 8^m$
	4 8		
M.T.G.	10 15 38		

Sid. time at noon	9h	12m	13s
Acceleration (15h)		2	28
(38m)			6
M.T.S.	11	30	0
Sid. time of observation	20	44	47

Entering Table IV. (Towson) with the nearest hour of sidereal time and the nearest latitude, the suitable stars are found to be—

East of meridian		2	
West	,,	7	8 9

Referring to map (p. 44, Towson), these numbers correspond to—

Capella (east of meridian).
α Urs. Maj., γ Urs. Maj., Arcturus (west of meridian).

In Fig. 217 NESW represents plane of horizon, NZS meridian, EQW equinoctial; A and B are circles of altitude drawn parallel to horizon, and distance 30° and 10° from it. Stars given in Towson which are suitable fall in the space between these circles of altitude. Set off the declination parallels of the stars 2, 7, and 9; and where the hour circles cut these parallels, give the positions of the stars as shown at 2, 7, and 9.

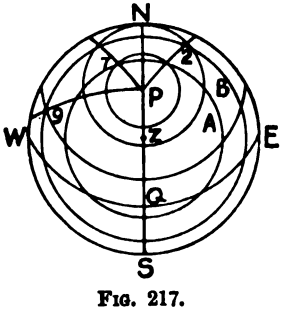


FIG. 217.

Hour angle of 2 Capella 9h 23m E.
,, ,, 7 Dubhe 9h 49m W.
,, ,, 9 Arcturus 6h 34m W.

Exercises.—

1. August 15th, 2h 30m A.M. at ship, in lat. 38° S., long. 25° E. What stars would be suitable for azimuths?

2. August 20th, 1h 16m A.M. apparent time at ship, in lat. 42° 30′ N., long. 130° W.

3. August 1st, 2h 15m A.M. mean time at ship, in lat. 21 S., long. 93° 30′ E. Give the names of suitable stars for determining the correction of the compass.

4. September 20th, 11h 48m P.M. mean time at ship, in lat. 49° S., long. 168° 15′ W. Name some stars which would be suitable for finding the correction of the compass.

5. September 23rd, 11h 58m P.M. mean time at ship, in lat. 35° N., long. 44° 36′ W. Give the names of stars suitable for finding the correction of the compass.

6. September 10th, 1h 45m P.M. apparent time at ship, in lat. 55° S., long. 135° W. Find what stars are suitable for azimuths.

92. **Example.**—Given the angle of heeling 15°, the course N.N.E. on the port tack, and the heeling error −12°, required the error when steering W.S.W. on the starboard tack and heeling 12°.

Since the amount of the heeling error varies directly as the heel, draw two concentric circles, whose radii CA and CB are in the ratio of the angles of heel, *i.e.* CA : CB :: 15 : 12,

$$\text{or } CB = \tfrac{12}{15}\, CA.$$

Draw CP, making with CA an angle equal to first direction of ship's head

(N.E.); and CX (W.S.W.) second direction of ship's head, X being on its own circle. Draw PQ and XY perpendiculars on AC. Then CQ represents the first heeling error 12°, and CY represents the second heeling error.

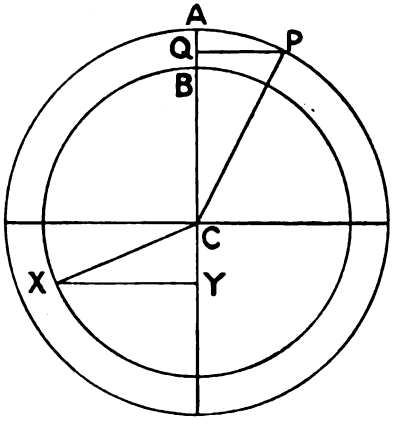


FIG. 218.

$$\begin{aligned}\text{Then CY} &= \text{CX} \cdot \cos \text{XCY} \\ &= \text{CB} \cdot \cos \text{XCY} \\ &= \tfrac{12}{15} \cdot \text{CA} \cdot \cos \text{XCY} \\ &= \tfrac{12}{15} \cdot \text{CP} \cdot \cos \text{XCY} \\ &= \tfrac{12}{15} \cdot \text{CQ} \cdot \text{Sec PCQ} \cdot \cos \text{XCY} \\ &= \frac{12 \cdot \cos \text{XCY} \cdot \text{CQ}}{15 \cdot \cos \text{PCQ}}\end{aligned}$$

This is expressed in a formula by—

$$\text{New error} = \frac{\text{new heel} \times \cos \text{new co.} \times \text{old error}}{\text{old heel} \times \cos \text{old co.}}$$

$$= \frac{{}^{\cdot 8}\!\not{12} \times \cos 6 \text{ pts.} \times 12}{\not{15} \times \cos 2 \text{ pts.}}$$

$$= 9{\cdot}6 \times \cos 6 \text{ pts} \times \sec 2 \text{ pts.}$$

apply logs.

9·6	log =	0·982271
6 pts.	L. cos =	9·582840
2 pts.	L. sec =	10·034385
3·977	log =	0·599496

The name changes with a change of tack, and also when the course is changed from the northern to the southern semicircle, or *vice versâ*.

A change of either changes the name.

A change in both does not change the name.

Here we have a change in hemisphere and tack, consequently the name remains the same (−).

$$\text{New error} = -4°$$

EXERCISES.

1. Given 20° heel on the starboard tack, steering N.W. by N., and observed heeling error +15°, required the error when steering W. by S. on the same tack, and heeling 17°.

2. Given 12° heel on the port tack, steering S.W. ½ W., and observed heeling error +9° 30′, required the error when steering S.W. by W. ¾ W. on the starboard tack, and heeling 14°.

3. Given 25° heel on the starboard tack, steering E. by N. ½ N., and observed heeling error +5° 30′, required the error when steering S. by W. ¼ W. on the port tack, and heeling 16°.

4. A ship steering S.E. ½ E., with the wind at S.S.W., and heeling 9°, the observed heeling error was +4°; the wind changes to W.N.W. in a squall, and the ship now lays S.W. ½ S., heeling 19°. Required the new heeling error.

5. Course N., wind E.N.E., heel 13°, and error observed +18°. Required the error when the ship is put on the other tack by change of wind, laying N.N.E., with a list of 20°.

6. Steering S.W. ¾ W., wind S.S.E., heel 12°, and observed heeling error −6°. Required the error when the wind changes to S.W., and the ship lays N.W. by W. ½ W., heeling 15°.

7. Steering N.W. ¾ N., heeling 12°, wind W.S.W., and error observed −7° 12′. Required the error when the course is changed to S., and heel 15°, with no change of wind.

8. Steering S. by E. ½ E., heeling 20° to starboard, heeling error observed −13° 30′. Required the error when steering E. on the same tack and heeling 18°.

9. Course N., wind W., the error observed when heeling 14° was 12° 30′. Required the error when on the other tack steering S. and heeling 16°.

10. Given heel 8½°, course N.W. by W. on the starboard tack, heeling error observed −2°, required the error when the wind veers so as to allow the ship to lay N. by W. on the same tack, heeling 8½° by clinometer.

11. Steering E. by S. ¾ S. on the port tack, and heeling 19°, the observed heeling error was +4° 40′. Required the error when steering S.S.W. on the same tack, the clinometer showing 13½°.

12. Heeling 23° when steering E. by N. ¼ N., and wind at N. by E., the heeling error observed was +4½°. Required the error when steering N.W. by W. ¾ W. on the starboard tack and heeling 17½°.

DESCRIPTION AND USE OF BEALL'S DEVIASCOPE.

This is a model representing the after-deck of a ship, with its binnacle and steering compass. A soft-iron bar at right angles to the deck represents the stern-post, and two soft-iron bars under the deck, the athwartship beams. These bars are to give the vertical and horizontal iron effects. To imitate the effects of sub-permanent magnetism in a ship, thin steel magnets are placed in grooves in the deck, which run fore and aft, athwartship, and at intermediate angles. By this means a counterpart of a ship built with her head in any given direction may be obtained. A "Pelorus" stands on the deck. The model has horizontal and heeling motions, with clamping screws, and a clinometer on the binnacle for showing the angle of heeling.

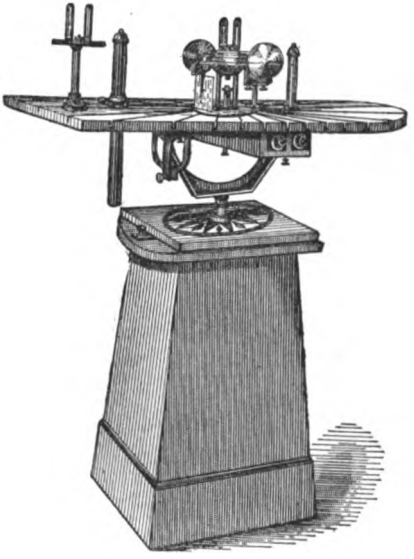
On the sides of the binnacle are jointed brass supports for the spherical soft-iron correctors. Under the binnacle is a tube for holding a small brass case, having holes for the reception of small round magnets for the compensation of heeling error. Other flat magnets are provided for compensating coefficients B and C, and a "Flinders" bar and dipping needle complete the outfit.

To explain, by the aid of the Deviascope, the ordinary "tentative" method of compensating a ship's compass—

The deck being horizontal by clinometer, the iron bars representing the stern-post and beams in position, and the magnets in one of the grooves, the sight vanes of the Pelorus should be clamped to the assumed correct magnetic bearing of an object as far removed as possible.

1. To compensate coefficient C.

Turn the Pelorus card round until the N. or S. point is at the lubber line, then turn the model until the object is seen through the sight vanes. This represents ship's head N. or S. correct magnetic. Now take up one of



BEALL'S DEVIASCOPE.

the compensating magnets and place it on the deck before or abaft the binnacle, at right angles to the midship groove, and move it towards or from the compass until the deviation disappears. Of course, the red end of the magnet must be towards the side which attracts the N. point of the compass.

2. To compensate coefficient B.

Bring the E. or W. point of the Pelorus card to the lubber line, and turn the model until the sight vanes are again directed to the object. Now place the "Flinders" bar on the fore side of the compass (because the vertical iron is abaft), and move it nearer until a part of the deviation is corrected (say one-half for the purpose of illustration); then place a magnet on the deck on either side of the binnacle at right angles to the athwartship groove, and move it towards or from the compass until the balance of the deviation from coefficient B is corrected.

3. To compensate coefficient D.

Turn the Pelorus card until the N.E. point is at the lubber line, and then turn the model until the object is seen through the sight vanes. If the deviation is E. on this course, it shows a coefficient +D ; but if W., a −D. The soft-iron correctors are then placed in position on their supports, and brought near enough to the compass to correct the deviation. For +D the correctors must be at the sides, and for −D they are moved round to the fore-and-aft line.

The joints in the brass supporting arms are to allow the correctors to be moved a little forward on one side, and aft on the other, for compensating coefficient E.

4. To compensate the vertical force producing heeling error.

(1) By heeling the model and observing the heeling error—

Bring the model's bow to N. or S. by compass (which will now show correct magnetic), and incline the deck (say 10° by clinometer), and observe the deviation. If the N. point is drawn to the higher side (which is usually the case), place the small vertical magnet underneath, with the red pole up, and move it up or down until the deviation is corrected; but if the N. point goes towards the lower side, the blue pole of the magnet must be uppermost.

(2) By dipping needle—

Bring the model's head to E. or W. magnetic, and, after removing the compass card and pivot, place the dipping needle[1] on the pillar in the bowl, with its red pole pointing N. If the needle dips, it shows a blue or +force below the compass ; but if it rises, it shows the force is red or −. In the former case the red pole of the magnet must be uppermost, and in the latter the blue pole. It should then be moved up or down until the needle is horizontal.

---

[1] The dipping needle is weighted to counterbalance the natural dip, so that it is horizontal when not on the model.

CHAPTER XXX.

MISCELLANEOUS PROBLEMS.

ART. 130. **Freights and Commissions.**—Freight is money paid to the owner of a ship for the carriage and safe delivery of goods, and its gross amount is reckoned at so much per ton. The gross freight is subject to additions and deductions, which are calculated at so much per cent., or per £100; and when made the net freight is the result. Primage is a charge paid by the shipper, over and above the freight, to the owner towards expenses of wharfage, pilotage, light dues, tug expenses, and other port charges. Commission and Brokerage are charges paid by the owner for expeditious loading of cargo, and for obtaining the contract.

Example.—Find the net freight on 5500 tons at 45 francs per ton. Brokerage $2\frac{1}{2}$ %; primage 6 %; exchange, 1 franc = $9\frac{3}{4}d$.

Gross freight = 5500×45 = 247,500 francs.

Brokerage and primage $(6-2\frac{1}{2})\% = 3\frac{1}{2}\% = \frac{247,500 \times 7}{2 \times 100}$ = 8662·5 ,,

Net freight = 256,162·5 ,,

= 2,497,584·375d.

= £10,406 12s.

Since the brokerage is a deduction from, and the primage an addition to the gross freight, the difference between them, $6 - 2\frac{1}{2} = 3\frac{1}{2}\%$, must be added to obtain the net amount.

ART. 131. **Areas, Volumes, Weights.**—The area of any surface is expressed in square measure, which in a *rectangle* is the product of the length and breadth, both dimensions being expressed in the same unit—feet, inches, etc.—when the area is expressed as square feet or square inches, etc.

For a *triangle* the area is half the product of base and the perpendicular height.

For a *trapezoid*, that is a four-sided figure having two sides parallel, the area is half the sum of the parallel sides multiplied by the perpendicular distance between them.

The circumference of a circle is given by πd.

The area of a *circle* by $\frac{\pi d^2}{4}$

where $\pi = 3{\cdot}1416$.

For the area of an irregular figure bounded on one side by a curve, such as the half-section of a boat or ship, Simpson's rules are employed; or by multiplying the area of the circumscribing rectangle by a constant called the coefficient of fineness of the plane. This constant varies in value from ·7 to ·85.

Examples.—

1. The circumference of an engine-room ventilator is 13 ft. 6 in., find its diameter and sectional area.

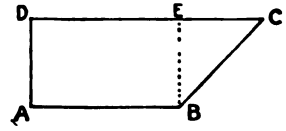
$$\text{Circ.} = 13{\cdot}5 \text{ ft.} \qquad \text{Area} = \frac{\pi d^2}{4}$$

$$\pi d = 13{\cdot}5 \qquad = \frac{3{\cdot}1416 \times (4{\cdot}3)^2}{4} \text{ sq. ft.}$$

$$\therefore d = \frac{13{\cdot}5}{3{\cdot}1416} \text{ ft.}$$

$$= 4{\cdot}3 \text{ ft.} \qquad = 14{\cdot}52 \text{ sq. ft.}$$

2. ABCD is a quadrilateral, AB is 20 ft., AD at right angles to AB is 12 ft., DC parallel to AB is 30 ft. Find its area.



$$\text{Area} = \tfrac{1}{2}(AB + CD) \cdot AD$$
$$= \tfrac{1}{2}(30 + 20) \cdot 12$$
$$= 300 \text{ sq. ft.}$$

Draw BE perpr. on CD.

Then area ABCD = area ABED + area BCE

$$= AB \cdot BE + \tfrac{1}{2} BE \cdot EC$$
$$= 20 \times 12 + \tfrac{1}{2} \cdot 12 \times 10 = 240 + 60 = 300 \text{ sq. ft.}$$

The volume of a rectangular block is the product of its length, breadth, and thickness or depth, and is expressed in cubic feet or cubic inches as case may be according to the unit used.

$$L \times B \times D = \text{Vol., and since } L \times B = \text{area}$$
$$\therefore \text{area} \times D = \text{Vol.,}$$
$$\text{and } D = \frac{\text{Vol.}}{\text{area}}, \text{ or area} = \frac{\text{Vol.}}{D}$$

The ratio of the volumes of displacement and of a block having same length, breadth, and draught as a vessel is called its block coefficient of fineness.

The volume of a cylinder or pipe is its sectional area multiplied by its length $= \frac{\pi d^2 . l}{4}$

The volume of a sphere $= \frac{\pi d^3}{6}$

When the volume of a solid is ascertained, its weight may be found from its specific gravity (S.G.), or in the case of a floating

body by finding the number of cubic feet of water displaced, allowing 35 cubic feet of salt water to weigh one ton. A cubic foot of fresh water weighs 1000 ounces, and contains $6\frac{1}{4}$ gallons approx. Hence taking the S.G. of cast iron as 7·207, the weight of a cubic foot would be 7·207 × 1000 ounces = 7207 ounces.

Examples.—

1. The displacement of a ship of 15,300 tons. Her dimensions are 462 ft. long, 45 ft. beam, and 32 ft. draught: find the coefficient of fineness.

$$\text{Coefft.} = \frac{\text{Displacement}}{\text{Contents of block}}$$

$$= \frac{\overset{1700}{\cancel{15,300}} \times \overset{1}{\cancel{35}}}{\underset{66}{\cancel{462}} \times \underset{9}{\cancel{45}} \times 32}$$

$$= \cdot 8.$$

2. The area of the water plane of a vessel is 21,000 sq. ft.; what are the tons per inch immersion at that plane, and how much will 300 tons of bunkers sink her?

When she sinks 1 in. or $\frac{1}{12}$ ft.

$$\text{The vol. displaced} = \frac{21,000}{12} \text{ cubic ft.}$$

$$\text{hence tons} = \frac{21,000}{12 \times 35} = 50$$

$$\text{or ton per inch immersion} = 50$$

$$\text{and 300 tons will put her down } \frac{300}{50} \text{ ins.}$$

$$= 6 \text{ ins.}$$

3. Allowing 45 cubic ft. of coal to a ton, how long must a bunker be made to contain 250 tons bunkers, beam ship 51 ft., height 'tween decks 6 ft. 6 in.?

$$\text{Coal will occupy } 45 \times 250 \text{ cubic ft.}$$

$$\text{Area end of bunker } 51 \times 6\tfrac{1}{2} \text{ sq. ft.}$$

$$\text{Hence length} = \frac{\text{volume}}{\text{area}}$$

$$= \frac{45 \times 250}{51 \times 6 \cdot 5} \text{ ft.}$$

$$= 33 \cdot 94 \text{ ft.}$$

4. Find the weight of a balk of timber 31 ft. long, 30 ins. wide, and 20 ins. deep, S.G. being ·72.

$$\text{Volume} = 31 \times 2\tfrac{1}{2} \times 1\tfrac{2}{3} \text{ cubic ft.}$$

$$\text{Weight of 1 c. ft.} = \cdot 72 \times 1000 = 720 \text{ oz.}$$

$$= 45 \text{ lbs.}$$

$$\text{Hence weight of log} = \frac{31 \times 5 \times 5 \times 45}{2 \times 3} \text{ lbs.}$$

$$= 5812 \cdot 5 \text{ lbs.}$$

5. How many gallons of oil will a cylindrical tank 4 ft. in diameter and 6 ft. long hold? What weight will the oil be, S.G. ·9?

$$\text{Volume tank} = \frac{4^2 \times 3{\cdot}1416 \times 6}{4} \text{ cubic ft.}$$
$$= 75{\cdot}4 \text{ cubic ft.}$$

each cubic ft. holds $6\frac{1}{4}$ gallons.

$$\text{Hence No. galls. in tank} = 75{\cdot}4 \times 6\tfrac{1}{4}$$
$$= 471\tfrac{1}{2}.$$

One gallon of water weighs 10 lbs., so the weight of 1 gallon of oil is $10 \times {\cdot}9 = 9$ lbs.

$$\therefore \text{ Weight of oil in tank} = 9 \times 471\tfrac{1}{2} \text{ lbs.}$$
$$= 4241\tfrac{1}{2} \text{ lbs.}$$

ART. 132. **Strength of Chain, Rope, and Wire.**—Chains may either be of open link or stayed link pattern, the former used for crane work, and the latter in cables. The relative strengths when of same iron may be taken as 6 to 9. For mentally estimating the safe working load in tons of chains, a simple rule is to square the number of eighths of an inch in the diameter, and divide by 10: thus for a chain $\frac{1}{2}$ inch diameter the safe working load would be $\frac{4^2}{10} = 1{\cdot}6$ tons.

The strength of ropes depends greatly on the quality of the hemp fibre of which it is made, and its ultimate strength is usually taken to be 6,400 lbs. per square inch of sectional area, the proof strength is one-half of the ultimate strength, and the safe working load may be considered to be one-fifth of the ultimate strength. The size of rope is measured by its circumference, and its strength deteriorates rapidly by age and use. The safe working load of a rope in tons is found by squaring the circumference and dividing by 7. Consider a rope C″ in circumference, its sectional area is $\frac{C^2}{4\pi}$ sq. inches;

$$\text{ultimate strength} = \frac{C^2 \times 6400}{4\pi \times 2240} \text{tons,}$$
$$\text{taking } \pi = \frac{22}{7}$$
$$= \frac{C^2 \times 7 \times 6400}{4 \times 22 \times 2240} \text{ tons,}$$
$$= \frac{C^2}{4{\cdot}4} \text{tons,}$$

which for a 6-inch rope would work out to about 8·2 tons.

Anderson, in his "Strength of Materials," says that from experiment the tenacity of this size rope is from 14 to 17 tons,

from which it seems that the ultimate strength is more than 6400 lbs. per sq. in. The rule given estimates the safe working load at $\frac{36}{7}$ tons = 5·2 tons approximately and appears to be rather excessive.

The safe working strain of a wire may be considered to be two-fifths of the square of the circumference in tons; which for a 3-inch wire would give $\frac{2 \times 3^2}{5}$ tons

$$= 1\frac{8}{5} = 3{\cdot}6 \text{ tons.}$$

The above rules may also be used to estimate the size rope or wire to carry a certain load.

For rope, multiply the load in tons by 7 and extract the square root.

For wire, multiply the load in tons by 5, divide by 2, and extract the square root.

When the size rope or wire to be used with a given purchase to take out a known weight has to be determined, first find the load on the hauling part, then the size to carry that load. To find the load on the haul, add one-tenth of the weight to be lifted for each sheave in the purchase, to the weight, and divide by the number of strings at the moving block. Thus to take out a 20-ton weight with a treble purchase and leading block: there are 7 sheaves: hence

$$\text{Load on haul} = \frac{\frac{7 \times 20}{10} + 20}{7} \text{ tons}$$

$$= \tfrac{34}{7} \text{ tons}$$

$$\text{Rope to lift this load} = \sqrt{\tfrac{34}{7} \times 7}$$

$$= \sqrt{34}$$

$$= 6 \text{ inches nearly;}$$

$$\text{or, Wire to lift this load} = \sqrt{\tfrac{34}{7} \times \tfrac{5}{2}}$$

$$= \sqrt{12}$$

$$= 3\tfrac{1}{2} \text{ inches nearly.}$$

Again, to find what weight can be lifted with a given purchase and a known rope or wire: first find the load such rope or wire will carry, multiply this load by the number of parts in the moving block, and allow one quarter of the result off for friction, etc. Thus using a 6-inch rope and treble purchase:

$$\text{Safe load for 6-inch rope} = \frac{6^2}{7} = \frac{36}{7} \text{ tons.}$$

There are 7 parts in moving block, hence total weight it can lift = $\frac{36}{7} \times 7 = 36$ tons; allow one quarter or 9 tons for friction, and remainder is 27 tons; or, using a $3\frac{1}{2}$ inch wire:

$$\text{Safe load} = \frac{2}{5} \times \left(\frac{7}{2}\right)^2 = \frac{2 \times 49}{5 \times 4} \text{ tons}$$

$$= 4{\cdot}9 \text{ tons.}$$

$$\text{Total lift} = 7 \times 4{\cdot}9 \text{ tons}$$

$$= 34{\cdot}3 \text{ tons.}$$

$$\text{For friction allow } \tfrac{1}{4} = 8{\cdot}6$$

$$\text{Remainder} = 25{\cdot}7 \text{ tons to be safely lifted.}$$

These rules evidently do not give similar results, but it must be remembered they are all approximations: to make them tally $\frac{9}{16}$ would certainly be a better multiplier than $\frac{3}{4}$; that is, the allowance for friction, etc., should be rather $\frac{7}{16}$ of the total lift.

ART. 133. **Speed, Consumption, and Displacement.**—When a screw turns in a solid bed, each complete turn advances the screw by an amount equal to the pitch; hence, if "N" be the number of revolutions per minute made, and "P" be the pitch of the screw in feet, the number of nautical miles per hour is $\frac{60 \cdot NP}{6080}$.

Since the water in which the screw of a ship turns is not solid, the actual speed over the ground, as determined by trial on measured miles, is less than this amount; the difference between the engine and actual speeds is called the "slip," and may be expressed as a percentage on the engine speed. Thus in a ship where the number of revolutions per minute is 57, the pitch 18 feet, determine the percentage of slip, if the trials give a speed of 9·5 knots.

$$\text{Engine speed} = \frac{60 \times 57 \times 18}{6080} \text{ knots}$$

$$= 10{\cdot}1 \text{ knots.}$$

$$\text{Actual speed} = 9{\cdot}5$$

$$\text{Slip} = {\cdot}6$$

$$\text{As } 10{\cdot}1 : 100 :: {\cdot}6 : —$$

$$\text{Hence percentage of slip} = 6\ \% \text{ roughly.}$$

The moving power in the ship is supplied by the burning of the coal, and it may be seen that the total consumption must depend on the speed, the time occupied, and the weight moved. From various experiments made it has been found that in the same ship, under the same conditions, the consumption varies as the cube of the speed obtained, if C represents consumption in tons, s the speed in knots, and t the time in hours:

C varies as $s^3 \times t$, and since $s \times t$ = distance "d"
C varies as $s^2 \times d$;

hence if the time is constant

Consumption varies as cube of speed;

if the distance is constant

Consumption varies as square of speed.

From observation it is found that in modern engines, with good coal, the consumption varies from 1·2 to 1·7 lbs. per hour per I.H.P., say 1½ lbs., and since the speed of the ship varies directly as the number of revolutions made per minute, hence the I.H.P. varies as cube of revolutions in a given time.

Example.—The daily consumption for a speed of 8·5 knots is 30 tons, what would it be for a speed of 10·5 knots?

Since the time is the same, consumption varies as cube of speed.

$$\text{As } 8{\cdot}5^3 : 10{\cdot}5^3 :: 30 : x$$

$$x = \frac{10{\cdot}5^3 \times 30}{8{\cdot}5^3} = \frac{34{,}728{\cdot}75}{614{\cdot}125}$$

$$= 56{\cdot}5 \text{ tons.}$$

Again, if in above ship the consumption is reduced to 20 tons per day, what speed would be developed?

$$\text{As } 30 : 20 :: 8{\cdot}5^3 : x^3$$

$$x^3 = \frac{20 \times 8{\cdot}5^3}{30} = 409{\cdot}4$$

$$\sqrt[3]{409{\cdot}4} \log = \frac{2{\cdot}612148}{3} \qquad \therefore x = \sqrt[3]{409{\cdot}4} = 7{\cdot}42 \text{ knots.}$$

$$= 0{\cdot}870716$$

$$= 7{\cdot}425 \log.$$

Again, if the ship has to make a voyage of 3570 miles, what would be the total consumption at the second speed?

As the distance is the same, consumption varies as square of speed.

$$\text{Number of days taken at } 8{\cdot}5 \text{ knots} = \frac{3570}{8{\cdot}5 \times 24} = 17{\cdot}5$$

$$\text{Total consumption} = 17{\cdot}5 \times 30 = 525 \text{ tons.}$$

$$\text{As } 8{\cdot}5^2 : 10{\cdot}5^2 :: 525 : x$$

$$x = \frac{10{\cdot}5^2 \times 525}{8{\cdot}5^2} = \frac{57{,}881{\cdot}25}{72{\cdot}25}$$

$$= 801 \text{ tons.}$$

$$\text{Number of days taken at } 10{\cdot}5 \text{ knots} = \frac{3570}{10{\cdot}5 \times 25} = 14\tfrac{1}{6}$$

$$\therefore \text{Total consumption at } 56{\cdot}5 \text{ tons per day} = 14\tfrac{1}{6} \times 56{\cdot}5$$

$$= 800{\cdot}5 \text{ tons,}$$

agreeing with amount obtained directly.

From the above example it is seen that 801 tons of coal will drive the ship 3570 miles at 10·5 knots, but if her speed is reduced to 8·5 knots the distance will be increased.

For 801 tons will be consumed in $\frac{801}{30}$ days at 8·5 knots;

$$\text{Hence distance} = \frac{801 \times 204}{30} \text{ miles}$$
$$= 5446{\cdot}8 \text{ miles.}$$

Again, if in the above ship, where the speed is 8·5 knots for a daily consumption of 30 tons, it is found that three days' coal are left at that speed when 1000 miles from port, to what speed must she be reduced to reach port?

Consumption varies as square of speed multiplied by distance.

$$\text{As } s^2 \times 1000 : 8{\cdot}5^2 \times 204 :: 90 : 30$$
$$\text{Hence } s^2 = \frac{8{\cdot}5^2 \times 204 \times \overset{3}{\cancel{90}}}{\underset{1}{\cancel{30}} \times 1000}$$
$$= 44{\cdot}217$$
$$\therefore s = 6{\cdot}65 \text{ knots.}$$

It is evident that the more deeply a ship is laden the more power is required to drive her through the water at the same speed, and it is found that the consumption or I.H.P. varies as the $\frac{2}{3}$rd power of the displacement, that is as the cube root of the square of the displacement. Thus if in a ship of 5000 tons displacement, 4000 tons are taken on at one port, and it is found that the consumption per day is 50 tons to drive her at a speed of 9 knots, what would the consumption be per day when the 1000 tons are taken on at another port to make the same speed?

$$\text{As } \sqrt[3]{4000^2} : \sqrt[3]{5000^2} :: 50 : x$$
$$x = \frac{50 \times \sqrt[3]{5000^2}}{\sqrt[3]{4000^2}}$$

$$\therefore \log x = \log 50 + \tfrac{2}{3} \log 5000 - \tfrac{2}{3} \log 4000$$
$$= \log 50 + \tfrac{2}{3} (\log 5000 - \log 4000)$$

5000 log =	3·698970
4000 log =	3·602060
	0·096910
	2
	3)0·193820
$\frac{2}{3}$ (log 5000 − log 4000) =	0·064607
50 log =	1·698970
58·02 log =	1·763577

$$\text{or } x = 58{\cdot}02 \text{ tons per day.}$$

Exercises.

1. Find net freight on 2400 tons at 17*s.* 6*d.* per ton, after allowing 6 % loading commission and 8 % primage.

2. Find the net freight of 5600 tons at 6 dollars per ton, allowing 5 % loading commission and 7 % primage, exchange being 4·85 dollars to £1.

3. Calculate net freight on 2700 tons at 22*s.* 6*d.* per ton, if primage is 7½ % and loading commission 3 %.

4. Convert £6836 10s. to dollars if £1 = 4·66 dollars. Convert 567832·5 francs to £'s. if £1 = 28·4 francs.

5. Wine costs 3·5 francs per litre, how much is this per gallon. The price of meat is 4 francs per kilogramme, how many pence per lb.?

6. Find displacement of a ship in tons whose dimensions are: Length 465 ft., breadth 50 ft., draught 28 ft., coefficient of fineness ·84.

7. Taking the height of a mercurial barometer to be 30 ins. when a water barometer reads 34 ft., find S.G. of mercury.

8. The sectional area at load water-level is 5600 sq. ft., how much will 320 tons of bunkers sink her?

9. Find coefficient of displacement of a liner 525 ft. long by 51 ft. broad by 25 ft. draught, her displacement being 15,300 tons.

10. A lifeboat is 24 ft. long, by 7 ft. 3 ins. wide, and 3 ft. 6 ins. deep: how many people will it carry, and what air-tight space must be allowed?

N.B.—Use ·6 as multiplier for finding cubical contents, and allow 10 cub. ft. per person, of which 1 cub. ft. must be air-tight.

11. How long must a bunker be made to contain 250 tons coal, allowing 42 cub. ft. to the ton, beam of ship 56 ft., height 'tween decks 6 ft. 6 ins.

12. The length of a rectangular field is to its breadth as 3 to 2, and its area is 11,094 sq. metres. Find the cost of surrounding it with a fence at 2·25 francs per metre length of boundary.

13. If 648 boxes are stowed close together on board a ship, find the total cost of freightage at 1s. 2*d.* per cub. yard, when each box measures 4½ ft. long, 3 ft. wide, and 2¼ ft. deep.

14. A rectangular tank 8 ft. long and 7 ft. wide holds when full 6¼ tons of water: find its depth.

15. If gold is worth 3 francs 10 centimes per gramme, find its value per ounce in English money, taking a franc as equal to 9¾*d.*, and a gramme as 15·43 grains.

16. How many tons of coal will a bunker 10 ft. 3 ins. deep, 42 ft. 6 ins. long, and breadth at two ends 8 ft. 6 ins., and in middle 10 ft., hold?

17. How many persons will a lifeboat hold whose length is 22 ft. 6 ins., breadth 6 ft. 6 ins., inside depth 3 ft. 6 ins., and what number of cubic feet of air-tight space should it contain?

18. A supposed cubical iceberg is 100 feet above level of sea, S.G. of ice ·921, sea water 1·026: find total height of berg.

19. If the displacement of a ship is 6800 tons, her length 340 ft., breadth 35 ft., and draught 22 ft., find coefficient of fineness.

20. If 150 tons of bunker sink a ship 5 ins., find her area at that water level.

21. Taking S.G. of iron as 7·7, find weight in pounds of a plate 20′ × 2′ × 1½″.

22. Length of ship 525 ft., beam 55 ft., coefficient of fineness ·9: how many tons per "inch" immersion?

23. Find approx. the daily consumption in a vessel of 5000 I.H.P.

24. Find safe working loads for chains of ⅜ in., 1½ in., and ⅞ in. diameter.

25. When chains are weakened by having been in use for a time, how is strength restored?

26. Find safe working strain of manilla ropes of 3½ ins., 5 ins., and 6½ ins. circumference.

27. Find the safe working strain of wires 1 in., 2½ ins., and 5 ins. circumference.

28. How many parts of a 3-in. rope will be equal in strength to a 6-in. rope?

29. What weight will a 3½-in. rope hang, when rove as a tackle having 6 parts at the travelling block?

30. What weight will a 2½-in. wire raise, using a double purchase?

31. What size rope or wire would you use to get out a 15-ton weight with a fourfold purchase and leading block?

32. What size rope and wire would be required to take out a 14-ton weight with a treble block?

33. If the breaking strain of a 2-in. chain is 72 tons, what is it of a 3-in. chain?

34. Mercury is 10,574 times as heavy as air, the S.G. of mercury is 13·597: find the weight in grains of 1 cub. ft. of air.

35. From Glazebrook's experiments the S.G. of air at 15° C. and 760 mm. pressure when ½ saturated was ·0012. Find weight in grains of 1 cub. ft. of this air.

36. Taking the composition of air to be 77 % of nitrogen, 21 % of oxygen, and the remainder carbonic acid gas, and Ganot's estimates that 1 litre of nitrogen at 0° C. and 760 mm. weighs 1·2561 grammes, of oxygen 1·4298 grammes, of carbonic acid 1·9774 grammes, find weight in grains of 1 cub. ft.

37. At 32° F. the velocity of sound is 1090 ft. per second, and the increase is at the rate of 10 ft. per second for each 9° F. rise in temperature.

You see the flash of a gun, and 6 seconds after hear the report, temperature 68° F.: how far away is the gun?

38. If the velocity of light be reckoned at 299,860 kilometres per sec., how many miles per sec. is this?

39. The pitch of a propeller is 15 ft., and it makes 60 revolutions per minute, allowing 5 % slip: find actual speed.

40. The pitch of a propeller is 16 ft., revolutions per minute 57, and time for measured mile is 7 minutes: find slip %.

41. The speed on the trials is 9·5 knots, propeller pitch 19 ft., revolutions per minute 56, find slip %.

42. For a speed of 10 knots, the daily consumption is 25 tons: what would the consumption be for speeds of 12 knots and 8 knots?

43. The daily consumption for a speed of 19 knots is 150 tons: what amount would be necessary for speeds of 21 and 15 knots?

44. For a daily consumption of 55 tons, the speed is 13 knots: what speed would be developed on consumption of 30 and 80 tons per day?

45. On a daily consumption of 84 tons, the speed is 15 knots: what would it be for consumptions of 2·5 and 5 tons per hour?

46. In a ship 16⅔ tons of fuel are consumed per hour for a speed of 24 knots: how many tons will be required for a voyage of 4½ days if the speed is increased 1 knot?

47. The total consumption for a certain distance at 9 knots is 240 tons: what would it be for same distance if speed is increased to 13·5 knots?

48. A ship steams 10 knots on 23 tons daily: how much coal will be required to steam 3036 miles in 11 days, and have 100 tons in reserve?

49. At 60 revolutions per minute the I.H.P. developed was 3500, what would the I.H.P. be when revolutions were increased to 75 per minute? How much coal per day is consumed under the two conditions?

50. For 1800 miles at 10 knots, the consumption is 500 tons: how much will be required for 2500 miles at 12 knots?

51. The consumption per day for a speed of 12 knots is found to have been 51 tons. There are 150 tons of coal left in bunkers, and 1152 miles to port, speed is reduced to 10 knots, how much will there be at the end of the voyage?

52. On a daily consumption of 80 tons, the vessel makes 15 knots; with only one day's coal left at this speed and 1000 miles to go, at what speed must she proceed to reach port?

53. There are 20 days' coal on board, when steaming at 9 knots on 35 tons per day, and 100 tons reserve. When 12 days out, owing to bad weather, it is found that 45 tons daily have been consumed: at what speed must she proceed to arrive with 50 tons in bunkers?

54. A vessel with a displacement of 3000 tons consumes 25 tons daily: what would consumption be at displacement of 2000 and 4000 tons?

55. A ship of 10,800 tons displacement burns 100 tons of coal daily, for a speed of 13·5 knots: what would be the effect on the consumption of taking in 300 tons extra cargo, and increasing speed $\frac{1}{2}$ knot?

56. A vessel of 8000 tons displacement has 7000 tons on board at one port, and develops a speed of 12 knots on 50 tons per day: what consumption will she have to make a speed of 13 knots, when the extra 1000 tons is taken in?

APPENDIX

CONTAINING EXTRACTS FROM

TIDE TABLES

AND

NAUTICAL ALMANAC

FOR THE SOLUTION OF ALL PROBLEMS IN THIS BOOK

USEFUL TABLES

AND

EXAMINATION PAPERS

AUGUST. I.

AT APPARENT NOON.

Date.		THE SUN'S				Sidereal Time of the Semi-diameter passing the Meridian.*	Equation of Time, *to be added to Apparent Time.*	Var. in 1 hour.
		Apparent Right Ascension.	Var. in 1 hour.	*Apparent* Declination.	Var. in 1 hour.			
		h. m. s.	s.	° ′ ″	″	m. s.	m. s.	s.
Wed.	1	8 42 55·43	9·724	N. 18 11 45·0	37·26	1 6·65	6 10·22	0·132
Thur.	2	8 46 48·49	9·698	17 56 41·9	38·00	1 6·56	6 6·74	0·158
Frid.	3	8 50 40·93	9·672	17 41 21·2	38·72	1 6·47	6 2·64	0·184
Sat.	4	8 54 32·76	9·647	17 25 43·3	39·43	1 6·39	5 57·93	0·209
Sun.	5	8 58 23·99	9·622	17 9 48·4	40·14	1 6·30	5 52·61	0·234
Mon.	6	9 2 14·61	9·597	16 53 36·8	40·82	1 6·21	5 46·70	0·258
Tues.	7	9 6 4·66	9·573	16 37 8·9	41·50	1 6·13	5 40·21	0·282
Wed.	8	9 9 54·13	9·549	16 20 24·9	42·16	1 6·04	5 33·14	0·307
Thur.	9	9 13 43·02	9·525	16 3 25·0	42·82	1 5·96	5 25·49	0·330
Frid.	10	9 17 31·34	9·502	15 46 9·5	43·46	1 5·87	5 17·28	0·354
Sat.	11	9 21 19·10	9·479	15 28 38·9	44·09	1 5·79	5 8·51	0·377
Sun.	12	9 25 6·31	9·456	15 10 53·3	44·70	1 5·71	4 59·19	0·400
Mon.	13	9 28 52·97	9·433	14 52 53·1	45·31	1 5·63	4 49·33	0·422
Tues.	14	9 32 39·09	9·411	14 34 38·6	45·90	1 5·55	4 38·93	0·445
Wed.	15	9 36 24·68	9·389	14 16 10·1	46·47	1 5·47	4 27·99	0·466
Thur.	16	9 40 9·75	9·367	13 57 28·0	47·03	1 5·39	4 16·54	0·488
Frid.	17	9 43 54·30	9·346	13 38 32·5	47·59	1 5·31	4 4·57	0·509
Sat.	18	9 47 38·34	9·325	13 19 23·9	48·12	1 5·23	3 52·09	0·530
Sun.	19	9 51 21·88	9·304	13 0 2·6	48·65	1 5·16	3 39·11	0·551
Mon.	20	9 55 4·92	9·283	12 40 28·9	49·16	1 5·09	3 25·64	0·571
Tues.	21	9 58 47·48	9·263	12 20 43·1	49·65	1 5·02	3 11·68	0·591
Wed.	22	10 2 29·56	9·244	12 0 45·7	50·13	1 4·95	2 57·25	0·611
Thur.	23	10 6 11·17	9·224	11 40 36·8	50·60	1 4·89	2 42·34	0·631
Frid.	24	10 9 52·32	9·205	11 20 16·9	51·05	1 4·82	2 26·98	0·649
Sat.	25	10 13 33·02	9·187	10 59 46·2	51·49	1 4·76	2 11·17	0·668
Sun.	26	10 17 13·28	9·169	10 39 5·2	51·92	1 4·70	1 54·92	0·686
Mon.	27	10 20 53·12	9·151	10 18 14·1	52·33	1 4·64	1 38·25	0·703
Tues.	28	10 24 32·55	9·135	9 57 13·2	52·73	1 4·58	1 21·17	0·720
Wed.	29	10 28 11·58	9·119	9 36 3·0	53·12	1 4·53	1 3·70	0·736
Thur.	30	10 31 50·24	9·104	9 14 43·6	53·49	1 4·48	0 45·86	0·751
Frid.	31	10 35 28·55	9·089	8 53 15·5	53·85	1 4·43	0 27·66	0·765
Sat.	32	10 39 6·52	9·075	N. 8 31 38·9	54·19	1 4·38	0 9·13	0·779

* *Mean* Time of the Semidiameter passing may be found by subtracting 0·18 from the *Sidereal* Time.

II. AUGUST.

AT MEAN NOON.

Date.		THE SUN'S			Equation of Time, *to be subtracted from Mean Time.*	Sidereal Time.
		Apparent. Right Ascension.	*Apparent* Declination.	Semidiameter.*		
		h. m. s.	° ′ ″	′ ″	m. s.	h. m. s.
Wed.	1	8 42 54·43	N. 18 11 48·8	15 47·14	6 10·24	8 36 44·19
Thur.	2	8 46 47·50	17 56 45·8	15 47·27	6 6·75	8 40 40·75
Frid.	3	8 50 39·96	17 41 25·1	15 47·40	6 2·66	8 44 37·30
Sat.	4	8 54 31·81	17 25 47·2	15 47·53	5 57·95	8 48 33·86
Sun.	5	8 58 23·05	17 9 52·4	15 47·67	5 52·64	8 52 30·41
Mon.	6	9 2 13·70	16 53 40·8	15 47·81	5 46·73	8 56 26·97
Tues.	7	9 6 3·76	16 37 12·8	15 47·95	5 40·24	9 0 23·52
Wed.	8	9 9 53·25	16 20 28·7	15 48·09	5 33·17	9 4 20·08
Thur.	9	9 13 42·16	16 3 28·8	15 48·24	5 25·52	9 8 16·63
Frid.	10	9 17 30·50	15 46 13·4	15 48·39	5 17·32	9 12 13·19
Sat.	11	9 21 18·29	15 28 42·7	15 48·54	5 8·55	9 16 9·74
Sun.	12	9 25 5·52	15 10 57·0	15 48·69	4 59·23	9 20 6·29
Mon.	13	9 28 52·22	14 52 56·8	15 48·85	4 49·37	9 24 2·85
Tues.	14	9 32 38·37	14 34 42·2	15 49·02	4 38·96	9 27 59·40
Wed.	15	9 36 23·98	14 16 13·6	15 49·19	4 28·02	9 31 55·96
Thur.	16	9 40 9·08	13 57 31·4	15 49·36	4 16·57	9 35 52·51
Frid.	17	9 43 53·66	13 38 35·7	15 49·54	4 4·60	9 39 49·07
Sat.	18	9 47 37·74	13 19 27·0	15 49·72	3 52·12	9 43 45·62
Sun.	19	9 51 21·31	13 0 5·5	15 49·90	3 39·14	9 47 42·17
Mon.	20	9 55 4·39	12 40 31·7	15 50·09	3 25·67	9 51 38·73
Tues.	21	9 58 46·99	12 20 45·8	15 50·29	3 11·71	9 55 35·28
Wed.	22	10 2 29·11	12 0 48·1	15 50·49	2 57·28	9 59 31·83
Thur.	23	10 6 10·76	11 40 39·0	15 50·69	2 42·37	10 3 28·39
Frid.	24	10 9 51·95	11 20 18·9	15 50·90	2 27·00	10 7 24·94
Sat.	25	10 13 32·69	10 59 48·1	15 51·11	2 11·19	10 11 21·50
Sun.	26	10 17 12·99	10 39 6·8	15 51·32	1 54·94	10 15 18·05
Mon.	27	10 20 52·87	10 18 15·5	15 51·54	1 38·27	10 19 14·60
Tues.	28	10 24 32·34	9 57 14·4	15 51·76	1 21·19	10 23 11·15
Wed.	29	10 28 11·42	9 36 3·9	15 51·99	1 3·71	10 27 7·71
Thur.	30	10 31 50·13	9 14 44·3	15 52·21	0 45·87	10 31 4·26
Frid.	31	10 35 28·48	8 53 15·9	15 52·44	0 27·67	10 35 0·81
Sat.	32	10 39 6·49	N. 8 31 39·1	15 52·67	0 9·13	10 38 57·37

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

AUGUST. III.

MEAN TIME.

Day.	THE SUN'S *Apparent*		Logarithm of the Radius Vector of the Earth.	Transit of the First Point of Aries.	THE MOON'S			
	Longitude.	Latitude.			Semidiameter.		Horizontal Parallax.	
	Noon.	*Noon.*	*Noon.*		*Noon.*	*Midnight.*	*Noon.*	*Midnight.*
	° ′ ″	″		h. m. s.	′ ″	′ ″	′ ″	′ ″
1	128 18 11·6	N. 0·11	0·0063890	15 20 44·55	16 20·78	16 20·09	59 53·36	59 50·83
2	129 15 35·7	0·00	·0063306	15 16 48·64	16 18·62	16 16·37	59 45·45	59 37·19
3	130 13 0·6	S. 0·14	·0062709	15 12 52·73	16 13·34	16 9·57	59 26·08	59 12·27
4	131 10 26·3	0·28	0·0062101	15 8 56·83	16 5·12	16 0·09	58 55·98	58 37·55
5	132 7 53·0	0·42	·0061481	15 5 0·92	15 54·56	15 48·67	58 17·30	57 55·68
6	133 5 20·7	0·55	·0060850	15 1 5·01	15 42·52	15 36·24	57 33·14	57 10·13
7	134 2 49·6	0·67	0·0060207	14 57 9·10	15 29·96	15 23·81	56 47·15	56 24·62
8	135 0 19·8	0·76	·0059551	14 53 13·19	15 17·90	15 12·33	56 2·95	55 42·51
9	135 57 51·3	0·83	·0058881	14 49 17·28	15 7·18	15 2·53	55 23·65	55 6·64
10	136 55 24·1	0·88	0·0058197	14 45 21·37	14 58·46	14 55·01	54 51·72	54 39·07
11	137 52 58·3	0·91	·0057498	14 41 25·46	14 52·22	14 50·12	54 28·85	54 21·16
12	138 50 33·9	0·90	·0056782	14 37 29·55	14 48·72	14 48·03	54 16·05	54 13·52
13	139 48 11·0	0·86	0·0056050	14 33 33·64	14 48·04	14 48·75	54 13·57	54 16·14
14	140 45 49·6	0·80	·0055300	14 29 37·74	14 50·11	14 52·10	54 21·14	54 28·44
15	141 43 29·6	0·71	·0054531	14 25 41·83	14 54·68	14 57·80	54 37·87	54 49·29
16	142 41 11·1	0·61	0·0053744	14 21 45·92	15 1·38	15 5·37	55 2·42	55 17·04
17	143 38 54·1	0·49	·0052936	14 17 50·01	15 9·70	15 14·30	55 32·91	55 49·73
18	144 36 38·5	0·36	·0052108	14 13 54·11	15 19·06	15 23·93	56 7·22	56 25·05
19	145 34 24·4	0·23	0·0051260	14 9 58·20	15 28·81	15 33·64	56 42·94	57 0·61
20	146 32 11·7	S. 0·09	·0050390	14 6 2·29	15 38·33	15 42·83	57 17·80	57 34·29
21	147 30 0·4	N. 0·03	·0049500	14 2 6·38	15 47·08	15 51·02	57 49·84	58 4·29
22	148 27 50·4	0·13	0·0048589	13 58 10·48	15 54·63	15 57·89	58 17·54	58 29·47
23	149 25 41·7	0·20	·0047659	13 54 14·57	16 0·78	16 3·30	58 40·08	58 49·32
24	150 23 34·2	0·24	·0046710	13 50 18·66	16 5·45	16 7·25	58 57·19	59 3·76
25	151 21 28·0	0·26	0·0045743	13 46 22·75	16 8·68	16 9·78	59 9·02	59 13·05
26	152 19 23·0	0·26	·0044761	13 42 26·85	16 10·55	16 10·98	59 15·85	59 17·45
27	153 17 19·2	0·21	·0043765	13 38 30·94	16 11·08	16 10·85	59 17·81	59 16·95
28	154 15 16·7	0·13	0·0042756	13 34 35·03	16 10·26	16 9·30	59 14·80	59 11·29
29	155 13 15·5	N. 0·03	·0041736	13 30 39·12	16 7·97	16 6·23	59 6·40	59 0·02
30	156 11 15·6	S. 0·10	·0040708	13 26 43·22	16 4·07	16 1·49	58 52·12	58 42·66
31	157 9 17·1	0·23	·0039672	13 22 47·31	15 58·48	15 55·06	58 31·65	58 19·13
32	158 7 20·1	S. 0·36	0·0038629	13 18 51·40	15 51·25	15 47·09	58 5·16	57 49·88

IV. AUGUST.

MEAN TIME.

Day.	THE MOON'S											
	Longitude.				Latitude.				Age.	Meridian Passage.		
	Noon.		*Midnight.*		*Noon.*		*Midnight.*		*Noon.*	*Upper.*		*Lower.*
	° ′ ″		° ′ ″		° ′ ″		° ′ ″		d.	h. m.		h. m.
1	267 36 27·5		274 51 41·4		N. 3 40 55·7		N. 3 10 27·6		10·96	9 36·2		22 6·3
2	282 5 51·2		289 18 15·8		2 36 59·9		2 1 9·3		11·96	10 36·3		23 6·1
3	296 28 14·8		303 35 10·3		1 23 35·1		N. 0 44 57·9		12·96	11 35·3		* *
4	310 38 27·3		317 37 34·9		N. 0 5 58·7		S. 0 32 43·1		13·96	12 31·6		0 3·8
5	324 32 7·9		331 21 46·3		S. 1 10 30·5		1 46 50·0		14·96	13 24·6		0 58·5
6	338 6 16·1		344 45 29·9		2 21 11·7		2 53 10·2		15·96	14 14·3		1 49·8
7	351 19 26·2		357 48 9·6		3 22 24·8		3 48 38·9		16·96	15 1·2		2 38·1
8	4 11 50·1		10 30 42·7		4 11 39·6		4 31 18·2		17·96	15 46·1		3 23·9
9	16 45 7·5		22 55 27·9		4 47 28·8		5 0 7·9		18·96	16 29·9		4 8·1
10	29 2 10·7		35 5 45·3		5 9 14·3		5 14 48·8		19·96	17 13·3		4 51·6
11	41 6 43·5		47 5 38·5		5 16 53·0		5 15 30·0		20·96	17 57·0		5 35·0
12	53 3 4·6		58 59 36·3		5 10 43·5		5 2 38·3		21·96	18 41·8		6 19·2
13	64 55 48·3		70 52 15·3		4 51 19·2		4 36 52·7		22·96	19 27·9		7 4·7
14	76 49 30·6		82 48 6·6		4 19 25·5		3 59 5·3		23·96	20 15·7		7 51·6
15	88 48 34·1		94 51 21·6		3 36 1·4		3 10 24·4		24·96	21 4·9		8 40·1
16	100 56 55·5		107 5 39·4		2 42 26·5		2 12 21·8		25·96	21 55·3		9 30·0
17	113 17 53·5		119 33 55·0		1 40 27·0		S. 1 7 0·9		26·96	22 46·2		10 20·7
18	125 53 57·0		132 18 8·7		S. 0 32 24·6		N. 0 2 57·9		27·96	23 37·1		11 11·7
19	138 46 35·5		145 19 18·4		N. 0 38 40·6		1 14 15·3		28·96	* *		12 2·5
20	151 56 14·0		158 37 15·4		1 49 11·5		2 22 58·5		0·44	0 27·7		12 52·7
21	165 22 11·8		172 10 48·4		2 55 4·4		3 24 57·6		1·44	1 17·7		13 42·5
22	179 2 48·8		185 57 53·6		3 52 7·6		4 16 5·7		2·44	2 7·4		14 32·2
23	192 55 41·3		199 55 50·1		4 36 25·7		4 52 45·1		3·44	2 57·2		15 22·4
24	206 57 57·0		214 1 40·0		5 4 45·0		5 12 11·2		4·44	3 47·8		16 13·6
25	221 6 36·7		228 12 26·4		5 14 54·0		5 12 49·1		5·44	4 39·8		17 6·6
26	235 18 48·9		242 25 25·6		5 5 56·9		4 54 23·5		6·44	5 33·8		18 1·6
27	249 31 58·6		256 38 10·7		4 38 19·2		4 17 59·5		7·44	6 29·9		18 58·6
28	263 43 46·1		270 48 28·8		3 53 44·3		3 25 57·9		8·44	7 27·6		19 56·8
29	277 52 2·9		284 54 12·5		2 55 7·5		2 21 44·1		9·44	8 26·1		20 55·2
30	291 54 41·0		298 53 11·9		1 46 20·8		N. 1 9 32·5		10·44	9 24·0		21 52·4
31	305 49 27·6		312 43 10·9		N. 0 31 55·2		S. 0 5 55·3		11·44	10 20·2		22 47·3
32	319 34 4·8		326 21 52·0		S. 0 43 23·4		S. 1 19 55·5		12·44	11 13·7		23 39·4

The Moon's Longitude and Latitude are from HANSEN's Tables; the Right Ascension and Declination contain NEWCOMB's corrections.

AUGUST. V.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Wednesday 1.			
	h. m. s.	s.	° ′ ″	″
0	17 49 48·87	25·622	S. 19 44 44·6	30·39
1	17 52 22·65	25·638	19 47 42·9	29·03
2	17 54 56·52	25·653	19 50 33·0	27·66
3	17 57 30·48	25·667	19 53 14·8	26·28
4	18 0 4·52	25·679	19 55 48·3	24·89
5	18 2 38·63	25·692	19 58 13·5	23·51
6	18 5 12·82	25·703	20 0 30·4	22·13
7	18 7 47·07	25·713	20 2 39·0	20·73
8	18 10 21·38	25·723	20 4 39·2	19·33
9	18 12 55·74	25·731	20 6 31·0	17·93
10	18 15 30·15	25·738	20 8 14·4	16·53
11	18 18 4·60	25·744	20 9 49·4	15·13
12	18 20 39·08	25·749	20 11 15·9	13·72
13	18 23 13·59	25·753	20 12 34·0	12·32
14	18 25 48·12	25·756	20 13 43·7	10·91
15	18 28 22·66	25·758	20 14 44·9	9·50
16	18 30 57·21	25·758	20 15 37·7	8·09
17	18 33 31·76	25·758	20 16 22·0	6·68
18	18 36 6·31	25·757	20 16 57·8	5·26
19	18 38 40·84	25·754	20 17 25·1	3·84
20	18 41 15·36	25·752	20 17 43·9	2·43
21	18 43 49·86	25·747	20 17 54·3	1·03
22	18 46 24·32	25·741	20 17 56·2	0·39
23	18 48 58·75	25·734	S. 20 17 49·6	1·81
	Thursday 2.			
0	18 51 33·13	25·726	S. 20 17 34·5	3·22
1	18 54 7·46	25·717	20 17 11·0	4·62
2	18 56 41·73	25·707	20 16 39·1	6·03
3	18 59 15·94	25·695	20 15 58·7	7·43
4	19 1 50·07	25·683	20 15 10·0	8·83
5	19 4 24·13	25·670	20 14 12·8	10·23
6	19 6 58·11	25·656	20 13 7·3	11·62
7	19 9 32·00	25·640	20 11 53·4	13·01
8	19 12 5·79	25·623	20 10 31·2	14·40
9	19 14 39·48	25·606	20 9 0·6	15·78
10	19 17 13·06	25·588	20 7 21·8	17·16
11	19 19 46·53	25·568	20 5 34·7	18·53
12	19 22 19·88	25·548	20 3 39·4	19·90
13	19 24 53·10	25·526	20 1 35·9	21·27
14	19 27 26·19	25·503	19 59 24·2	22·63
15	19 29 59·13	25·478	19 57 4·3	23·99
16	19 32 31·93	25·454	19 54 36·3	25·33
17	19 35 4·58	25·429	19 52 0·3	26·68
18	19 37 37·08	25·402	19 49 16·2	28·02
19	19 40 9·41	25·374	19 46 24·1	29·34
20	19 42 41·57	25·346	19 43 24·1	30·67
21	19 45 13·56	25·317	19 40 16·1	31·98
22	19 47 45·37	25·287	19 37 0·3	33·29
23	19 50 17·00	25·255	19 33 36·6	34·60
24	19 52 48·43	25·223	S. 19 30 5·1	35·89

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Friday 3.			
	h. m. s.	s.	° ′ ″	″
0	19 52 48·43	25·223	S. 19 30 5·1	35·89
1	19 55 19·67	25·190	19 26 25·9	37·18
2	19 57 50·71	25·156	19 22 39·0	38·46
3	20 0 21·54	25·121	19 18 44·4	39·73
4	20 2 52·16	25·086	19 14 42·3	40·98
5	20 5 22·57	25·050	19 10 32·6	42·24
6	20 7 52·76	25·013	19 6 15·4	43·48
7	20 10 22·72	24·975	19 1 50·8	44·72
8	20 12 52·46	24·937	18 57 18·8	45·94
9	20 15 21·96	24·898	18 52 39·5	47·15
10	20 17 51·23	24·858	18 47 53·0	48·36
11	20 20 20·25	24·817	18 42 59·2	49·56
12	20 22 49·03	24·776	18 37 58·3	50·74
13	20 25 17·56	24·733	18 32 50·3	51·93
14	20 27 45·83	24·690	18 27 35·2	53·09
15	20 30 13·84	24·647	18 22 13·2	54·24
16	20 32 41·59	24·603	18 16 44·3	55·38
17	20 35 9·07	24·558	18 11 8·6	56·53
18	20 37 36·29	24·513	18 5 26·0	57·65
19	20 40 3·23	24·467	17 59 36·8	58·75
20	20 42 29·89	24·420	17 53 41·0	59·86
21	20 44 56·28	24·375	17 47 38·5	60·95
22	20 47 22·39	24·328	17 41 29·6	62·03
23	20 49 48·21	24·279	S. 17 35 14·2	63·09
	Saturday 4.			
0	20 52 13·74	24·231	S. 17 28 52·5	64·14
1	20 54 38·98	24·183	17 22 24·5	65·18
2	20 57 3·93	24·133	17 15 50·3	66·22
3	20 59 28·58	24·084	17 9 9·9	67·23
4	21 1 52·94	24·035	17 2 23·5	68·23
5	21 4 17·00	23·985	16 55 31·1	69·23
6	21 6 40·76	23·934	16 48 32·7	70·23
7	21 9 4·21	23·883	16 41 28·4	71·19
8	21 11 27·36	23·833	16 34 18·4	72·14
9	21 13 50·20	23·781	16 27 2·7	73·09
10	21 16 12·73	23·729	16 19 41·3	74·03
11	21 18 34·95	23·678	16 12 14·4	74·95
12	21 20 56·87	23·627	16 4 41·9	75·86
13	21 23 18·47	23·574	15 57 4·1	76·75
14	21 25 39·76	23·522	15 49 20·9	77·63
15	21 28 0·73	23·468	15 41 32·5	78·50
16	21 30 21·38	23·416	15 33 38·9	79·36
17	21 32 41·72	23·364	15 25 40·2	80·20
18	21 35 1·75	23·311	15 17 36·5	81·03
19	21 37 21·45	23·257	15 9 27·9	81·85
20	21 39 40·83	23·204	15 1 14·3	82·66
21	21 41 59·90	23·152	14 52 56·0	83·45
22	21 44 18·65	23·098	14 44 32·9	84·23
23	21 46 37·07	23·044	14 36 5·2	85·00
24	21 48 55·18	22·992	S. 14 27 32·9	85·75

VI. AUGUST.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Sunday 5.			
	h. m. s.	s.	° ′ ″	″
0	21 48 55·18	22·992	S. 14 27 32·9	85·75
1	21 51 12·97	22·938	14 18 56·2	86·49
2	21 53 30·43	22·884	14 10 15·0	87·22
3	21 55 47·58	22·832	14 1 29·5	87·93
4	21 58 4·41	22·778	13 52 39·8	88·63
5	22 0 20·92	22·725	13 43 45·9	89·33
6	22 2 37·11	22·672	13 34 47·8	90·01
7	22 4 52·98	22·619	13 25 45·8	90·67
8	22 7 8·54	22·567	13 16 39·8	91·33
9	22 9 23·78	22·513	13 7 29·9	91·97
10	22 11 38·70	22·461	12 58 16·2	92·59
11	22 13 53·31	22·408	12 48 58·8	93·21
12	22 16 7·60	22·356	12 39 37·7	93·81
13	22 18 21·58	22·304	12 30 13·1	94·40
14	22 20 35·25	22·253	12 20 44·9	94·98
15	22 22 48·61	22·200	12 11 13·4	95·53
16	22 25 1·65	22·148	12 1 38·5	96·09
17	22 27 14·39	22·098	11 52 0·3	96·63
18	22 29 26·82	22·047	11 42 18·9	97·16
19	22 31 38·95	21·997	11 32 34·4	97·68
20	22 33 50·78	21·946	11 22 46·8	98·18
21	22 36 2·30	21·895	11 12 56·3	98·67
22	22 38 13·52	21·845	11 3 2·8	99·15
23	22 40 24·44	21·796	S. 10 53 6·5	99·62
	Monday 6.			
0	22 42 35·07	21·747	S. 10 43 7·4	100·08
1	22 44 45·40	21·698	10 33 5·6	100·52
2	22 46 55·44	21·649	10 23 1·2	100·94
3	22 49 5·19	21·600	10 12 54·3	101·37
4	22 51 14·64	21·552	10 2 44·8	101·78
5	22 53 23·81	21·505	9 52 32·9	102·18
6	22 55 32·70	21·458	9 42 18·7	102·56
7	22 57 41·30	21·410	9 32 2·2	102·93
8	22 59 49·62	21·363	9 21 43·5	103·30
9	23 1 57·66	21·318	9 11 22·6	103·65
10	23 4 5·43	21·272	9 0 59·7	103·99
11	23 6 12·92	21·226	8 50 34·7	104·33
12	23 8 20·14	21·181	8 40 7·8	104·64
13	23 10 27·09	21·137	8 29 39·0	104·95
14	23 12 33·78	21·093	8 19 8·4	105·24
15	23 14 40·20	21·048	8 8 36·1	105·53
16	23 16 46·36	21·005	7 58 2·1	105·81
17	23 18 52·26	20·962	7 47 26·4	106·08
18	23 20 57·90	20·919	7 36 49·2	106·33
19	23 23 3·29	20·878	7 26 10·5	106·57
20	23 25 8·43	20·836	7 15 30·4	106·81
21	23 27 13·32	20·795	7 4 48·8	107·03
22	23 29 17·97	20·755	6 54 6·0	107·24
23	23 31 22·38	20·714	6 43 21·9	107·44
24	23 33 26·54	20·674	S. 6 32 36·7	107·63

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Tuesday 7.			
	h. m. s.	s.	° ′ ″	″
0	23 33 26·54	20·674	S. 6 32 36·7	107·63
1	23 35 30·47	20·635	6 21 50·3	107·82
2	23 37 34·16	20·596	6 11 2·9	107·99
3	23 39 37·62	20·558	6 0 14·4	108·16
4	23 41 40·86	20·521	5 49 25·0	108·31
5	23 43 43·87	20·483	5 38 34·7	108·46
6	23 45 46·65	20·446	5 27 43·5	108·59
7	23 47 49·22	20·410	5 16 51·6	108·72
8	23 49 51·57	20·374	5 5 58·9	108·84
9	23 51 53·71	20·338	4 55 5·5	108·94
10	23 53 55·63	20·303	4 44 11·6	109·04
11	23 55 57·35	20·270	4 33 17·0	109·13
12	23 57 58·87	20·236	4 22 22·0	109·21
13	0 0 0·18	20·203	4 11 26·5	109·28
14	0 2 1·30	20·170	4 0 30·6	109·34
15	0 4 2·22	20·138	3 49 34·4	109·40
16	0 6 2·95	20·106	3 38 37·8	109·45
17	0 8 3·49	20·075	3 27 41·0	109·48
18	0 10 3·85	20·044	3 16 44·0	109·52
19	0 12 4·02	20·014	3 5 46·8	109·53
20	0 14 4·02	19·984	2 54 49·6	109·54
21	0 16 3·83	19·954	2 43 52·3	109·56
22	0 18 3·47	19·927	2 32 54·9	109·55
23	0 20 2·95	19·899	S. 2 21 57·7	109·53
	Wednesday 8.			
0	0 22 2·26	19·872	S. 2 11 0·6	109·51
1	0 24 1·41	19·844	2 0 3·6	109·48
2	0 26 0·39	19·818	1 49 6·8	109·44
3	0 27 59·22	19·792	1 38 10·3	109·40
4	0 29 57·89	19·767	1 27 14·0	109·35
5	0 31 56·42	19·743	1 16 18·1	109·28
6	0 33 54·80	19·718	1 5 22·6	109·22
7	0 35 53·03	19·693	0 54 27·5	109·15
8	0 37 51·12	19·671	0 43 32·8	109·07
9	0 39 49·08	19·648	0 32 38·7	108·98
10	0 41 46·90	19·626	0 21 45·1	108·87
11	0 43 44·59	19·604	S. 0 10 52·2	108·78
12	0 45 42·15	19·583	N. 0 0 0·2	108·67
13	0 47 39·59	19·563	0 10 51·8	108·54
14	0 49 36·90	19·543	0 21 42·7	108·42
15	0 51 34·10	19·524	0 32 32·8	108·28
16	0 53 31·19	19·505	0 43 22·1	108·14
17	0 55 28·16	19·487	0 54 10·5	108·00
18	0 57 25·03	19·469	1 4 58·1	107·85
19	0 59 21·79	19·451	1 15 44·7	107·68
20	1 1 18·44	19·434	1 26 30·3	107·52
21	1 3 15·00	19·419	1 37 14·9	107·35
22	1 5 11·47	19·403	1 47 58·5	107·18
23	1 7 7·84	19·388	1 58 41·0	106·98
24	1 9 4·13	19·374	N. 2 9 22·2	106·78

AUGUST. VII.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Thursday 9.			
	h. m. s.	s.	° ′ ″	″
0	1 9 4·13	19·374	N. 2 9 22·2	106·78
1	1 11 0·33	19·360	2 20 2·3	106·58
2	1 12 56·45	19·346	2 30 41·2	106·38
3	1 14 52·48	19·333	2 41 18·9	106·18
4	1 16 48·44	19·321	2 51 55·3	105·96
5	1 18 44·33	19·309	3 2 30·4	105·73
6	1 20 40·15	19·298	3 13 4·1	105·50
7	1 22 35·90	19·287	3 23 36·4	105·26
8	1 24 31·59	19·277	3 34 7·2	105·02
9	1 26 27·22	19·267	3 44 36·6	104·78
10	1 28 22·79	19·257	3 55 4·5	104·52
11	1 30 18·30	19·248	4 5 30·8	104·26
12	1 32 13·76	19·240	4 15 55·6	103·99
13	1 34 9·18	19·233	4 26 18·7	103·72
14	1 36 4·55	19·225	4 36 40·2	103·44
15	1 37 59·88	19·218	4 47 0·0	103·15
16	1 39 55·17	19·212	4 57 18·0	102·86
17	1 41 50·42	19·207	5 7 34·3	102·57
18	1 43 45·65	19·202	5 17 48·8	102·27
19	1 45 40·84	19·196	5 28 1·5	101·96
20	1 47 36·00	19·192	5 38 12·3	101·65
21	1 49 31·14	19·189	5 48 21·3	101·33
22	1 51 26·27	19·186	5 58 28·3	101·00
23	1 53 21·37	19·182	N. 6 8 33·3	100·68
	Friday 10.			
0	1 55 16·45	19·180	N. 6 18 36·4	100·34
1	1 57 11·53	19·178	6 28 37·4	99·99
2	1 59 6·59	19·177	6 38 36·3	99·65
3	2 1 1·65	19·177	6 48 33·2	99·30
4	2 2 56·71	19·177	6 58 27·9	98·94
5	2 4 51·77	19·177	7 8 20·5	98·58
6	2 6 46·83	19·177	7 18 10·8	98·20
7	2 8 41·89	19·178	7 27 58·9	97·83
8	2 10 36·97	19·180	7 37 44·8	97·46
9	2 12 32·05	19·182	7 47 28·4	97·08
10	2 14 27·15	19·184	7 57 9·7	96·69
11	2 16 22·26	19·187	8 6 48·7	96·29
12	2 18 17·40	19·192	8 16 25·2	95·88
13	2 20 12·56	19·195	8 25 59·3	95·48
14	2 22 7·74	19·199	8 35 31·0	95·08
15	2 24 2·95	19·203	8 45 0·3	94·67
16	2 25 58·18	19·208	8 54 27·0	94·24
17	2 27 53·45	19·215	9 3 51·2	93·82
18	2 29 48·76	19·222	9 13 12·8	93·38
19	2 31 44·11	19·228	9 22 31·8	92·95
20	2 33 39·50	19·235	9 31 48·2	92·51
21	2 35 34·93	19·243	9 41 1·9	92·06
22	2 37 30·41	19·251	9 50 12·9	91·61
23	2 39 25·94	19·259	9 59 21·2	91·15
24	2 41 21·52	19·268	N.10 8 26·7	90·69

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Saturday 11.			
	h. m. s.	s.	° ′ ″	″
0	2 41 21·52	19·268	N.10 8 26·7	90·69
1	2 43 17·15	19·277	10 17 29·5	90·23
2	2 45 12·84	19·287	10 26 29·4	89·75
3	2 47 8·59	19·297	10 35 26·5	89·28
4	2 49 4·40	19·307	10 44 20·7	88·79
5	2 51 0·27	19·318	10 53 12·0	88·31
6	2 52 56·21	19·329	11 2 0·4	87·82
7	2 54 52·22	19·341	11 10 45·8	87·32
8	2 56 48·30	19·353	11 19 28·2	86·81
9	2 58 44·45	19·365	11 28 7·5	86·30
10	3 0 40·68	19·378	11 36 43·8	85·79
11	3 2 36·98	19·391	11 45 17·0	85·28
12	3 4 33·37	19·405	11 53 47·1	84·75
13	3 6 29·84	19·418	12 2 14·0	84·23
14	3 8 26·39	19·433	12 10 37·8	83·69
15	3 10 23·03	19·448	12 18 58·3	83·15
16	3 12 19·76	19·463	12 27 15·6	82·62
17	3 14 16·58	19·478	12 35 29·7	82·07
18	3 16 13·49	19·493	12 43 40·4	81·51
19	3 18 10·50	19·509	12 51 47·8	80·95
20	3 20 7·60	19·526	12 59 51·8	80·38
21	3 22 4·81	19·543	13 7 52·4	79·82
22	3 24 2·11	19·559	13 15 49·7	79·25
23	3 25 59·52	19·577	N.13 23 43·4	78·67
	Sunday 12.			
0	3 27 57·03	19·594	N.13 31 33·7	78·09
1	3 29 54·65	19·613	13 39 20·5	77·50
2	3 31 52·38	19·631	13 47 3·7	76·90
3	3 33 50·22	19·649	13 54 43·3	76·30
4	3 35 48·17	19·668	14 2 19·3	75·70
5	3 37 46·24	19·688	14 9 51·7	75·09
6	3 39 44·42	19·707	14 17 20·4	74·48
7	3 41 42·72	19·727	14 24 45·4	73·86
8	3 43 41·14	19·747	14 32 6·7	73·24
9	3 45 39·68	19·767	14 39 24·3	72·61
10	3 47 38·34	19·787	14 46 38·0	71·98
11	3 49 37·12	19·808	14 53 48·0	71·34
12	3 51 36·03	19·829	15 0 54·1	70·69
13	3 53 35·07	19·850	15 7 56·3	70·04
14	3 55 34·23	19·872	15 14 54·6	69·39
15	3 57 33·53	19·894	15 21 49·0	68·73
16	3 59 32·96	19·916	15 28 39·4	68·07
17	4 1 32·52	19·938	15 35 25·8	67·40
18	4 3 32·21	19·960	15 42 8·2	66·73
19	4 5 32·04	19·983	15 48 46·5	66·04
20	4 7 32·01	20·007	15 55 20·7	65·36
21	4 9 32·12	20·029	16 1 50·8	64·67
22	4 11 32·36	20·052	16 8 16·7	63·97
23	4 13 32·74	20·076	16 14 38·4	63·28
24	4 15 33·27	20·100	N.16 20 56·0	62·58

VIII. AUGUST.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Monday 13.			
	h. m. s.	s.	° ′ ″	″
0	4 15 33·27	20·100	N. 16 20 56·0	62·58
1	4 17 33·94	20·123	16 27 9·3	61·86
2	4 19 34·75	20·147	16 33 18·3	61·14
3	4 21 35·70	20·171	16 39 23·0	60·43
4	4 23 36·80	20·196	16 45 23·4	59·71
5	4 25 38·05	20·220	16 51 19·5	58·98
6	4 27 39·44	20·244	16 57 11·1	58·23
7	4 29 40·98	20·269	17 2 58·8	57·49
8	4 31 42·67	20·294	17 8 41·0	56·74
9	4 33 44·51	20·319	17 14 19·2	55·99
10	4 35 46·50	20·343	17 19 52·9	55·24
11	4 37 48·63	20·368	17 25 22·1	54·48
12	4 39 50·92	20·394	17 30 46·7	53·72
13	4 41 53·36	20·419	17 36 6·7	52·94
14	4 43 55·95	20·445	17 41 22·0	52·17
15	4 45 58·70	20·470	17 46 32·7	51·39
16	4 48 1·59	20·495	17 51 38·7	50·61
17	4 50 4·64	20·521	17 56 40·0	49·82
18	4 52 7·84	20·547	18 1 36·5	49·02
19	4 54 11·20	20·573	18 6 28·2	48·22
20	4 56 14·71	20·598	18 11 15·1	47·41
21	4 58 18·37	20·623	18 15 57·1	46·60
22	5 0 22·19	20·649	18 20 34·3	45·78
23	5 2 26·16	20·675	N. 18 25 6·5	44·96
	Tuesday 14.			
0	5 4 30·29	20·701	N. 18 29 33·8	44·14
1	5 6 34·57	20·727	18 33 56·2	43·32
2	5 8 39·01	20·753	18 38 13·6	42·48
3	5 10 43·60	20·778	18 42 26·0	41·64
4	5 12 48·34	20·803	18 46 33·3	40·79
5	5 14 53·24	20·829	18 50 35·5	39·95
6	5 16 58·29	20·855	18 54 32·7	39·10
7	5 19 3·50	20·880	18 58 24·7	38·23
8	5 21 8·85	20·905	19 2 11·5	37·38
9	5 23 14·36	20·932	19 5 53·2	36·51
10	5 25 20·03	20·957	19 9 29·6	35·63
11	5 27 25·84	20·981	19 13 0·8	34·76
12	5 29 31·80	21·007	19 16 26·7	33·88
13	5 31 37·92	21·032	19 19 47·3	32·99
14	5 33 44·18	21·056	19 23 2·6	32·11
15	5 35 50·59	21·082	19 26 12·6	31·21
16	5 37 57·16	21·107	19 29 17·1	30·31
17	5 40 3·87	21·130	19 32 16·3	29·41
18	5 42 10·72	21·154	19 35 10·0	28·50
19	5 44 17·72	21·179	19 37 58·3	27·59
20	5 46 24·87	21·203	19 40 41·1	26·68
21	5 48 32·16	21·228	19 43 18·4	25·75
22	5 50 39·60	21·251	19 45 50·1	24·83
23	5 52 47·17	21·274	19 48 16·3	23·90
24	5 54 54·89	21·298	N. 19 50 36·9	22·97

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Wednesday 15.			
	h. m. s.	s.	° ′ ″	″
0	5 54 54·89	21·298	N. 19 50 36·9	22·97
1	5 57 2·75	21·322	19 52 51·9	22·03
2	5 59 10·75	21·344	19 55 1·3	21·09
3	6 1 18·88	21·367	19 57 5·0	20·14
4	6 3 27·15	21·390	19 59 3·0	19·19
5	6 5 35·56	21·413	20 0 55·3	18·24
6	6 7 44·10	21·434	20 2 41·9	17·29
7	6 9 52·77	21·456	20 4 22·8	16·33
8	6 12 1·57	21·478	20 5 57·9	15·37
9	6 14 10·50	21·499	20 7 27·2	14·40
10	6 16 19·56	21·521	20 8 50·7	13·43
11	6 18 28·75	21·542	20 10 8·4	12·46
12	6 20 38·06	21·563	20 11 20·2	11·48
13	6 22 47·50	21·583	20 12 26·2	10·51
14	6 24 57·06	21·603	20 13 26·3	9·52
15	6 27 6·73	21·623	20 14 20·4	8·53
16	6 29 16·53	21·643	20 15 8·6	7·54
17	6 31 26·44	21·662	20 15 50·9	6·56
18	6 33 36·47	21·681	20 16 27·3	5·56
19	6 35 46·61	21·699	20 16 57·6	4·56
20	6 37 56·86	21·718	20 17 22·0	3·57
21	6 40 7·22	21·736	20 17 40·4	2·56
22	6 42 17·69	21·753	20 17 52·7	1·55
23	6 44 28·26	21·770	N. 20 17 59·0	0·54
	Thursday 16.			
0	6 46 38·93	21·788	N. 20 17 59·2	0·47
1	6 48 49·71	21·805	20 17 53·4	1·48
2	6 51 0·59	21·821	20 17 41·5	2·50
3	6 53 11·56	21·837	20 17 23·4	3·52
4	6 55 22·63	21·853	20 16 59·3	4·53
5	6 57 33·79	21·868	20 16 29·0	5·56
6	6 59 45·05	21·883	20 15 52·6	6·58
7	7 1 56·39	21·898	20 15 10·1	7·60
8	7 4 7·82	21·912	20 14 21·4	8·63
9	7 6 19·33	21·926	20 13 26·5	9·66
10	7 8 30·93	21·939	20 12 25·5	10·69
11	7 10 42·60	21·952	20 11 18·2	11·73
12	7 12 54·35	21·965	20 10 4·8	12·75
13	7 15 6·18	21·978	20 8 45·2	13·78
14	7 17 18·09	21·990	20 7 19·4	14·83
15	7 19 30·06	22·002	20 5 47·3	15·87
16	7 21 42·11	22·013	20 4 9·0	16·90
17	7 23 54·22	22·023	20 2 24·5	17·93
18	7 26 6·39	22·034	20 0 33·8	18·97
19	7 28 18·63	22·045	19 58 36·9	20·01
20	7 30 30·93	22·054	19 56 33·7	21·06
21	7 32 43·28	22·063	19 54 24·2	22·10
22	7 34 55·69	22·073	19 52 8·5	23·13
23	7 37 8·15	22·081	19 49 46·6	24·18
24	7 39 20·66	22·089	N. 19 47 18·4	25·23

AUGUST. IX.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Friday 17.			
	h. m. s.	s.	° ′ ″	″
0	7 39 20·66	22·089	N. 19 47 18·4	25·23
1	7 41 33·22	22·098	19 44 44·0	26·26
2	7 43 45·83	22·105	19 42 3·3	27·30
3	7 45 58·48	22·112	19 39 16·4	28·34
4	7 48 11·17	22·118	19 36 23·2	29·38
5	7 50 23·90	22·125	19 33 23·8	30·42
6	7 52 36·67	22·131	19 30 18·2	31·45
7	7 54 49·47	22·136	19 27 6·4	32·49
8	7 57 2·30	22·141	19 23 48·3	33·53
9	7 59 15·16	22·146	19 20 24·0	34·57
10	8 1 28·05	22·151	19 16 53·5	35·60
11	8 3 40·97	22·154	19 13 16·8	36·63
12	8 5 53·90	22·158	19 9 33·9	37·67
13	8 8 6·86	22·162	19 5 44·8	38·69
14	8 10 19·84	22·164	19 1 49·6	39·73
15	8 12 32·83	22·167	18 57 48·1	40·76
16	8 14 45·84	22·168	18 53 40·5	41·78
17	8 16 58·85	22·170	18 49 26·8	42·80
18	8 19 11·88	22·172	18 45 6·9	43·83
19	8 21 24·92	22·173	18 40 40·9	44·85
20	8 23 37·96	22·173	18 36 8·7	45·87
21	8 25 51·00	22·174	18 31 30·5	46·88
22	8 28 4·05	22·175	18 26 46·2	47·89
23	8 30 17·10	22·174	N. 18 21 55·8	48·90
	Saturday 18.			
0	8 32 30·14	22·173	N. 18 16 59·4	49·91
1	8 34 43·18	22·173	18 11 56·9	50·92
2	8 36 56·21	22·171	18 6 48·4	51·92
3	8 39 9·23	22·170	18 1 33·9	52·92
4	8 41 22·25	22·168	17 56 13·4	53·91
5	8 43 35·25	22·165	17 50 47·0	54·89
6	8 45 48·23	22·163	17 45 14·7	55·88
7	8 48 1·20	22·161	17 39 36·4	56·87
8	8 50 14·16	22·158	17 33 52·2	57·85
9	8 52 27·09	22·153	17 28 2·2	58·83
10	8 54 40·00	22·150	17 22 6·3	59·80
11	8 56 52·89	22·147	17 16 4·6	60·77
12	8 59 5·76	22·143	17 9 57·1	61·73
13	9 1 18·60	22·138	17 3 43·8	62·69
14	9 3 31·41	22·133	16 57 24·8	63·65
15	9 5 44·20	22·128	16 51 0·0	64·61
16	9 7 56·95	22·123	16 44 29·5	65·55
17	9 10 9·67	22·118	16 37 53·4	66·48
18	9 12 22·36	22·113	16 31 11·7	67·43
19	9 14 35·02	22·107	16 24 24·3	68·36
20	9 16 47·64	22·100	16 17 31·4	69·28
21	9 19 0·22	22·093	16 10 32·9	70·21
22	9 21 12·76	22·088	16 3 28·9	71·13
23	9 23 25·27	22·082	15 56 19·4	72·03
24	9 25 37·74	22·075	N. 15 49 4·5	72·93

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Sunday 19.			
	h. m. s.	s.	° ′ ″	″
0	9 25 37·74	22·075	N. 15 49 4·5	72·93
1	9 27 50·17	22·068	15 41 44·2	73·83
2	9 30 2·55	22·060	15 34 18·5	74·73
3	9 32 14·89	22·053	15 26 47·4	75·62
4	9 34 27·18	22·045	15 19 11·0	76·50
5	9 36 39·43	22·038	15 11 29·4	77·38
6	9 38 51·64	22·031	15 3 42·5	78·25
7	9 41 3·80	22·023	14 55 50·4	79·12
8	9 43 15·91	22·014	14 47 53·1	79·98
9	9 45 27·97	22·007	14 39 50·7	80·82
10	9 47 39·99	21·998	14 31 43·3	81·66
11	9 49 51·95	21·990	14 23 30·8	82·51
12	9 52 3·87	21·983	14 15 13·2	83·34
13	9 54 15·74	21·973	14 6 50·7	84·16
14	9 56 27·55	21·965	13 58 23·3	84·98
15	9 58 39·32	21·957	13 49 51·0	85·78
16	10 0 51·03	21·948	13 41 13·9	86·58
17	10 3 2·70	21·940	13 32 32·0	87·38
18	10 5 14·31	21·931	13 23 45·3	88·18
19	10 7 25·87	21·922	13 14 53·9	88·95
20	10 9 37·37	21·913	13 5 57·9	89·72
21	10 11 48·83	21·905	12 56 57·3	90·48
22	10 14 0·23	21·896	12 47 52·1	91·24
23	10 16 11·58	21·888	N. 12 38 42·4	91·98
	Monday 20.			
0	10 18 22·88	21·879	N. 12 29 28·3	92·73
1	10 20 34·13	21·870	12 20 9·7	93·47
2	10 22 45·32	21·862	12 10 46·7	94·19
3	10 24 56·47	21·854	12 1 19·4	94·90
4	10 27 7·57	21·845	11 51 47·9	95·61
5	10 29 18·61	21·837	11 42 12·1	96·32
6	10 31 29·61	21·828	11 32 32·1	97·01
7	10 33 40·55	21·820	11 22 48·0	97·69
8	10 35 51·45	21·812	11 12 59·8	98·37
9	10 38 2·29	21·803	11 3 7·6	99·03
10	10 40 13·09	21·796	10 53 11·4	99·69
11	10 42 23·84	21·788	10 43 11·3	100·34
12	10 44 34·55	21·781	10 33 7·3	100·98
13	10 46 45·21	21·773	10 22 59·5	101·61
14	10 48 55·82	21·765	10 12 48·0	102·23
15	10 51 6·39	21·758	10 2 32·8	102·84
16	10 53 16·92	21·751	9 52 13·9	103·45
17	10 55 27·40	21·744	9 41 51·4	104·04
18	10 57 37·85	21·738	9 31 25·4	104·63
19	10 59 48·25	21·730	9 20 55·9	105·20
20	11 1 58·61	21·723	9 10 23·0	105·77
21	11 4 8·93	21·718	8 59 46·7	106·33
22	11 6 19·22	21·712	8 49 7·1	106·88
23	11 8 29·47	21·705	8 38 24·2	107·41
24	11 10 39·68	21·699	N. 8 27 38·2	107·93

X. AUGUST.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Tuesday 21.			
	h. m s.	s.	° ′ ″	″
0	11 10 39·68	21·699	N. 8 27 38·2	107·93
1	11 12 49·86	21·694	8 16 49·0	108·45
2	11 15 0·01	21·689	8 5 56·8	108·96
3	11 17 10·13	21·683	7 55 1·5	109·46
4	11 19 20·21	21·678	7 44 3·3	109·94
5	11 21 30·27	21·674	7 33 2·2	110·43
6	11 23 40·30	21·670	7 21 58·2	110·89
7	11 25 50·31	21·667	7 10 51·5	111·35
8	11 28 0·30	21·663	6 59 42·0	111·80
9	11 30 10·26	21·658	6 48 29·9	112·23
10	11 32 20·20	21·656	6 37 15·2	112·67
11	11 34 30·13	21·653	6 25 57·9	113·08
12	11 36 40·03	21·649	6 14 38·2	113·48
13	11 38 49·92	21·648	6 3 16·1	113·88
14	11 40 59·80	21·645	5 51 51·6	114·27
15	11 43 9·66	21·643	5 40 24·9	114·64
16	11 45 19·52	21·643	5 28 55·9	115·01
17	11 47 29·37	21·641	5 17 24·8	115·36
18	11 49 39·21	21·640	5 5 51·6	115·71
19	11 51 49·05	21·640	4 54 16·3	116·04
20	11 53 58·89	21·640	4 42 39·1	116·37
21	11 56 8·73	21·640	4 30 59·9	116·68
22	11 58 18·57	21·640	4 19 18·9	116·98
23	12 0 28·41	21·641	N. 4 7 36·2	117·27
	Wednesday 22.			
0	12 2 38·26	21·643	N. 3 55 51·7	117·55
1	12 4 48·12	21·644	3 44 5·6	117·82
2	12 6 57·99	21·646	3 32 17·9	118·08
3	12 9 7·87	21·648	3 20 28·7	118·33
4	12 11 17·77	21·651	3 8 38·0	118·56
5	12 13 27·68	21·653	2 56 46·0	118·78
6	12 15 37·61	21·657	2 44 52·7	118·99
7	12 17 47·56	21·661	2 32 58·1	119·20
8	12 19 57·54	21·666	2 21 2·3	119·39
9	12 22 7·55	21·670	2 9 5·4	119·57
10	12 24 17·58	21·674	1 57 7·5	119·73
11	12 26 27·64	21·679	1 45 8·6	119·88
12	12 28 37·73	21·685	1 33 8·9	120·03
13	12 30 47·86	21·692	1 21 8·3	120·17
14	12 32 58·03	21·698	1 9 6·9	120·29
15	12 35 8·24	21·705	0 57 4·8	120·41
16	12 37 18·49	21·712	0 45 2·0	120·51
17	12 39 28·78	21·720	0 32 58·7	120·59
18	12 41 39·13	21·728	0 20 54·9	120·67
19	12 43 49·52	21·737	N. 0 8 50·7	120·73
20	12 45 59·97	21·746	S. 0 3 13·9	120·79
21	12 48 10·47	21·756	0 15 18·8	120·83
22	12 50 21·04	21·766	0 27 23·9	120·86
23	12 52 31·66	21·775	0 39 29·1	120·88
24	12 54 42·34	21·786	S. 0 51 34·4	120·89

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Thursday 23.			
	h. m. s.	s.	° ′ ″	″
0	12 54 42.34	21·786	S. 0 51 34·4	120·89
1	12 56 53·09	21·798	1 3 39·8	120·89
2	12 59 3·91	21·808	1 15 45·1	120·87
3	13 1 14·79	21·820	1 27 50·2	120·84
4	13 3 25·75	21·833	1 39 55·2	120·81
5	13 5 36·79	21·846	1 51 59·9	120·76
6	13 7 47·90	21·859	2 4 4·3	120·70
7	13 9 59·10	21·873	2 16 8·3	120·63
8	13 12 10·38	21·888	2 28 11·8	120·53
9	13 14 21·75	21·902	2 40 14·7	120·43
10	13 16 33·20	21·916	2 52 17·0	120·33
11	13 18 44·74	21·932	3 4 18·7	120·21
12	13 20 56·38	21·948	3 16 19·5	120·07
13	13 23 8·11	21·964	3 28 19·5	119·93
14	13 25 19·95	21·981	3 40 18·6	119·78
15	13 27 31·88	21·998	3 52 16·8	119·61
16	13 29 43·92	22·015	4 4 13·9	119·42
17	13 31 56·06	22·033	4 16 9·8	119·23
18	13 34 8·31	22·051	4 28 4·6	119·03
19	13 36 20·67	22·070	4 39 58·1	118·81
20	13 38 33·15	22·089	4 51 50·3	118·58
21	13 40 45·74	22·108	5 3 41·1	118·34
22	13 42 58·45	22·129	5 15 30·4	118·08
23	13 45 11·29	22·149	S. 5 27 18·1	117·82
	Friday 24.			
0	13 47 24·24	22·169	S. 5 39 4·2	117·54
1	13 49 37·32	22·192	5 50 48·6	117·26
2	13 51 50·54	22·213	6 2 31·3	116·96
3	13 54 3·88	22·235	6 14 12·1	116·65
4	13 56 17·36	22·258	6 25 51·1	116·33
5	13 58 30·97	22·280	6 37 28·0	115·98
6	14 0 44·72	22·303	6 49 2·9	115·64
7	14 2 58·61	22·328	7 0 35·7	115·28
8	14 5 12·65	22·352	7 12 6·2	114·90
9	14 7 26·83	22·375	7 23 34·5	114·52
10	14 9 41·15	22·400	7 35 0·5	114·13
11	14 11 55·63	22·425	7 46 24·0	113·71
12	14 14 10·25	22·450	7 57 45·0	113·29
13	14 16 25·03	22·476	8 9 3·5	112·86
14	14 18 39·96	22·503	8 20 19·3	112·41
15	14 20 55·06	22·529	8 31 32·4	111·95
16	14 23 10·31	22·555	8 42 42·7	111·48
17	14 25 25·72	22·583	8 53 50·2	111·00
18	14 27 41·30	22·610	9 4 54·7	110·51
19	14 29 57·04	22·638	9 15 56·3	110·01
20	14 32 12·95	22·666	9 26 54·8	109·49
21	14 34 29·03	22·694	9 37 50·2	108·96
22	14 36 45·28	22·723	9 48 42·3	108·42
23	14 39 1·71	22·753	9 59 31·2	107·87
24	14 41 18·31	22·782	S. 10 10 16·7	107·30

AUGUST. XI.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Saturday 25.			
	h. m. s.	s.	° ′ ″	″
0	14 41 18·31	22·782	S. 10 10 16·7	107·30
1	14 43 35·09	22·811	10 20 58·8	106·73
2	14 45 52·04	22·841	10 31 37·4	106·13
3	14 48 9·18	22·871	10 42 12·4	105·53
4	14 50 26·49	22·901	10 52 43·7	104·92
5	14 52 43·99	22·932	11 3 11·4	104·29
6	14 55 1·67	22·963	11 13 35·2	103·65
7	14 57 19·54	22·994	11 23 55·2	103·01
8	14 59 37·60	23·025	11 34 11·3	102·35
9	15 1 55·84	23·056	11 44 23·4	101·68
10	15 4 14·27	23·088	11 54 31·4	100·98
11	15 6 32·89	23·119	12 4 35·2	100·29
12	15 8 51·70	23·151	12 14 34·9	99·59
13	15 11 10·70	23·183	12 24 30·3	98·87
14	15 13 29·90	23·216	12 34 21·3	98·13
15	15 15 49·29	23·248	12 44 7·9	97·38
16	15 18 8·88	23·281	12 53 49·9	96·63
17	15 20 28·66	23·313	13 3 27·4	95·87
18	15 22 48·63	23·346	13 13 0·3	95·09
19	15 25 8·81	23·379	13 22 28·5	94·30
20	15 27 29·18	23·412	13 31 51·9	93·50
21	15 29 49·75	23·444	13 41 10·5	92·69
22	15 32 10·51	23·478	13 50 24·2	91·87
23	15 34 31·48	23·511	S. 13 59 32·9	91·03
	Sunday 26.			
0	15 36 52·64	23·543	S. 14 8 36·5	90·18
1	15 39 14·00	23·578	14 17 35·0	89·33
2	15 41 35·57	23·611	14 26 28·4	88·46
3	15 43 57·33	23·643	14 35 16·5	87·58
4	15 46 19·29	23·676	14 43 59·3	86·68
5	15 48 41·44	23·709	14 52 36·7	85·78
6	15 51 3·80	23·743	15 1 8·7	84·88
7	15 53 26·36	23·776	15 9 35·2	83·95
8	15 55 49·11	23·808	15 17 56·1	83·01
9	15 58 12·06	23·842	15 26 11·3	82·07
10	16 0 35·21	23·874	15 34 20·9	81·12
11	16 2 58·55	23·907	15 42 24·7	80·15
12	16 5 22·09	23·939	15 50 22·7	79·18
13	16 7 45·82	23·972	15 58 14·8	78·18
14	16 10 9·75	24·004	16 6 0·9	77·18
15	16 12 33·87	24·035	16 13 41·0	76·18
16	16 14 58·17	24·067	16 21 15·1	75·17
17	16 17 22·67	24·099	16 28 43·1	74·14
18	16 19 47·36	24·130	16 36 4·8	73·10
19	16 22 12·23	24·161	16 43 20·3	72·06
20	16 24 37·29	24·192	16 50 29·5	71·00
21	16 27 2·53	24·223	16 57 32·3	69·93
22	16 29 27·96	24·253	17 4 28·6	68·85
23	16 31 53·56	24·283	17 11 18·5	67·78
24	16 34 19·35	24·313	S. 17 18 1·9	66·68

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Monday 27.			
	h. m. s.	s.	° ′ ″	″
0	16 34 19·35	24·313	S. 17 18 1·9	66·68
1	16 36 45·31	24·342	17 24 38·7	65·58
2	16 39 11·45	24·371	17 31 8·8	64·47
3	16 41 37·76	24·399	17 37 32·3	63·35
4	16 44 4·24	24·428	17 43 49·0	62·22
5	16 46 30·89	24·456	17 49 58·9	61·08
6	16 48 57·71	24·483	17 56 2·0	59·93
7	16 51 24·68	24·509	18 1 58·1	58·78
8	16 53 51·82	24·537	18 7 47·3	57·62
9	16 56 19·12	24·563	18 13 29·5	56·45
10	16 58 46·57	24·588	18 19 4·7	55·27
11	17 1 14·17	24·613	18 24 32·7	54·08
12	17 3 41·92	24·638	18 29 53·6	52·88
13	17 6 9·82	24·662	18 35 7·3	51·68
14	17 8 37·86	24·686	18 40 13·8	50·48
15	17 11 6·05	24·709	18 45 13·0	49·26
16	17 13 34·37	24·731	18 50 4·9	48·04
17	17 16 2·82	24·753	18 54 49·5	46·81
18	17 18 31·40	24·774	18 59 26·6	45·57
19	17 21 0·11	24·795	19 3 56·3	44·33
20	17 23 28·94	24·816	19 8 18·6	43·08
21	17 25 57·90	24·836	19 12 33·3	41·83
22	17 28 26·97	24·853	19 16 40·5	40·57
23	17 30 56·14	24·872	S. 19 20 40·1	39·29
	Tuesday 28.			
0	17 33 25·43	24·890	S. 19 24 32·0	38·02
1	17 35 54·82	24·907	19 28 16·3	36·75
2	17 38 24·31	24·923	19 31 53·0	35·47
3	17 40 53·90	24·939	19 35 21·9	34·18
4	17 43 23·58	24·953	19 38 43·1	32·88
5	17 45 53·34	24·967	19 41 56·5	31·58
6	17 48 23·18	24·981	19 45 2·1	30·28
7	17 50 53·11	24·994	19 47 59·9	28·98
8	17 53 23·11	25·005	19 50 49·9	27·68
9	17 55 53·17	25·016	19 53 32·0	26·36
10	17 58 23·30	25·027	19 56 6·2	25·04
11	18 0 53·49	25·037	19 58 32·5	23·72
12	18 3 23·74	25·046	20 0 50·9	22·40
13	18 5 54·04	25·053	20 3 1·3	21·08
14	18 8 24·38	25·060	20 5 3·8	19·75
15	18 10 54·76	25·067	20 6 58·3	18·43
16	18 13 25·18	25·073	20 8 44·9	17·09
17	18 15 55·63	25·077	20 10 23·4	15·75
18	18 18 26·10	25·081	20 11 53·9	14·42
19	18 20 56·60	25·084	20 13 16·4	13·08
20	18 23 27·11	25·087	20 14 30·9	11·74
21	18 25 57·64	25·088	20 15 37·3	10·40
22	18 28 28·17	25·088	20 16 35·7	9·06
23	18 30 58·70	25·088	20 17 26·0	7·71
24	18 33 29·23	25·088	S. 20 18 8·2	6·37

XII. AUGUST.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Wednesday 29.			
	h. m. s.	s.	° ′ ″	″
0	18 33 29·23	25·088	S. 20 18 8·2	6·37
1	18 35 59·75	25·085	20 18 42·4	5·03
2	18 38 30·25	25·083	20 19 8·5	3·68
3	18 41 0·74	25·079	20 19 26·6	2·34
4	18 43 31·20	25·074	20 19 36·6	1·00
5	18 46 1·63	25·069	20 19 38·6	0·34
6	18 48 32·03	25·063	20 19 32·5	1·68
7	18 51 2·38	25·055	20 19 18·4	3·03
8	18 53 32·69	25·048	20 18 56·2	4·37
9	18 56 2·95	25·039	20 18 26·0	5·70
10	18 58 33·16	25·030	20 17 47·8	7·03
11	19 1 3·31	25·018	20 17 1·7	8·36
12	19 3 33·38	25·007	20 16 7·5	9·69
13	19 6 3·39	24·995	20 15 5·4	11·02
14	19 8 33·32	24·982	20 13 55·3	12·35
15	19 11 3·17	24·968	20 12 37·2	13·67
16	19 13 32·93	24·953	20 11 11·3	14·98
17	19 16 2·60	24·938	20 9 37·4	16·30
18	19 18 32·18	24·921	20 7 55·7	17·61
19	19 21 1·65	24·903	20 6 6·1	18·92
20	19 23 31·02	24·886	20 4 8·7	20·22
21	19 26 0·28	24·868	20 2 3·5	21·52
22	19 28 29·43	24·848	19 59 50·5	22·82
23	19 30 58·45	24·827	S. 19 57 29·7	24·11
	Thursday 30.			
0	19 33 27·35	24·806	S. 19 55 1·2	25·39
1	19 35 56·12	24·784	19 52 25·0	26·68
2	19 38 24·76	24·761	19 49 41·1	27·95
3	19 40 53·25	24·737	19 46 49·6	29·22
4	19 43 21·60	24·713	19 43 50·5	30·48
5	19 45 49·81	24·689	19 40 43·9	31·73
6	19 48 17·87	24·663	19 37 29·7	32·99
7	19 50 45·76	24·636	19 34 8·0	34·23
8	19 53 13·50	24·609	19 30 38·9	35·47
9	19 55 41·07	24·581	19 27 2·3	36·71
10	19 58 8·47	24·553	19 23 18·4	37·93
11	20 0 35·70	24·523	19 19 27·2	39·14
12	20 3 2·75	24·493	19 15 28·7	40·36
13	20 5 29·62	24·463	19 11 22·9	41·57
14	20 7 56·30	24·432	19 7 9·9	42·76
15	20 10 22·80	24·400	19 2 49·8	43·94
16	20 12 49·10	24·367	18 58 22·6	45·13
17	20 15 15·20	24·334	18 53 48·3	46·30
18	20 17 41·11	24·301	18 49 7·0	47·47
19	20 20 6·81	24·266	18 44 18·7	48·63
20	20 22 32·30	24·232	18 39 23·5	49·77
21	20 24 57·59	24·197	18 34 21·5	50·91
22	20 27 22·66	24·161	18 29 12·6	52·04
23	20 29 47·52	24·124	18 23 57·0	53·16
24	20 32 12·15	24·087	S. 18 18 34·7	54·27

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Friday 31.			
	h. m. s.	s.	° ′ ″	″
0	20 32 12·15	24·087	S. 18 18 34·7	54·27
1	20 34 36·56	24·050	18 13 5·8	55·37
2	20 37 0·75	24·013	18 7 30·3	56·47
3	20 39 24·71	23·974	18 1 48·2	57·56
4	20 41 48·44	23·935	17 55 59·6	58·63
5	20 44 11·93	23·896	17 50 4·7	59·68
6	20 46 35·19	23·857	17 44 3·4	60·74
7	20 48 58·21	23·817	17 37 55·8	61·79
8	20 51 20·99	23·777	17 31 41·9	62·83
9	20 53 43·53	23·736	17 25 21·9	63·85
10	20 56 5·82	23·695	17 18 55·7	64·87
11	20 58 27·87	23·654	17 12 23·5	65·87
12	21 0 49·67	23·613	17 5 45·3	66·87
13	21 3 11·22	23·570	16 59 1·1	67·85
14	21 5 32·51	23·528	16 52 11·1	68·82
15	21 7 53·55	23·486	16 45 15·3	69·78
16	21 10 14·34	23·443	16 38 13·7	70·73
17	21 12 34·87	23·399	16 31 6·5	71·68
18	21 14 55·13	23·356	16 23 53·6	72·61
19	21 17 15·14	23·313	16 16 35·2	73·53
20	21 19 34·88	23·268	16 9 11·3	74·43
21	21 21 54·36	23·225	16 1 42·0	75·33
22	21 24 13·58	23·181	15 54 7·4	76·21
23	21 26 32·53	23·136	S. 15 46 27·5	77·08
	Saturday, SEPT. 1.			
0	21 28 51·21	23·091	S. 15 38 42·4	77·94

PHASES OF THE MOON.

		h. m.
Aug. 4	○ Full Moon . .	0 59·7
11	☾ *Last Quarter* . .	14 47·5
19	● New Moon . .	13 27·5
26	☽ *First Quarter* .	12 42·5

		h.
Aug. 12	☾ Apogee . . .	17·8
26	☾ Perigee . . .	21·5

AUGUST. XIII.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.		Noon.	P.L. of diff.	IIIh.	P.L. of diff.	VIh.	P.L. of diff.	IXh.	P.L. of diff.
			° ′ ″		° ′ ″		° ′ ″		° ′ ″	
1	Spica	W.	65 16 56	2222	67 4 51	2221	68 52 47	2220	70 40 44	2220
	Fomalhaut	E.	68 13 17	2476	66 31 30	2485	64 49 55	2495	63 8 34	2506
	Saturn	E.	76 35 21	2167	74 46 3	2168	72 56 47	2168	71 7 31	2169
	α Pegasi	E.	83 40 36	2541	82 0 20	2545	80 20 9	2550	78 40 4	2556
2	Spica	W.	79 40 6	2231	81 27 48	2235	83 15 24	2239	85 2 55	2243
	Antares	W.	34 21 41	2372	36 5 56	2363	37 50 24	2355	39 35 4	2349
	Saturn	E.	62 1 48	2181	60 12 52	2185	58 24 1	2189	56 35 17	2194
	α Pegasi	E.	70 22 16	2603	68 43 25	2618	67 4 54	2634	65 26 44	2651
	α Arietis	E.	118 21 20	2380	111 37 16	2379	109 53 11	2380	108 9 7	2381
3	Spica	W.	93 58 30	2274	95 45 8	2282	97 31 34	2290	99 17 48	2299
	Antares	W.	48 19 51	2342	50 4 50	2344	51 49 45	2348	53 34 35	2352
	Saturn	E.	47 33 38	2225	45 45 47	2233	43 58 8	2241	42 10 41	2249
	α Pegasi	E.	57 22 37	2768	55 47 27	2798	54 12 56	2831	52 39 8	2868
	α Arietis	E.	99 29 37	2400	97 46 2	2406	96 2 36	2413	94 19 20	2421
4	Spica	W.	108 5 27	2351	109 50 12	2363	111 34 40	2375	113 18 50	2388
	Antares	W.	62 16 45	2387	64 0 39	2395	65 44 21	2405	67 27 49	2415
	Saturn	E.	33 16 47	2298	31 30 45	2309	29 44 59	2321	27 59 30	2333
	α Arietis	E.	85 46 7	2471	84 4 13	2483	82 22 36	2496	80 41 17	2509
	Aldebaran	E.	117 43 6	2315	115 57 29	2326	114 12 8	2337	112 27 3	2349
5	Antares	W.	76 1 16	2473	77 43 7	2486	79 24 40	2499	81 5 54	2513
	α Arietis	E.	72 19 38	2586	70 40 24	2604	69 1 34	2622	67 23 8	2640
	Aldebaran	E.	103 46 4	2413	102 2 48	2427	100 19 52	2441	98 37 15	2455
6	Antares	W.	89 27 11	2586	91 6 25	2601	92 45 19	2616	94 23 52	2632
	α Aquilæ	W.	47 56 31	3623	49 14 41	3584	50 33 33	3550	51 53 2	3520
	α Arietis	E.	59 17 39	2745	57 41 59	2769	56 6 50	2793	54 32 12	2818
	Aldebaran	E.	90 9 20	2530	88 28 48	2544	86 48 36	2559	85 8 45	2575
	Jupiter	E.	113 11 10	2600	111 32 15	2616	109 53 42	2632	108 15 30	2647
7	Antares	W.	102 31 12	2713	104 7 35	2729	105 43 36	2746	107 19 15	2762
	α Aquilæ	W.	58 37 6	3430	59 58 49	3421	61 20 42	3413	62 42 44	3408
	α Arietis	E.	46 47 45	2962	45 16 45	2996	43 46 27	3031	42 16 53	3069
	Aldebaran	E.	76 54 52	2653	75 17 9	2668	73 39 46	2684	72 2 44	2699
	Jupiter	E.	100 9 43	2725	98 33 36	2741	96 57 51	2756	95 22 25	2772
8	α Aquilæ	W.	69 33 54	3402	70 56 9	3405	72 18 20	3408	73 40 27	3412
	Saturn	W.	20 39 30	2755	22 14 57	2769	23 50 5	2784	25 24 54	2797
	Aldebaran	E.	64 2 40	2775	62 27 39	2788	60 52 56	2802	59 18 31	2817
	Jupiter	E.	87 30 19	2847	85 56 52	2862	84 23 45	2876	82 50 56	2891
	Pollux	E.	108 4 3	2826	106 30 9	2839	104 56 32	2852	103 23 12	2866
	Sun	E.	130 41 2	3114	129 13 9	3129	127 45 35	3144	126 18 19	3160
9	α Aquilæ	W.	80 29 35	3445	81 51 2	3453	83 12 19	3461	84 33 27	3470
	Saturn	W.	33 14 42	2862	34 47 49	2874	36 20 41	2886	37 53 18	2897
	Aldebaran	E.	51 30 56	2883	49 58 16	2895	48 25 51	2908	46 53 42	2919
	Jupiter	E.	75 11 16	2958	73 40 11	2970	72 9 21	2982	70 38 46	2994
	Pollux	E.	95 40 47	2930	94 9 6	2942	92 37 41	2954	91 6 31	2965
	Sun	E.	119 6 30	3232	117 40 59	3245	116 15 43	3258	114 50 42	3270

XIV. AUGUST.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	*Midnight.*	P.L. of diff.	XV^h.	P.L. of diff.	XVIII^h.	P.L. of diff.	XXI^h.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
1	Spica W.	72 28 41	2222	74 16 36	2223	76 4 29	2225	77 52 19	2228
	Fomalhaut E.	61 27 29	2519	59 46 42	2533	58 6 15	2549	56 26 10	2568
	Saturn E.	69 18 16	2170	67 29 4	2172	65 39 55	2175	63 50 49	2178
	α Pegasi E.	77 0 8	2562	75 20 21	2570	73 40 45	2580	72 1 23	2591
2	Spica W.	86 50 19	2248	88 37 35	2254	90 24 43	2260	92 11 41	2267
	Antares W.	41 19 53	2344	43 4 48	2341	44 49 48	2339	46 34 50	2340
	Saturn E.	54 46 40	2199	52 58 11	2205	51 9 51	2211	49 21 40	2217
	α Pegasi E.	63 48 58	2670	62 11 38	2690	60 34 45	2713	58 58 23	2740
	α Arietis E.	106 25 4	2383	104 41 5	2386	102 57 10	2389	101 13 20	2394
3	Spica W.	101 3 49	2309	102 49 36	2319	104 35 8	2329	106 20 25	2340
	Antares W.	55 19 19	2358	57 3 55	2364	58 48 22	2371	60 32 39	2379
	Saturn E.	40 23 26	2258	38 36 25	2268	36 49 38	2277	35 3 5	2287
	α Pegasi E.	51 6 8	2908	49 33 59	2952	48 2 46	3000	46 32 33	3053
	α Arietis E.	92 36 15	2429	90 53 22	2439	89 10 43	2449	87 28 18	2459
4	Spica W.	115 2 42	2401	116 46 15	2416	118 29 27	2430	120 12 19	2444
	Antares W.	69 11 2	2426	70 54 0	2437	72 36 42	2449	74 19 7	2460
	Saturn E.	26 14 18	2345	24 29 24	2358	22 44 49	2371	21 0 33	2384
	α Arietis E.	79 0 16	2523	77 19 35	2538	75 39 15	2553	73 59 16	2569
	Aldebaran E.	110 42 15	2362	108 57 45	2374	107 13 33	2387	105 29 39	2400
5	Antares W.	82 46 49	2527	84 27 24	2541	86 7 40	2556	87 47 36	2571
	α Arietis E.	65 45 8	2660	64 7 34	2681	62 30 28	2701	60 53 49	2722
	Aldebaran E.	96 54 59	2470	95 13 3	2485	93 31 28	2499	91 50 14	2514
6	Antares W.	96 2 3	2649	97 39 52	2665	99 17 20	2681	100 54 26	2696
	α Aquilæ W.	53 13 4	3496	54 33 33	3475	55 54 25	3457	57 15 37	3442
	α Arietis E.	52 58 8	2844	51 24 37	2872	49 51 42	2901	48 19 24	2931
	Aldebaran E.	83 29 16	2591	81 50 8	2607	80 11 22	2622	78 32 57	2637
	Jupiter E.	106 37 39	2663	105 0 9	2678	103 22 59	2694	101 46 11	2709
7	Antares W.	108 54 33	2779	110 29 29	2795	112 4 4	2811	113 38 17	2828
	α Aquilæ W.	64 4 52	3404	65 27 5	3401	66 49 20	3400	68 11 37	3400
	α Arietis E.	40 48 6	3110	39 20 8	3153	37 53 1	3199	36 26 50	3249
	Aldebaran E.	70 26 3	2715	68 49 43	2729	67 13 42	2744	65 38 1	2760
	Jupiter E.	93 47 20	2787	92 12 35	2802	90 38 10	2818	89 4 5	2832
8	α Aquilæ W.	75 2 30	3418	76 24 27	3424	77 46 16	3430	79 7 59	3437
	Saturn W.	26 59 26	2811	28 33 40	2824	30 7 37	2836	31 41 18	2849
	Aldebaran E.	57 44 25	2831	56 10 37	2844	54 37 7	2857	53 3 53	2870
	Jupiter E.	81 18 25	2905	79 46 12	2919	78 14 17	2931	76 42 38	2945
	Pollux E.	101 50 10	2879	100 17 25	2892	98 44 56	2905	97 12 44	2917
	Sun E.	124 51 22	3175	123 24 43	3190	121 58 22	3204	120 32 18	3218
9	α Aquilæ W.	85 54 25	3479	87 15 13	3488	88 35 50	3498	89 56 17	3508
	Saturn W.	39 25 41	2908	40 57 49	2919	42 29 44	2929	44 1 25	2939
	Aldebaran E.	45 21 47	2931	43 50 7	2942	42 18 41	2952	40 47 28	2962
	Jupiter E.	69 8 26	3006	67 38 21	3018	66 8 30	3028	64 38 52	3039
	Pollux E.	89 35 35	2977	88 4 54	2988	86 34 26	2999	85 4 12	3009
	Sun E.	113 25 56	3283	112 1 25	3295	110 37 8	3306	109 13 4	3317

AUGUST. XV.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.		Noon.			P.L. of diff.	IIIh.			P.L. of diff.	VIh.			P.L. of diff.	IXh.			P.L. of diff.
			°	′	″		°	′	″		°	′	″		°	′	″	
10	α Aquilæ	W.	91	16	32	3519	92	36	35	3529	93	56	27	3540	95	16	7	3551
	Fomalhaut	W.	56	56	54	3327	58	20	34	3324	59	44	17	3323	61	8	2	3322
	Saturn	W.	45	32	54	2949	47	4	10	2958	48	35	15	2967	50	6	9	2976
	α Pegasi	W.	43	47	13	3765	45	2	52	3733	46	19	4	3705	47	35	46	3680
	Aldebaran	E.	39	16	28	2972	37	45	41	2981	36	15	5	2990	34	44	40	2999
	Jupiter	E.	63	9	27	3049	61	40	15	3058	60	11	14	3067	58	42	24	3076
	Pollux	E.	83	34	10	3019	82	4	21	3029	80	34	45	3039	79	5	20	3047
	Sun	E.	107	49	12	3328	106	25	33	3338	105	2	6	3348	103	38	50	3358
11	α Aquilæ	W.	101	51	17	3611	103	9	39	3624	104	27	48	3637	105	45	42	3651
	Fomalhaut	W.	68	6	56	3321	69	30	43	3322	70	54	29	3321	72	18	16	3321
	Saturn	W.	57	38	12	3010	59	8	12	3017	60	38	4	3022	62	7	50	3026
	α Pegasi	W.	54	5	7	3587	55	23	55	3573	56	42	59	3560	58	2	17	3549
	Aldebaran	E.	27	15	7	3035	25	45	38	3042	24	16	17	3047	22	47	2	3052
	Jupiter	E.	51	20	48	3114	49	52	55	3120	48	25	10	3126	46	57	32	3131
	Pollux	E.	71	40	49	3086	70	12	23	3093	68	44	5	3100	67	15	55	3105
	Sun	E.	96	44	58	3396	95	22	37	3403	94	0	24	3408	92	38	17	3413
12	Fomalhaut	W.	79	17	7	3322	80	40	53	3322	82	4	39	3321	83	28	26	3321
	Saturn	W.	69	35	23	3043	71	4	43	3045	72	34	0	3046	74	3	16	3047
	α Pegasi	W.	64	41	40	3501	66	2	3	3494	67	22	34	3486	68	43	14	3478
	Jupiter	E.	39	40	48	3151	38	13	41	3154	36	46	37	3157	35	19	36	3159
	Pollux	E.	59	56	46	3130	58	29	14	3134	57	1	46	3138	55	34	22	3142
	Sun	E.	85	48	58	3432	84	27	18	3434	83	5	40	3436	81	44	4	3437
13	Fomalhaut	W.	90	27	33	3316	91	51	26	3314	93	15	21	3313	94	39	18	3311
	Saturn	W.	81	29	31	3044	82	58	49	3042	84	28	9	3039	85	57	33	3037
	α Pegasi	W.	75	28	35	3444	76	50	1	3438	78	11	35	3431	79	33	16	3425
	α Arietis	W.	32	5	5	3652	33	22	43	3606	34	41	11	3565	36	0	24	3527
	Jupiter	E.	28	5	5	3167	26	38	16	3168	25	11	29	3168	23	44	42	3169
	Pollux	E.	48	18	22	3155	46	51	19	3158	45	24	20	3160	43	57	23	3163
	Sun	E.	74	56	10	3434	73	34	32	3432	72	12	52	3430	70	51	9	3427
14	Fomalhaut	W.	101	39	36	3301	103	3	47	3299	104	28	0	3296	105	52	16	3294
	Saturn	W.	93	25	33	3016	94	55	26	3011	96	25	25	3005	97	55	31	2998
	α Pegasi	W.	86	23	28	3394	87	45	52	3388	89	8	22	3382	90	30	59	3376
	α Arietis	W.	42	45	43	3382	44	8	20	3359	45	31	23	3337	46	54	52	3316
	Pollux	E.	36	43	31	3180	35	16	58	3185	33	50	31	3192	32	24	12	3200
	Sun	E.	64	1	33	3405	62	39	22	3399	61	17	4	3393	59	54	39	3386
15	Saturn	W.	105	28	10	2962	106	59	10	2954	108	30	20	2946	110	1	41	2936
	α Pegasi	W.	97	25	47	3347	98	49	4	3342	100	12	28	3337	101	35	57	3332
	α Arietis	W.	53	58	0	3224	55	23	41	3207	56	49	41	3190	58	16	2	3174
	Sun	E.	53	0	30	3347	51	37	13	3338	50	13	45	3328	48	50	6	3319
16	α Arietis	W.	65	32	28	3098	67	0	40	3083	68	29	10	3069	69	57	58	3055
	Sun	E.	41	49	2	3267	40	24	12	3255	38	59	8	3244	37	33	51	3233
17	α Arietis	W.	77	26	17	2985	78	56	49	2972	80	27	37	2958	81	58	42	2945
	Sun	E.	30	24	0	3173	28	57	18	3161	27	30	22	3148	26	3	11	3137
22	Sun	W.	30	47	40	2723	32	23	49	2716	34	0	7	2709	35	36	35	2702

XVI. AUGUST.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	*Midnight.*	P.L. of diff.	XV^h.	P.L. of diff.	XVIII^h.	P.L. of diff.	XXI^h.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
10	α Aquilæ W.	96 35 35	3563	97 54 50	3575	99 13 52	3587	100 32 41	3598
	Fomalhaut W.	62 31 48	3322	63 55 34	3321	65 19 21	3321	66 43 8	3320
	Saturn W.	51 36 52	2984	53 7 25	2991	54 37 49	2997	56 8 5	3004
	α Pegasi W.	48 52 54	3657	50 10 27	3637	51 28 21	3619	52 46 35	3602
	Aldebaran E.	33 14 26	3008	31 44 23	3015	30 14 29	3022	28 44 44	3029
	Jupiter E.	57 13 45	3085	55 45 17	3093	54 16 58	3101	52 48 49	3107
	Pollux E.	77 36 5	3056	76 7 1	3065	74 38 8	3072	73 9 24	3079
	Sun E.	102 15 45	3366	100 52 50	3374	99 30 4	3382	98 7 27	3389
11	α Aquilæ W.	107 3 22	3665	108 20 46	3679	109 37 55	3694	110 54 48	3709
	Fomalhaut W.	73 42 8	3322	75 5 49	3322	76 29 35	3322	77 53 21	3322
	Saturn W.	63 37 30	3030	65 7 4	3034	66 36 34	3037	68 6 0	3040
	α Pegasi W.	59 21 47	3538	60 41 29	3528	62 1 22	3518	63 21 26	3509
	Aldebaran E.	21 17 54	3056	19 48 51	3061	18 19 54	3065	16 51 2	3069
	Jupiter E.	45 30 0	3136	44 2 35	3140	42 35 15	3144	41 7 59	3148
	Pollux E.	65 47 52	3111	64 19 56	3117	62 52 7	3122	61 24 24	3126
	Sun E.	91 16 15	3418	89 54 19	3422	88 32 28	3426	87 10 41	3429
12	Fomalhaut W.	84 52 13	3320	86 16 1	3319	87 39 50	3318	89 3 41	3317
	Saturn W.	75 32 31	3047	77 1 45	3047	78 30 59	3046	80 0 14	3045
	α Pegasi W.	70 4 3	3471	71 24 59	3464	72 46 3	3457	74 7 15	3450
	Jupiter E.	33 52 38	3161	32 25 42	3163	30 58 48	3164	29 31 56	3165
	Pollux E.	54 7 3	3145	52 39 48	3148	51 12 36	3150	49 45 27	3153
	Sun E.	80 22 29	3438	79 0 55	3438	77 39 21	3437	76 17 46	3436
13	Fomalhaut W.	96 3 17	3309	97 27 18	3307	98 51 22	3305	100 15 28	3303
	Saturn W.	87 27 0	3034	88 56 31	3030	90 26 6	3026	91 55 46	3021
	α Pegasi W.	80 55 4	3419	82 16 59	3412	83 39 2	3406	85 1 12	3400
	α Arietis W.	37 20 18	3493	38 40 50	3463	40 1 56	3434	41 23 34	3407
	Jupiter E.	22 17 56	3171	20 51 12	3173	19 24 31	3177	17 57 54	3182
	Pollux E.	42 30 30	3166	41 3 40	3169	39 36 53	3172	38 10 10	3176
	Sun E.	69 29 23	3423	68 7 33	3419	66 45 38	3415	65 23 38	3410
14	Fomalhaut W.	107 16 35	3292	108 40 56	3290	110 5 19	3288	111 29 45	3286
	Saturn W.	99 25 46	2992	100 56 9	2985	102 26 40	2978	103 57 20	2970
	α Pegasi W.	91 53 43	3370	93 16 34	3364	94 39 32	3358	96 2 36	3352
	α Arietis W.	48 18 45	3296	49 43 1	3277	51 7 39	3259	52 32 39	3241
	Pollux E.	30 58 2	3209	29 32 3	3221	28 6 19	3236	26 40 52	3253
	Sun E.	58 32 6	3379	57 9 25	3372	55 46 36	3364	54 23 38	3355
15	Saturn W.	111 33 14	2927	113 4 58	2918	114 36 54	2908	116 9 3	2898
	α Pegasi W.	102 59 32	3327	104 23 13	3323	105 46 58	3319	107 10 48	3314
	α Arietis W.	59 42 42	3158	61 9 41	3143	62 36 58	3128	64 4 34	3113
	Sun E.	47 26 17	3310	46 2 17	3299	44 38 4	3288	43 13 39	3278
16	α Arietis W.	71 27 3	3041	72 56 25	3026	74 26 5	3012	75 56 2	2998
	Sun E.	36 8 21	3221	34 42 37	3209	33 16 39	3198	31 50 27	3185
17	α Arietis W.	83 30 4	2933	85 1 41	2920	86 33 35	2907	88 5 45	2894
	Sun E.	24 35 46	3124	23 8 6	3112	21 40 11	3101	20 12 2	3088
22	Sun W.	37 13 13	2695	38 49 59	2689	40 26 54	2683	42 3 58	2677

AUGUST. XVII.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	Noon.	P.L. of diff.	IIIh.	P.L. of d.ff.	VIh.	P.L. of diff.	IXh.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
22	α Aquilæ E.	114 52 11	3081	113 23 38	3057	111 54 36	3035	110 25 7	3015
23	Sun W.	43 41 9	2671	45 18 28	2666	46 55 53	2660	48 33 26	2655
	α Aquilæ E.	102 51 54	2935	101 20 19	2922	99 48 28	2912	98 16 24	2903
24	Sun W.	56 42 45	2635	58 20 53	2631	59 59 6	2627	61 37 24	2624
	α Aquilæ E.	90 33 31	2872	89 0 36	2869	87 27 37	2867	85 54 36	2867
25	Sun W.	69 49 55	2610	71 28 36	2608	73 7 20	2606	74 46 7	2604
	α Aquilæ E.	78 9 50	2881	76 37 7	2887	75 4 32	2895	73 32 7	2905
	Fomalhaut E.	111 52 35	2588	110 13 24	2580	108 34 2	2573	106 54 30	2566
26	Sun W.	83 0 34	2597	84 39 33	2596	86 18 33	2596	87 57 34	2595
	Spica W.	33 30 38	2393	35 14 23	2384	36 58 21	2375	38 42 31	2368
	α Aquilæ E.	65 53 38	2975	64 22 54	2995	62 52 35	3017	61 22 43	3041
	Fomalhaut E.	98 34 46	2542	96 54 31	2539	95 14 12	2537	93 33 50	2535
	Saturn E.	107 10 29	2267	105 23 41	2266	103 36 52	2265	101 50 1	2264
	α Pegasi E.	113 16 54	2708	111 40 25	2697	110 3 41	2687	108 26 44	2679
27	Sun W.	96 12 53	2593	97 51 58	2594	99 31 1	2594	101 10 5	2594
	Spica W.	47 25 35	2344	49 10 30	2341	50 55 30	2339	52 40 34	2337
	α Aquilæ E.	54 2 2	3209	52 36 3	3255	51 10 59	3305	49 46 53	3360
	Fomalhaut E.	85 11 41	2536	83 31 17	2538	81 50 56	2540	80 10 38	2543
	Saturn E.	92 55 33	2262	91 8 38	2263	89 21 44	2264	87 34 51	2263
	α Pegasi E.	100 19 27	2649	98 41 38	2645	97 3 44	2643	95 25 47	2640
28	Sun W.	109 25 9	2599	111 4 5	2601	112 42 59	2602	114 21 51	2604
	Spica W.	61 26 25	2332	63 11 38	2331	64 56 52	2332	66 42 5	2332
	Fomalhaut E.	71 50 31	2569	70 10 53	2577	68 31 26	2585	66 52 11	2594
	Saturn E.	78 40 32	2267	76 53 44	2268	75 6 58	2270	73 20 14	2272
	α Pegasi E.	87 15 46	2643	85 37 50	2647	83 59 59	2651	82 22 13	2655
29	Sun W.	122 35 24	2618	124 13 55	2621	125 52 22	2624	127 30 45	2628
	Spica W.	75 27 49	2340	77 12 51	2342	78 57 49	2344	80 42 44	2347
	Antares W.	30 18 55	2525	31 59 34	2507	33 40 37	2493	35 22 0	2481
	Fomalhaut E.	58 39 37	2660	57 2 2	2677	55 24 51	2696	53 48 6	2716
	Saturn E.	64 27 17	2283	62 40 52	2285	60 54 31	2288	59 8 14	2291
	α Pegasi E.	74 15 18	2692	72 38 27	2702	71 1 49	2713	69 25 26	2726
	α Arietis E.	117 25 13	2499	115 43 59	2497	114 2 42	2495	112 21 22	2494
30	Spica W.	89 26 6	2366	91 10 30	2371	92 54 46	2376	94 38 56	2381
	Antares W.	43 52 12	2450	45 34 36	2448	47 17 3	2446	48 59 32	2445
	Saturn E.	50 18 7	2311	48 32 23	2315	46 46 46	2320	45 1 16	2325
	α Pegasi E.	61 28 22	2810	59 54 7	2831	58 20 20	2855	56 47 4	2882
	α Arietis E.	103 54 30	2497	102 13 12	2499	100 31 58	2502	98 50 48	2506
31	Spica W.	103 17 40	2412	105 0 57	2420	106 44 3	2427	108 26 59	2435
	Antares W.	57 31 39	2456	59 13 54	2460	60 56 3	2465	62 38 6	2470
	Saturn E.	36 15 45	2355	34 31 6	2362	32 46 36	2368	31 2 16	2376
	α Pegasi E.	49 10 8	3054	47 41 2	3099	46 12 51	3149	44 45 40	3204
	α Arietis E.	90 26 27	2532	88 45 58	2539	87 5 39	2546	85 25 30	2554
	Aldebaran E.	122 33 24	2379	120 49 19	2385	119 5 23	2392	117 21 37	2399

XVIII. AUGUST.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	*Midnight.*	P.L. of diff.	XV^h.	P.L. of diff.	XVIII^h.	P.L. of diff.	XXI^h.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
22	α Aquilæ E.	108 55 13	2996	107 24 55	2978	105 54 14	2962	104 23 13	2948
23	Sun W.	50 11 6	2651	51 48 52	2646	53 26 44	2642	55 4 42	2638
	α Aquilæ E.	96 44 8	2894	95 11 41	2887	93 39 5	2881	92 6 21	2876
24	Sun W.	63 15 47	2621	64 54 13	2618	66 32 43	2615	68 11 17	2612
	α Aquilæ E.	84 21 35	2867	82 48 34	2869	81 15 35	2872	79 42 40	2876
25	Sun W.	76 24 56	2603	78 3 47	2601	79 42 41	2599	81 21 37	2597
	α Aquilæ E.	71 59 54	2915	70 27 54	2927	68 56 10	2942	67 24 44	2958
	Fomalhaut E.	105 14 49	2560	103 34 59	2554	101 55 1	2549	100 14 56	2545
26	Sun W.	89 36 37	2594	91 15 40	2593	92 54 44	2593	94 33 48	2593
	Spica W.	40 26 52	2362	42 11 22	2357	43 55 59	2352	45 40 44	2347
	α Aquilæ E.	59 53 21	3068	58 24 32	3099	56 56 21	3132	55 28 50	3168
	Fomalhaut E.	91 53 25	2534	90 12 59	2534	88 32 33	2534	86 52 7	2534
	Saturn E.	100 3 9	2264	98 16 16	2263	96 29 22	2263	94 42 28	2262
	α Pegasi E.	106 49 36	2671	105 12 17	2663	103 34 48	2657	101 57 11	2653
27	Sun W.	102 49 8	2595	104 28 10	2596	106 7 11	2596	107 46 11	2598
	Spica W.	54 25 40	2335	56 10 49	2334	57 55 59	2333	59 41 12	2332
	α Aquilæ E.	48 23 51	3422	47 1 59	3491	45 41 25	3568	44 22 16	3653
	Fomalhaut E.	78 30 24	2547	76 50 15	2551	75 10 13	2556	73 30 18	2562
	Saturn E.	85 47 57	2264	84 1 4	2264	82 14 12	2265	80 27 21	2266
	α Pegasi E.	93 47 47	2640	92 9 46	2640	90 31 45	2640	88 53 45	2641
28	Sun W.	116 0 40	2607	117 39 26	2609	119 18 9	2612	120 56 48	2615
	Spica W.	68 27 18	2333	70 12 29	2335	71 57 38	2336	73 42 45	2338
	Fomalhaut E.	65 13 8	2605	63 34 19	2617	61 55 47	2630	60 17 32	2644
	Saturn E.	71 33 33	2274	69 46 54	2276	68 0 19	2278	66 13 46	2280
	α Pegasi E.	80 44 33	2660	79 7 0	2667	77 29 36	2675	75 52 21	2683
29	Sun W.	129 9 2	2632	130 47 13	2636	132 25 19	2641	134 3 19	2646
	Spica W.	82 27 35	2351	84 12 21	2355	85 57 1	2358	87 41 36	2362
	Antares W.	37 3 40	2471	38 45 34	2464	40 27 39	2458	42 9 52	2453
	Fomalhaut E.	52 11 48	2740	50 36 1	2766	49 0 48	2794	47 26 12	2825
	Saturn E.	57 22 2	2295	55 35 55	2298	53 49 53	2302	52 3 57	2307
	α Pegasi E.	67 49 21	2740	66 13 34	2755	64 38 7	2772	63 3 3	2790
	α Arietis E.	110 40 0	2493	108 58 37	2493	107 17 14	2493	105 35 51	2495
30	Spica W.	96 22 58	2387	98 6 52	2393	99 50 37	2399	101 34 13	2405
	Antares W.	50 42 2	2447	52 24 30	2448	54 6 56	2450	55 49 19	2453
	Saturn E.	43 15 53	2331	41 30 38	2337	39 45 32	2342	38 0 34	2348
	α Pegasi E.	55 14 22	2910	53 42 16	2941	52 10 49	2975	50 40 5	3013
	α Arietis E.	97 9 42	2510	95 28 42	2515	93 47 50	2520	92 7 5	2525
31	Spica W.	110 9 44	2444	111 52 17	2453	113 34 37	2461	115 16 45	2470
	Antares W.	64 20 3	2476	66 1 52	2481	67 43 32	2488	69 25 2	2494
	Saturn E.	29 18 7	2383	27 34 8	2391	25 50 21	2399	24 6 45	2408
	α Pegasi E.	43 19 35	3265	41 54 42	3333	40 31 7	3408	39 8 59	3492
	α Arietis E.	83 45 32	2563	82 5 46	2572	80 26 12	2582	78 46 51	2592
	Aldebaran E.	115 38 1	2407	113 54 36	2415	112 11 22	2423	110 28 19	2431

SEPTEMBER. I.

AT APPARENT NOON.

Date.		THE SUN'S				Sidereal Time of the Semi-diameter passing the Meridian.*	Equation of Time, *to be added to* / *subtracted from Apparent Time.*	Var. in 1 hour.
		Apparent Right Ascension.	Var. in 1 hour.	*Apparent* Declination.	Var. in 1 hour.			
		h. m. s	s.	° ′ ″	″	m. s.	m. s.	s.
Sat.	1	10 39 6·52	9·075	N. 8 31 38·9	54·19	1 4·38	0 9·13	0·779
Sun.	2	10 42 44·17	9·063	8 9 54·2	54·53	1 4.34	0 9·73	0·792
Mon.	3	10 46 21·53	9·051	7 48 1·6	54·85	1 4·30	0 28·87	0·803
Tues.	4	10 49 58·62	9·040	7 26 1·4	55·16	1 4·26	0 48·28	0·814
Wed.	5	10 53 35·45	9·030	7 3 54·0	55·45	1 4·22	1 7·94	0·824
Thur.	6	10 57 12·06	9·021	6 41 39·6	55·74	1 4·18	1 27·83	0·833
Frid.	7	11 0 48·46	9·012	6 19 18·6	56·01	1 4·15	1 47·93	0·841
Sat.	8	11 4 24·66	9·005	5 56 51·3	56·26	1 4·12	2 8·22	0·849
Sun.	9	11 8 0·70	8·999	5 34 18·0	56·51	1 4·10	2 28·67	0·855
Mon.	10	11 11 36·59	8·993	5 11 39·0	56·74	1 4·08	2 49·28	0·861
Tues.	11	11 15 12·35	8·988	4 48 54·6	56·95	1 4·06	3 10·02	0·866
Wed.	12	11 18 48·00	8·983	4 26 5·2	57·16	1 4·04	3 30·87	0·871
Thur.	13	11 22 23·55	8·980	4 3 11·1	57·35	1 4·02	3 51·81	0·874
Frid.	14	11 25 59·03	8·977	3 40 12·6	57·52	1 4·01	4 12·83	0·877
Sat.	15	11 29 34·45	8·975	3 17 10·0	57·69	1 4·00	4 33·90	0·879
Sun.	16	11 33 9·84	8·974	2 54 3·7	57·83	1 3·99	4 55·01	0·880
Mon.	17	11 36 45·20	8·973	2 30 54·1	57·96	1 3·99	5 16·14	0·881
Tues.	18	11 40 20·55	8·973	2 7 41·4	58·08	1 3·99	5 37·28	0·881
Wed.	19	11 43 55·91	8·974	1 44 26·1	58·19	1 3·99	5 58·41	0·880
Thur.	20	11 47 31·30	8·975	1 21 8·4	58·28	1 4·00	6 19·52	0·879
Frid.	21	11 51 6·73	8·977	0 57 48·9	58·35	1 4·01	6 40·58	0·876
Sat.	22	11 54 42·22	8·980	0 34 27·7	58·41	1 4·02	7 1·59	0·874
Sun.	23	11 58 17·79	8·984	N. 0 11 5·3	58·45	1 4·04	7 22·52	0·870
Mon.	24	12 1 53·44	8·988	S. 0 12 17·9	58·48	1 4·06	7 43·36	0·866
Tues.	25	12 5 29·21	8·993	0 35 41·7	58·49	1 4·08	8 4·09	0·861
Wed.	26	12 9 5·11	8·999	0 59 5·6	58·49	1 4·10	8 24·68	0·855
Thur.	27	12 12 41·17	9·006	1 22 29·3	58·48	1 4·13	8 45·12	0·848
Frid.	28	12 16 17·40	9·014	1 45 52·5	58·45	1 4·16	9 5·38	0·840
Sat.	29	12 19 53·84	9·023	2 9 14·9	58·41	1 4·19	9 25·44	0·831
Sun.	30	12 23 30·50	9·033	2 32 36·0	58·35	1 4·23	9 45·28	0·822
Mon.	31	12 27 7·40	9·043	S. 2 55 55·7	58·28	1 4·27	10 4·88	0·811

* *Mean* Time of the Semidiameter passing may be found by subtracting 0ˢ·18 from the *Sidereal* Time.

II. SEPTEMBER.

AT MEAN NOON.

Date.		THE SUN'S Apparent Right Ascension.	Apparent Declination.	Semidiameter.*	Equation of Time, to be subtracted from / added to Mean Time.	Sidereal Time.
		h. m. s.	° ′ ″	′ ″	m. s.	h. m. s.
Sat.	1	10 39 6·49	N. 8 31 39·1	15 52·67	0 9·13	10 38 57·37
Sun.	2	10 42 44·19	8 9 54·0	15 52·90	0 9·73	10 42 53·92
Mon.	3	10 46 21·60	7 48 1·1	15 53·13	0 28·87	10 46 50·47
Tues.	4	10 49 58·74	7 26 0·7	15 53·36	0 48·29	10 50 47·03
Wed.	5	10 53 35·62	7 3 52·9	15 53·60	1 7·95	10 54 43·58
Thur.	6	10 57 12·28	6 41 38·2	15 53·83	1 27·85	10 58 40·13
Frid.	7	11 0 48·73	6 19 16·9	15 54·07	1 47·95	11 2 36·68
Sat.	8	11 4 24·99	5 56 49·3	15 54·30	2 8·25	11 6 33·24
Sun.	9	11 8 1·08	5 34 15·7	15 54·54	2 28·71	11 10 29·79
Mon.	10	11 11 37·02	5 11 36·3	15 54·78	2 49·32	11 14 26·34
Tues.	11	11 15 12·83	4 48 51·6	15 55·03	3 10·06	11 18 22·89
Wed.	12	11 18 48·53	4 26 1·9	15 55·27	3 30·92	11 22 19·44
Thur.	13	11 22 24·13	4 3 7·4	15 55·52	3 51·87	11 26 16·00
Frid.	14	11 25 59·66	3 40 8·5	15 55·77	4 12·89	11 30 12·55
Sat.	15	11 29 35·13	3 17 5·6	15 56·03	4 33·97	11 34 9·10
Sun.	16	11 33 10·57	2 53 59·0	15 56·28	4 55·08	11 38 5·65
Mon.	17	11 36 45·98	2 30 49·0	15 56·54	5 16·22	11 42 2·21
Tues.	18	11 40 21·39	2 7 36·0	15 56·80	5 37·36	11 45 58·76
Wed.	19	11 43 56·81	1 44 20·3	15 57·06	5 58·50	11 49 55·31
Thur.	20	11 47 32·25	1 21 2·3	15 57·33	6 19·61	11 53 51·86
Frid.	21	11 51 7·73	0 57 42·4	15 57·60	6 40·68	11 57 48·41
Sat.	22	11 54 43·27	0 34 20·9	15 57·87	7 1·69	12 1 44·97
Sun.	23	11 58 18·89	N. 0 10 58·1	15 58·15	7 22·63	12 5 41·52
Mon.	24	12 1 54·60	S. 0 12 25·4	15 58·42	7 43·47	12 9 38·07
Tues.	25	12 5 30·42	0 35 49·5	15 58·70	8 4·20	12 13 34·62
Wed.	26	12 9 6·37	0 59 13·8	15 58·98	8 24·80	12 17 31·17
Thur.	27	12 12 42·48	1 22 37·9	15 59·26	8 45·24	12 21 27·73
Frid.	28	12 16 18·77	1 46 1·4	15 59·53	9 5·51	12 25 24·28
Sat.	29	12 19 55·26	2 9 24·1	15 59·81	9 25·58	12 29 20·83
Sun.	30	12 23 31·97	2 32 45·6	16 0·09	9 45·42	12 33 17·38
Mon.	31	12 27 8·92	S. 2 56 5·5	16 0·37	10 5·01	12 37 13·94

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

SEPTEMBER. III.

MEAN TIME.

Day.	THE SUN'S *Apparent* Longitude. *Noon.*	THE SUN'S *Apparent* Latitude. *Noon.*	Logarithm of the Radius Vector of the Earth. *Noon.*	Transit of the First Point of Aries.	THE MOON'S Semidiameter. *Noon.*	THE MOON'S Semidiameter. *Midnight.*	THE MOON'S Horizontal Parallax. *Noon.*	THE MOON'S Horizontal Parallax. *Midnight.*
	° ′ ″	″		h. m. s.	′ ″	′ ″	′ ″	′ ″
1	158 7 20·1	S. 0·36	0·0038629	13 18 51·40	15 51·25	15 47·09	58 5·16	57 49·88
2	159 5 24·6	0·50	·0037580	13 14 55·50	15 42·60	15 37·85	57 33·43	57 16·04
3	160 3 30·7	0·62	·0036526	13 10 59·59	15 32·92	15 27·88	56 57·98	56 39·53
4	161 1 38·6	0·72	0·0035466	13 7 3·68	15 22·80	15 17·79	56 20·93	56 2·58
5	161 59 48·4	0·81	·0034402	13 3 7·78	15 12·94	15 8·31	55 44·75	55 27·79
6	162 58 0·0	0·86	·0033332	12 59 11·87	15 3·99	15 0·06	55 11·96	54 57·57
7	163 56 13·5	0·88	0·0032256	12 55 15·96	14 56·59	14 53·64	54 44·86	54 34·06
8	164 54 29·0	0·88	·0031173	12 51 20·05	14 51·27	14 49·52	54 25·37	54 18·99
9	165 52 46·6	0·85	·0030082	12 47 24·15	14 48·43	14 48·03	54 15·00	54 13·53
10	166 51 6·2	0·81	0·0028984	12 43 28·24	14 48·34	14 49·37	54 14·66	54 18·42
11	167 49 27·9	0·74	·0027877	12 39 32·34	14 51·09	14 53·52	54 24·74	54 33·64
12	168 47 51·7	0·63	·0026761	12 35 36·43	14 56·63	15 0·36	54 45·01	54 58·68
13	169 46 17·6	0·51	0·0025635	12 31 40·52	15 4·68	15 9·53	55 14·51	55 32·26
14	170 44 45·5	0·38	·0024498	12 27 44·62	15 14·83	15 20·49	55 51·69	56 12·45
15	171 43 15·6	0·25	·0023349	12 23 48·71	15 26·42	15 32·51	56 34·16	56 56·48
16	172 41 47·8	S. 0·12	0·0022189	12 19 52·80	15 38·64	15 44·70	57 18·95	57 41·14
17	173 40 21·9	0·00	·0021016	12 15 56·90	15 50·55	15 56·08	58 2·59	58 22·85
18	174 38 58·1	N. 0·11	·0019830	12 12 0·99	16 1·18	16 5·75	58 41·54	58 58·27
19	175 37 36·2	0·19	0·0018631	12 8 5·09	16 9·70	16 12·96	59 12·73	59 24·70
20	176 36 16·2	0·24	·0017419	12 4 9·18	16 15·50	16 17·31	59 34·01	59 40·62
21	177 34 57·9	0·28	·0016196	12 0 13·27	16 18·37	16 18·73	59 44·52	59 45·84
22	178 33 41·5	0·27	0·0014963	11 56 17·37	16 18·43	16 17·52	59 44·73	59 41·40
23	179 32 26·7	0·24	·0013721	11 52 21·46	16 16·06	16 14·15	59 36·06	59 29·03
24	180 31 13·6	0·17	·0012471	11 48 25·55	16 11·82	16 9·15	59 20·51	59 10·75
25	181 30 2·2	N. 0·06	0·0011216	11 44 29·65	16 6·22	16 3·07	59 0·01	58 48·45
26	182 28 52·6	S. 0·05	·0009957	11 40 33·74	15 59·72	15 56·22	58 36·18	58 23·38
27	183 27 44·6	0·18	·0008696	11 36 37·84	15 52·61	15 48·89	58 10·13	57 56·50
28	184 26 38·3	0·31	0·0007434	11 32 41·93	15 45·09	15 41·19	57 42·55	57 28·28
29	185 25 33·8	0·43	·0006174	11 28 46·02	15 37·22	15 33·20	57 13·75	56 59·02
30	186 24 31·2	0·55	·0004917	11 24 50·12	15 29·13	15 25·03	56 44·11	56 29·09
31	187 23 30·5	S. 0·66	0·0003662	11 20 54·21	15 20·93	15 16·85	56 14·06	55 59·11

IV. SEPTEMBER

MEAN TIME.

Day.	THE MOON'S Longitude. *Noon.*	*Midnight.*	Latitude. *Noon.*	*Midnight.*	Age. *Noon.*	Meridian Passage. *Upper.*	*Lower.*
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	d.	h. m.	h. m.
1	319 34 4·8	326 21 52·0	S.0 43 23·4	S.1 19 55·5	12·44	11 13·7	23 39·4
2	333 6 17·3	339 47 6·2	1 55 0·4	2 28 9·3	13·44	12 4·4	* *
3	346 24 7·0	352 57 10·5	2 58 57·2	3 27 2·8	14·44	12 52·4	0 28·7
4	359 26 10·6	5 51 4·7	3 52 8·3	4 14 0·1	15·44	13 38·4	1 15·6
5	12 11 54·8	18 28 46·4	4 32 28·0	4 47 25·4	16·44	14 23·0	2 0·8
6	24 41 49·2	30 51 17·0	4 58 48·6	5 6 36·7	17·44	15 6·9	2 45·0
7	36 57 28·0	43 0 43·4	5 10 50·7	5 11 33·6	18·44	15 50·9	3 28·9
8	49 1 28·5	55 0 11·2	5 8 49·7	5 2 44·6	19·44	16 35·4	4 13·0
9	60 57 22·2	66 53 34·6	4 53 24·9	4 40 57·5	20·44	17 21·0	4 58·0
10	72 49 23·5	78 45 24·8	4 25 30·2	4 7 11·6	21·44	18 7·8	5 44·2
11	84 42 15·9	90 40 33·9	3 46 10·5	3 22 36·9	22·44	18 56·1	6 31·8
12	96 40 56·5	102 44 0·3	2 56 41·7	2 28 36·9	23·44	19 45·6	7 20·7
13	108 50 20·4	115 0 30·2	1 58 36·2	1 26 54·8	24·44	20 35·9	8 10·7
14	121 15 0·1	127 34 16·6	S.0 53 50·5	S.0 19 42·8	25·44	21 26·7	9 1·3
15	133 58 41·9	140 28 33·0	N.0 15 5·3	N.0 50 9·1	26·44	22 17·5	9 52·1
16	147 4 0·7	153 45 8·8	1 25 0·4	1 59 9·3	27·44	23 8·2	10 42·9
17	160 31 53·3	167 24 2·8	2 32 3·7	3 3 10·0	28·44	23 58·8	11 33·5
18	174 21 18·1	181 23 11·4	3 31 54·3	3 57 43·6	29·44	* *	12 24·2
19	188 29 9·2	195 38 31·2	4 20 6·4	4 38 35·0	0·98	0 49·8	13 15·5
20	202 50 33·3	210 4 28·5	4 52 45·0	5 2 18·2	1·98	1 41·4	14 7·7
21	217 19 28·7	224 34 47·9	5 7 2·2	5 6 50·6	2·98	2 34·3	15 1·4
22	231 49 42·0	239 3 31·8	5 1 44·1	4 51 49·6	3·98	3 28·9	15 56·9
23	246 15 43·0	253 25 47·8	4 37 19·7	4 18 32·6	4·98	4 25·2	16 53·9
24	260 33 24·3	267 38 16·8	3 55 50·2	3 29 38·8	5·98	5 22·9	17 52·0
25	274 40 14·8	281 39 12·6	3 0 26·7	2 28 44·7	6·98	6 21·1	18 50·0
26	288 35 8·1	295 28 1·6	1 55 4·5	1 19 58·7	7·98	7 18·5	19 46·7
27	302 17 56·0	309 4 54·0	N.0 43 59·9	N.0 7 40·4	8·98	8 14·3	20 41·2
28	315 48 59·0	322 30 13·6	S.0 28 28·3	S.1 3 56·0	9·98	9 7·5	21 33·1
29	329 8 39·7	335 44 17·4	1 38 14·1	2 10 55·7	10·98	9 58·0	22 22·4
30	342 17 6·2	348 47 4·4	2 41 36·4	3 9 54·8	11·98	10 46·1	23 9·4
31	355 14 9·2	1 38 18·0	S.3 35 31·3	S.3 58 10·3	12·98	11 32·3	23 54·8

The Moon's Longitude and Latitude are from HANSEN'S Tables; the Right Ascension and Declination contain NEWCOMB'S corrections.

JUPITER'S ELEMENTS.

Date.	Apparent Right Ascension. Noon.	Apparent Declination. Noon.	Polar S.D.	Hor. Par.	Meridian Passage.
	h. m. s.	° ′ ″	″	″	h. m.
Aug. 7	6 6 35·2	N. 23 7 48·2	15·7	1·5	21 3·5
8	6 7 26·4	23 7 43·5	15·7	1·5	21 0·4
9	6 8 17·2	23 7 37·9	15·7	1·5	20 57·3
10	6 9 7·8	23 7 31·3	15·7	1·5	20 54·2
11	6 9 58·0	23 7 23·8	15·8	1·5	20 51·1

SATURN'S ELEMENTS.

	h. m. s.	° ′ ″	″	″	h. m.
Aug. 9	23 2 32·3	S. 8 21 10·0	8·7	1·0	13 51·9
10	23 2 18·4	8 22 46·9	8·8	1·0	13 47·7
11	23 2 4·3	8 24 24·9	8·8	1·0	13 43·5
12	23 1 50·0	8 26 4·0	8·8	1·0	13 39·4
13	23 1 35·5	8 27 44·1	8·8	1·0	13 35·2

MEAN PLACES OF STARS.

Star's Name.	Mag.	Right Ascension.	Declination.
		h. m. s.	° ′ ″
α Andromedæ ...	2·1	0 3 31·6	N. 28 34 17·8
γ Pegasi	2.9	0 8 23·6	N. 14 39 39·6
α Phœnicis	2·4	0 21 38·4	S. 42 48 59·3
β Ceti	2·2	0 38 52·3	S. 18 30 8·6
α Ursæ Minoris ...	2·1	1 25 8·2	N. 88 48 18·9
α Eridani	0·5	1 34 12·8	S. 57 42 51·3
γ^1 Andromedæ ...	2·2	1 58 7·5	N. 41 52 44·3
α Arietis	2·2	2 1 52·3	N. 23 1 5·7
α Ceti	2·8	2 57 21·9	N. 3 43 16·4
α Persei	1·9	3 17 36·4	N. 49 31 37·6
α Tauri	1·1	4 30 31·5	N. 16 19 14·8

MEAN PLACES OF STARS (*continued*).

Star's Name.	Mag.	Right Ascension.			Declination.		
		h.	m.	s.	°	′	″
α Aurigæ	0·2	5	9	44·6	N. 45	54	10·8
β Orionis	0·3	5	10	1·2	S. 8	18	35·3
γ Orionis	1·7	5	20	5·3	N. 6	15	53·8
β Tauri	1·8	5	20	20·9	N. 28	31	42·8
ε Orionis	1·7	5	31	26·6	S. 1	15	41·4
α Orionis	var.	5	50	4·9	N. 7	23	24·0
β Aurigæ	2·1	5	52	38·0	N. 44	56	18·6
β Canis Majoris ...	2·0	6	18	33·6	S. 17	54	31·7
α Argûs	−1·0	6	21	51·9	S. 52	38	39·0
γ Geminorum ...	1·9	6	32	16·9	N. 16	28	47·9
α Canis Majoris ...	−1·4	6	41	0·2	S. 16	35	12·2
ε Canis Majoris ...	1·6	6	54	55·9	S. 28	50	37·6
δ Canis Majoris ...	2·0	7	4	34·1	S. 26	14	36·9
α Geminorum ...	2·0	7	28	36·2	N. 32	5	43·4
α Canis Minoris ...	0·5	7	34	22·9	N. 5	27	57·3
β Geminorum ...	1·2	7	39	33·9	N. 28	15	13·3
ε Argûs	1·7	8	20	35·1	S. 59	12	24·6
δ Argûs	2·0	8	42	6·3	S. 54	21	49·9
β Argûs	1·7	9	12	10·3	S. 69	19	47·8
α Hydræ	2·2	9	22	58·1	S. 8	15	2·9
α Leonis	1·3	10	3	22·0	N. 12	25	36·7
α Ursæ Majoris ...	2·0	10	57	56·1	N. 62	15	31·0
α^1 Crucis	1·0	12	21	21·8	S. 62	35	41·5
α^2 Crucis	1½	Δα	+	0·6	Δδ	−	3·1
γ Crucis	1·6	12	25	56·7	S. 56	35	12·3
β Crucis	1·5	12	42	13·4	S. 59	10	30·1
ε Ursæ Majoris ...	1·8	12	49	53·8	N. 56	28	11·9
α Virginis	1·2	13	20	14·4	S. 10	40	14·8
η Ursæ Majoris ...	1·9	13	43	50·3	N. 49	46	56·0
β Centauri	0·8	13	57	11·0	S. 59	55	11·0
α Boötis	0·3	14	11	22·4	N. 19	40	17·6
α^2 Centauri	1	14	33	13·2	S. 60	26	43·3
α Scorpii	1·3	16	23	38·5	S. 26	13	25·7
α Trianguli Australis	1·9	16	38	42·2	S. 68	51	20·9
λ Scorpii	1·8	17	27	13·5	S. 37	2	8·2
α Ophiuchi	2·1	17	30	34·2	N. 12	37	40·7
ε Sagittarii	1·9	18	17	56·0	S. 34	25	45·8
α Lyræ	0·1	18	33	45·4	N. 38	41	45·0
α Aquilæ	0·9	19	46	11·8	N. 8	37	10·6
α Pavonis	2·0	20	18	12·9	S. 57	2	12·3
γ Cygni	2·3	20	18	51·3	N. 39	57	19·7
α Cygni	1·3	20	38	13·6	N. 44	56	38·9
α Cephei	2·6	21	16	20·2	N. 62	11	13·7
ε Pegasi	2·5	21	39	34·1	N. 9	26	37·5
α Gruis	1·9	22	2	18·7	S. 47	24	59·7
β Gruis	2·1	22	37	3·5	S. 47	22	35·0
α Piscis Australis ...	1·3	22	52	27·5	S. 30	7	14·2
α Pegasi	2·6	23	0	4·7	N. 14	41	57·8

THE CORRECTION FOR SECOND DIFFERENCES, IN FINDING THE GREENWICH TIME CORRESPONDING TO A REDUCED LUNAR DISTANCE.

Arguments:—Interval and Mean difference of Proportional Logarithms.

Interval.		Mean difference of the Proportional Logarithms in the Ephemeris.																							
		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52
h. m.	h. m.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.
0 0	3 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 10	2 50	0	0	0	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3
0 20	2 40	0	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6
0 30	2 30	0	1	1	2	2	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9
0 40	2 20	0	1	1	2	2	3	3	3	4	4	5	5	6	6	6	7	7	8	8	9	9	10	10	10	11	11
0 50	2 10	1	1	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8	9	9	10	10	11	12	12	13	13
1 0	2 0	1	1	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	10	10	11	12	12	13	13	14	14
1 10	1 50	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10	11	11	12	12	13	14	14	15	15
1 20	1 40	1	1	2	3	3	4	4	5	6	6	7	7	8	9	9	10	10	11	12	12	13	14	14	15	15	16
1 30	1 30	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10	11	11	12	12	13	14	14	15	16	16

Interval.		Mean difference of the Proportional Logarithms in the Ephemeris.																							
		54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102
h. m.	h. m.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.
0 0	3 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 10	2 50	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7
0 20	2 40	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	10	11	11	11	11	12	12	12	12	13
0 30	2 30	9	10	10	10	11	11	12	12	12	13	13	13	14	14	14	14	15	15	16	16	16	17	17	17	18
0 40	2 20	12	12	13	13	13	14	14	15	15	16	16	16	17	17	18	18	19	19	19	20	20	21	21	22	22
0 50	2 10	14	14	15	15	16	16	16	17	17	18	19	19	20	20	21	21	22	22	22	23	23	24	24	25	26
1 0	2 0	15	16	16	17	17	18	18	19	19	20	21	21	22	22	23	23	24	24	25	25	26	27	27	28	28
1 10	1 50	16	17	17	18	18	19	19	20	21	21	22	22	23	24	24	25	25	26	27	27	28	28	29	30	30
1 20	1 40	17	17	18	19	19	20	20	21	21	22	23	23	24	25	25	26	26	27	28	28	29	29	30	31	31
1 30	1 30	17	18	18	19	19	20	21	21	22	23	23	24	24	25	25	26	27	27	28	29	29	30	31	31	32

Interval.		Mean difference of the Proportional Logarithms in the Ephemeris.																			
		104	106	108	110	112	114	116	118	120	122	124	126	128	130	132	134	136	138		
h. m.	h. m.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.		
0 0	3 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
0 10	2 50	7	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9		
0 20	2 40	13	13	13	14	14	14	14	15	15	15	15	15	16	16	16	16	17	17		
0 30	2 30	18	18	19	19	19	20	20	20	21	21	21	22	22	22	23	23	24	24		
0 40	2 20	22	23	23	24	24	25	25	25	26	26	27	27	28	28	28	29	29	30		
0 50	2 10	26	26	27	27	28	29	29	29	30	30	31	31	32	32	33	33	34	34		
1 0	2 0	29	29	30	30	31	31	32	33	33	34	34	35	35	36	37	37	38	38		
1 10	1 50	31	31	32	32	33	34	34	35	35	36	37	37	38	38	39	40	40	41		
1 20	1 40	32	33	33	34	34	35	35	36	37	38	38	39	39	40	41	41	42	42		
1 30	1 30	32	33	34	34	35	35	36	36	37	38	39	39	40	40	41	42	42	43		

The Correction is to be *added* to the approximate Greenwich Time when the Proportional Logarithms in the Ephemeris are *decreasing*, and *subtracted* when they are *increasing*.

TABLES USED IN DETERMINING THE LATITUDE BY OBSERVATIONS OF THE POLE STAR OUT OF THE MERIDIAN.

TABLE I.

Containing the *First* Correction.

Arguments:—Sidereal Time of Observation.

Sidereal Time.	Correction.	Sidereal Time.	Sidereal Time.	Correction.	Sidereal Time.
h. m.	° ′ ″	h. m.	h. m.	° ′ ″	h. m.
0 0	− 1 7 6 +	12 0	6 0	− 0 26 5 +	18 0
10	1 8 11	10	10	0 23 8	10
20	1 9 7	20	20	0 20 9	20
30	1 9 56	30	30	0 17 7	30
40	1 10 37	40	40	0 14 3	40
50	1 11 10	50	50	0 10 57	50
1 0	1 11 35	13 0	7 0	0 7 50	19 0
10	1 11 51	10	10	0 4 42	10
20	1 11 59	20	20	− 0 1 34 +	20
30	1 11 59	30	30	+ 0 1 34 −	30
40	1 11 51	40	40	0 4 42	40
50	1 11 35	50	50	0 7 50	50
2 0	1 11 10	14 0	8 0	0 10 57	20 0
10	1 10 37	10	10	0 14 3	10
20	1 9 56	20	20	0 17 7	20
30	1 9 7	30	30	0 20 9	30
40	1 8 11	40	40	0 23 8	40
50	1 7 6	50	50	0 26 5	50
3 0	1 5 54	15 0	9 0	0 29 0	21 0
10	1 4 34	10	10	0 31 51	10
20	1 3 7	20	20	0 34 38	20
30	1 1 33	30	30	0 37 21	30
40	0 59 52	40	40	0 40 0	40
50	0 58 4	50	50	0 42 34	50
4 0	0 56 9	16 0	10 0	0 45 4	22 0
10	0 54 8	10	10	0 47 29	10
20	0 52 1	20	20	0 49 48	20
30	0 49 48	30	30	0 52 1	30
40	0 47 29	40	40	0 54 8	40
50	0 45 4	50	50	0 56 9	50
5 0	0 42 34	17 0	11 0	0 58 4	23 0
10	0 40 0	10	10	0 59 52	10
20	0 37 21	20	20	1 1 34	20
30	0 34 38	30	30	1 3 7	30
40	0 31 51	40	40	1 4 34	40
50	0 29 0	50	50	1 5 54	50
6 0	− 0 26 5 +	18 0	12 0	+ 1 7 6 −	24 0

TABLE II.

Containing the *Second* Correction (*always to be added*).

Arguments:—Sidereal Time and Altitude.

Sidereal Time.	Altitude.																Sidereal Time.
	0°		5°		10°		15°		20°		25°		30°		35°		
h. m.	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	h. m.
0 0	0	0	0	1	0	1	0	2	0	2	0	3	0	3	0	4	12 0
30	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	2	30
1 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13 0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30
2 0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	14 0
30	0	0	0	0	0	1	0	1	0	1	0	2	0	2	0	3	30
3 0	0	0	0	1	0	1	0	2	0	3	0	3	0	4	0	5	15 0
30	0	0	0	1	0	2	0	3	0	4	0	6	0	7	0	9	30
4 0	0	0	0	2	0	3	0	5	0	6	0	8	0	10	0	12	16 0
30	0	0	0	2	0	4	0	6	0	9	0	11	0	14	0	17	30
5 0	0	0	0	3	0	5	0	8	0	11	0	14	0	17	0	21	17 0
30	0	0	0	3	0	6	0	9	0	13	0	16	0	20	0	24	30
6 0	0	0	0	3	0	7	0	11	0	14	0	18	0	23	0	28	18 0
30	0	0	0	4	0	7	0	11	0	16	0	20	0	25	0	30	30
7 0	0	0	0	4	0	8	0	12	0	16	0	21	0	26	0	31	19 0
30	0	0	0	4	0	8	0	12	0	16	0	21	0	26	0	32	30
8 0	0	0	0	4	0	8	0	12	0	16	0	21	0	25	0	31	20 0
30	0	0	0	4	0	7	0	11	0	15	0	19	0	24	0	29	30
9 0	0	0	0	3	0	7	0	10	0	14	0	18	0	22	0	27	21 0
30	0	0	0	3	0	6	0	9	0	12	0	15	0	19	0	23	30
10 0	0	0	0	2	0	5	0	7	0	10	0	13	0	16	0	19	22 0
30	0	0	0	2	0	4	0	6	0	8	0	10	0	12	0	15	30
11 0	0	0	0	1	0	3	0	4	0	6	0	7	0	9	0	11	23 0
30	0	0	0	1	0	2	0	3	0	4	0	5	0	6	0	7	30
12 0	0	0	0	0	0	1	0	2	0	2	0	3	0	3	0	4	24 0

TABLE III.

Containing the *Third* Correction (*always to be added*).

Arguments:—Sidereal Time and Date.

Sidereal Time.	Jan. 1.		Feb. 1.		March 1.		April 1.		May 1.		June 1.		July 1.	
h.	′	″	′	″	′	″	′	″	′	″	′	″	′	″
0	1	35	1	31	1	24	1	15	1	7	1	2	1	3
2	1	30	1	32	1	29	1	21	1	12	1	4	1	0
4	1	17	1	24	1	26	1	21	1	14	1	5	0	57
6	1	0	1	10	1	15	1	16	1	12	1	4	0	54
8	0	43	0	53	1	1	1	7	1	7	1	2	0	53
10	0	30	0	38	0	47	0	56	1	0	1	0	0	54
12	0	25	0	29	0	36	0	45	0	53	0	58	0	57
14	0	30	0	28	0	31	0	39	0	48	0	56	1	0
16	0	43	0	36	0	34	0	39	0	46	0	55	1	3
18	1	0	0	50	0	45	0	44	0	48	0	56	1	6
20	1	17	1	7	0	59	0	53	0	53	0	58	1	7
22	1	30	1	22	1	13	1	4	1	0	1	0	1	6
24	1	35	1	31	1	24	1	15	1	7	1	2	1	3

TABLE II.

Containing the *Second* Correction (*always to be added*).

Arguments:—Sidereal Time and Altitude.

Sidereal Time.	Altitude.								Sidereal Time.
	35°	40°	45°	50°	55°	60°	65°	70°	
h. m.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	h. m.
0 0	0 4	0 5	0 6	0 7	0 8	0 10	0 13	0 16	12 0
30	0 2	0 2	0 3	0 3	0 4	0 4	0 5	0 7	30
1 0	0 0	0 0	0 1	0 1	0 1	0 1	0 1	0 1	13 0
30	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	30
2 0	0 1	0 1	0 1	0 1	0 2	0 2	0 2	0 3	14 0
30	0 3	0 3	0 4	0 4	0 5	0 6	0 8	0 10	30
3 0	0 5	0 6	0 7	0 9	0 10	0 13	0 16	0 20	15 0
30	0 9	0 10	0 12	0 14	0 17	0 21	0 26	0 34	30
4 0	0 12	0 15	0 18	0 21	0 25	0 31	0 38	0 49	16 0
30	0 17	0 20	0 24	0 28	0 34	0 41	0 51	1 5	30
5 0	0 21	0 25	0 29	0 35	0 42	0 51	1 3	1 21	17 0
30	0 24	0 29	0 35	0 41	0 50	1 0	1 14	1 36	30
6 0	0 28	0 33	0 39	0 47	0 56	1 8	1 24	1 48	18 0
30	0 30	0 36	0 43	0 51	1 1	1 14	1 31	1 57	30
7 0	0 31	0 38	0 45	0 53	1 4	1 17	1 36	2 3	19 0
30	0 32	0 38	0 45	0 54	1 5	1 18	1 37	2 4	30
8 0	0 31	0 37	0 44	0 53	1 3	1 16	1 35	2 2	20 0
30	0 29	0 35	0 42	0 50	1 0	1 12	1 29	1 55	30
9 0	0 27	0 32	0 38	0 45	0 54	1 6	1 21	1 44	21 0
30	0 23	0 28	0 33	0 39	0 47	0 57	1 11	1 31	30
10 0	0 19	0 23	0 28	0 33	0 39	0 48	0 59	1 16	22 0
30	0 15	0 18	0 22	0 26	0 31	0 37	0 46	0 59	30
11 0	0 11	0 13	0 16	0 19	0 23	0 27	0 34	0 43	23 0
30	0 7	0 9	0 10	0 12	0 15	0 18	0 22	0 29	30
12 0	0 4	0 5	0 6	0 7	0 8	0 10	0 13	0 16	24 0

TABLE III.

Containing the *Third* Correction (*always to be added*).

Arguments:—Sidereal Time and Date.

Sidereal Time.	July 1.	Aug. 1.	Sept. 1.	Oct. 1.	Nov. 1.	Dec. 1.	Dec. 31.
h.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	1 3	1 10	1 20	1 31	1 42	1 50	1 53
2	1 0	1 2	1 8	1 17	1 28	1 39	1 47
4	0 57	0 53	0 54	0 58	1 7	1 17	1 28
6	0 54	0 46	0 41	0 40	0 44	0 51	1 1
8	0 53	0 43	0 33	0 27	0 25	0 27	0 34
10	0 54	0 45	0 33	0 23	0 15	0 12	0 14
12	0 57	0 50	0 40	0 29	0 18	0 10	0 7
14	1 0	0 58	0 52	0 43	0 32	0 21	0 13
16	1 3	1 7	1 6	1 2	0 53	0 43	0 32
18	1 6	1 14	1 19	1 20	1 16	1 9	0 59
20	1 7	1 17	1 27	1 33	1 35	1 33	1 26
22	1 6	1 15	1 27	1 37	1 45	1 48	1 46
24	1 3	1 10	1 20	1 31	1 42	1 50	1 53

For converting INTERVALS of MEAN SOLAR Time into Equivalent INTERVALS of SIDEREAL Time.

HOURS.		MINUTES.				SECONDS.			
Hours of Mean Time.	Equivalents in Sidereal Time.	Minutes of Mean Time.	Equivalents in Sidereal Time.	Minutes of Mean Time.	Equivalents in Sidereal Time.	Seconds of Mean Time.	Equivalents in Sidereal Time.	Seconds of Mean Time.	Equivalents in Sidereal Time.
	h. m. s.		m. s.		m. s.		s.		s.
1	1 0 9·8565	1	1 0·1643	31	31 5·0925	1	1·0027	31	31·0849
2	2 0 19·7130	2	2 0·3286	32	32 5·2568	2	2·0055	32	32·0876
3	3 0 29·5694	3	3 0·4928	33	33 5·4211	3	3·0082	33	33·0904
4	4 0 39·4259	4	4 0·6571	34	34 5·5853	4	4·0110	34	34·0931
5	5 0 49·2824	5	5 0·8214	35	35 5·7496	5	5·0137	35	35·0958
6	6 0 59·1388	6	6 0·9857	36	36 5·9139	6	6·0164	36	36·0986
7	7 1 8·9953	7	7 1·1499	37	37 6·0782	7	7·0192	37	37·1013
8	8 1 18·8518	8	8 1·3142	38	38 6·2424	8	8·0219	38	38·1040
9	9 1 28·7083	9	9 1·4785	39	39 6·4067	9	9·0246	39	39·1068
10	10 1 38·5647	10	10 1·6428	40	40 6·5710	10	10·0274	40	40·1095
11	11 1 48·4212	11	11 1·8070	41	41 6·7353	11	11·0301	41	41·1123
12	12 1 58·2777	12	12 1·9713	42	42 6·8995	12	12·0329	42	42·1150
13	13 2 8·1342	13	13 2·1356	43	43 7·0638	13	13·0356	43	43·1177
14	14 2 17·9906	14	14 2·2998	44	44 7·2281	14	14·0383	44	44·1205
15	15 2 27·8471	15	15 2·4641	45	45 7·3924	15	15·0411	45	45·1232
16	16 2 37·7036	16	16 2·6284	46	46 7·5566	16	16·0438	46	46·1259
17	17 2 47·5600	17	17 2·7927	47	47 7·7209	17	17·0465	47	47·1287
18	18 2 57·4165	18	18 2·9569	48	48 7·8852	18	18·0493	48	48·1314
19	19 3 7·2730	19	19 3·1212	49	49 8·0495	19	19·0520	49	49·1342
20	20 3 17·1295	20	20 3·2855	50	50 8·2137	20	20·0548	50	50·1369
21	21 3 26·9859	21	21 3·4498	51	51 8·3780	21	21·0575	51	51·1396
22	22 3 36·8424	22	22 3·6140	52	52 8·5423	22	22·0602	52	52·1424
23	23 3 46·6989	23	23 3·7783	53	53 8·7066	23	23·0630	53	53·1451
24	24 3 56·5554	24	24 3·9426	54	54 8·8708	24	24·0657	54	54·1479
		25	25 4·1069	55	55 9·0351	25	25·0685	55	55·1506
		26	26 4·2711	56	56 9·1994	26	26·0712	56	56·1533
		27	27 4·4354	57	57 9·3637	27	27·0739	57	57·1561
		28	28 4·5997	58	58 9·5279	28	28·0767	58	58·1588
		29	29 4·7640	59	59 9·6922	29	29·0794	59	59·1615
		30	30 4·9282	60	60 9·8565	30	30·0821	60	60·1643

SUMMARY OF USEFUL FORMULÆ.

I. *Plane Trigonometry.*

1. $\text{Sin A} = \dfrac{1}{\text{cosec A}}$, $\cos A = \dfrac{1}{\sec A}$, $\tan A = \dfrac{1}{\cot A}$

   $\text{Cosec A} = \dfrac{1}{\sin A}$, $\sec A = \dfrac{1}{\cos A}$, $\cot A = \dfrac{1}{\tan A}$

2. $\text{Tan A} = \dfrac{\sin A}{\cos A}$, $\cot A = \dfrac{\cos A}{\sin A}$

3. $\text{Sin}^2 A + \cos^2 A = 1$
   whence, $\sin^2 A = 1 - \cos^2 A$, and $\cos^2 A = 1 - \sin^2 A$

4. $\text{Sec}^2 A = 1 + \tan^2 A$, and therefore $\tan^2 A = \sec^2 A - 1$
   $\text{Cosec}^2 A = 1 + \cot^2 A$, and therefore $\cot^2 A = \text{cosec}^2 A - 1$

5. $\text{Sin A} = \cos(90° - A) = \sin(180° - A)$
   $\text{Cos A} = \sin(90° - A) = -\cos(180° - A)$

6. $\left.\begin{aligned} \text{Sin}(A + B) &= \sin A \,.\, \cos B + \cos A \,.\, \sin B \\ \text{Sin}(A - B) &= \sin A \,.\, \cos B - \cos A \,.\, \sin B \\ \text{Cos}(A + B) &= \cos A \,.\, \cos B - \sin A \,.\, \sin B \\ \text{Cos}(A - B) &= \cos A \,.\, \cos B + \sin A \,.\, \sin B \end{aligned}\right\}$

7. $\left.\begin{aligned} \text{Tan}(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \,.\, \tan B} \\ \text{Tan}(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \,.\, \tan B} \\ \text{Cot}(A + B) &= \frac{\cot A \,.\, \cot B - 1}{\cot A + \cot B} \\ \text{Cot}(A - B) &= \frac{\cot A \,.\, \cot B + 1}{\cot B - \cot A} \end{aligned}\right\}$

8. $\left.\begin{aligned} \text{Sin A} + \sin B &= 2 \sin \tfrac{1}{2}(A + B) \,.\, \cos \tfrac{1}{2}(A - B) \\ \text{Sin A} - \sin B &= 2 \cos \tfrac{1}{2}(A + B) \,.\, \sin \tfrac{1}{2}(A - B) \\ \text{Cos A} + \cos B &= 2 \cos \tfrac{1}{2}(A + B) \,.\, \cos \tfrac{1}{2}(A - B) \\ \text{Cos A} - \cos B &= 2 \sin \tfrac{1}{2}(A + B) \,.\, \sin \tfrac{1}{2}(B - A) \end{aligned}\right\}$

9. $\text{Sin } 2A = 2 \sin A \,.\, \cos A$

10. $\left.\begin{aligned} \text{Cos } 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A \\ &= 2 \cos^2 A - 1 \\ \text{Cos A} &= 2 \cos^2 \tfrac{1}{2} A - 1 \\ &= 1 - 2 \sin^2 \tfrac{1}{2} A \\ \text{Cos}^2 \tfrac{1}{2} A &= \tfrac{1}{2}(1 + \cos A) \\ \text{Sin}^2 \tfrac{1}{2} A &= \tfrac{1}{2}(1 - \cos A) \end{aligned}\right\}$

NOTE.—These formulæ are equally true of the angles of spherical triangles.

Relations of sides and angles of a plane triangle whose angles are A, B, C, and opposite sides a, b, c.

11. $A + B + C = 180°$.

12. $\dfrac{\text{Sin A}}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

13. $\text{Cos } A = \dfrac{b^2 + c^2 - a^2}{2bc}$

14. $$\left.\begin{aligned} \text{Cos } \tfrac{1}{2}A &= \sqrt{\frac{s \cdot (s-a)}{bc}} \\ \text{Sin } \tfrac{1}{2}A &= \sqrt{\frac{(s-b) \cdot (s-c)}{bc}} \\ \text{Tan } \tfrac{1}{2}A &= \sqrt{\frac{(s-b) \cdot (s-c)}{s \cdot (s-a)}} \end{aligned}\right\} \text{where } s = \tfrac{1}{2}(a+b+c)$$

15. $\text{Cot } \frac{1}{2}A = \dfrac{b+c}{b-c} \cdot \tan \frac{1}{2}(B - C)$

II. *Spherical Trigonometry.*

Relations of sides and angles of an oblique-angled spherical triangle.

1. $$\left.\begin{aligned} \text{Cos } A &= \frac{\cos a - \cos b \cdot \cos c}{\sin b \cdot \sin c} \\ \text{Cos } a &= \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \end{aligned}\right\}$$

2. $\dfrac{\text{Sin } A}{\sin a} = \dfrac{\sin B}{\sin b} = \dfrac{\sin C}{\sin c}$

3. $$\left.\begin{aligned} \text{Sin } \tfrac{1}{2}A &= \sqrt{\frac{\sin (s-b) \cdot \sin (s-c)}{\sin b \cdot \sin c}} \\ \text{Cos } \tfrac{1}{2}A &= \sqrt{\frac{\sin s \cdot \sin (s-a)}{\sin b \cdot \sin c}} \\ \text{Tan } \tfrac{1}{2}A &= \sqrt{\frac{\sin (s-b) \cdot \sin (s-c)}{\sin s \cdot \sin (s-a)}} \end{aligned}\right\} \text{where } s = \tfrac{1}{2}(a+b+c)$$

4. $$\left.\begin{aligned} \text{Tan } \tfrac{1}{2}(A+B) &= \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cdot \cot \tfrac{1}{2}C \\ \text{Tan } \tfrac{1}{2}(A-B) &= \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cdot \cot \tfrac{1}{2}C \end{aligned}\right\}$$

5. $$\left.\begin{aligned} \text{Tan } \tfrac{1}{2}(a+b) &= \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \cdot \tan \tfrac{1}{2}c \\ \text{Tan } \tfrac{1}{2}(a-b) &= \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \cdot \tan \tfrac{1}{2}c \end{aligned}\right\}$$

6. $$\left.\begin{aligned} \text{Sin } \tfrac{1}{2}c &= \frac{\sin \frac{1}{2}(a+b)}{\cos \frac{1}{2}(A+B)} \cdot \sin \tfrac{1}{2}C \\ \cos \tfrac{1}{2}c &= \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(A+B)} \cdot \sin \tfrac{1}{2}C \end{aligned}\right\}$$

WEIGHTS AND MEASURES.

Angular.

60 seconds (″) = 1 minute.
60 minutes (′) = 1 degree.
90 degrees (°) = 1 quadrant.
4 quadrants = 1 circumference.

Apothecaries' (dry).

20 grains = 1 scruple (℈).
3 scruples = 1 drachm (ʒ).
8 drachms = 1 ounce (℥).
12 ounces = 1 pound.

Apothecaries' (wet).

60 minims or drops = 1 drachm.
8 drachms = 1 ounce (oz.).
20 ounces = 1 pint.
8 pints = 1 gallon.
4 drachms = 1 tablespoonful.
2 ounces = 1 wineglassful.
3 ounces = 1 teacupful.

Avoirdupois.

16 drachms (dr.) = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
28 pounds = 1 quarter (qr.).
4 quarters = 1 hundredweight (cwt.).
20 hundredweights = 1 ton.

Troy.

24 grains (gr.) = 1 pennyweight (dwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound.
7000 grains Troy = 1 lb. Avoirdupois.

Lineal.

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards = 1 rod, pole, or perch (po.).
40 poles = 1 furlong (fur.).
8 furlongs = 1 mile (m.).

Square.

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod, pole or perch (sq. po.).
40 poles = 1 rood (rd.).
4 roods = 1 acre (ac.).
640 acres = 1 square mile (sq. m.).

Cubic or solid.

1728 cubic inches (cub. in.) = 1 cubic foot (cub. ft.).
27 cubic feet = 1 cubic yard (cub. yd.).

Capacity or dry.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
2 gallons = 1 peck (pk.).
4 pecks = 1 bushel (bush.).
8 bushels = 1 quarter (qr.).
36 bushels = 1 chaldron (chal.).

Wine or Ale.

4 noggins = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
9 gallons = 1 firkin (fir.).
18 gallons = 1 kilderkin (kil.) ale.
= 1 runlet (run.) spirit.
36 gallons = 1 barrel (bar.) ale.
54 gallons = 1 hogshead (hhd.) ale.
2 barrels = 1 puncheon (pun.) ale.
3 barrels = 1 butt (ale).
42 gallons = 1 Tierce (wine).
63 gallons = 1 hogshead (wine).
84 gallons = 1 puncheon (wine).
126 gallons = 1 pipe.
252 gallons = 1 tun.

Pipes of wine vary in quantity.

Metric system derives its name from the metre which is the unit of length in the system, and is equal to a ten-millionth part of the distance from the equator to the pole, or 39·3708 inches.

To indicate multiples of the unit Greek prefixes are used thus—

Deka	means	10	times unit
Hekto	,,	100	,, ,,
Kilo	,,	1000	,, ,,
Myria	,,	10,000	,, ,,

To indicate sub-multiples Latin prefixes are used thus—

Deci	means	$\frac{1}{10}$ of unit
Centi	,,	$\frac{1}{100}$,,
Milli	,,	$\frac{1}{1000}$,,

Unit of weight is gramme of water at 4° C. = 1 cu. centimetre = 15·43 grains
Unit of capacity is litre = 1 cu. decimetre = 1·76 pints
1 litre of water weighs 1 kilogram = 2·2 lbs.

Thus

10 millimetres	make	1 centimetre
10 centimetres	,,	1 decimetre
10 decimetres	,,	1 metre
10 metres	,,	1 dekametre
10 dekametres	,,	1 hektometre
10 hektometres	,,	1 kilometre
10 kilometres	,,	1 myriametre.

Specific gravity of solids at zero as compared with distilled water at 4° C.

Platinum	22·069
Gold	19·362
Silver	10·474
Steel	7·816
Cast iron	7·207
Marble	2·837
Common salt	2·220
Sugar	1·600
Ice	·930
Beech	·852
Oak	·845
Elm	·800
Pine	·657
Cork	·240
Snow	·183

Specific gravity of liquids under same conditions—

Mercury	13·598
Glycerine	1·260
Blood	1·060
Milk	1·029
Olive oil	·915
Turpentine	·870
Petroleum	·836
Alcohol	·793

Vapour density under same conditions—

Air	1·0
Hydrogen	·0693
Nitrogen	·8714
Oxygen	1·1057
Carbonic acid	1·5291
Water in a saturated space	·6225

	Boiling points.	Freezing points.
Alcohol	78° C.	− 202° C.
Turpentine	157	− 27
Mercury	358	− 38·8
Salt water	102	− 3·3

EXAMINATION PAPERS.

PAPER I.

1. Multiply 142·07 by 0·00347, by logs.
2. Divide 0·0004629 by 0·00000542, by logs.
3.

Hour.	Courses.	Knts.	10ths.	Winds.	Leeway.	Deviation.	Remarks.
					Pts.		
1	S.	7	2	W.S.W.	¾	3° W.	
2		7	2				
3		6	9				
4		6	7				
5	W. by S. ½ S.	6	8	S.	¼	8° W.	
6		7	3				
7		8	4				
8		6	5				
9	S.W. by W. ½ W.	8	—	N.W.	¼	7° W.	A point in lat. 50° 44′ N., long. 0° 13′ E.; bearing by compass N.E. ¾ E.; ship's head, W. by S. ½ S.; deviation as per log, distant 12 miles. Variation, 18° W.
10		7	2				
11		7	7				
12		7	1				
1	N.E. ½ N.	6	2	N.N.W.	1¾	—	
2		6	5				
3		7	3				
4	W. by N. ½ N.	7	8	N.	¼	7° W.	
5		7	6				
6		6	9				
7		6	4				
8		6	3				
9	N.E. ½ E.	7	4	N. ½ W.	1¼	2° E.	
10		6	5				
11		6	3				
12		6	8				

By observation the lat. and long. of the ship was 49° 30′ N., 0° 45′ W. Find set and drift of the current.

4. August 3rd, in long. 68° 45′ E., the observed mer. alt. of the Sun's L.L. bearing S. was 58° 14′ 40″; ind. err. − 3′ 25″, height of eye, 27 ft. Required the latitude.

5. In lat. 51° 30′ S., long. 37° E., the departure made good eastward was 306·5 miles. Required the long. in by parallel sailing.

6. Required the compass course and distance by Mercator's sailing—

From A, lat. 30° 27′ N. Long. 18° 18′ W.
To B, ,, 10 45 S. ,, 32 54 W.
Variation, 13 15 W. Deviation, 24 30 W.

7. January 19th, find the M.T.G. times of high water at Rotterdam.

8. August 5th, at ship, in lat. 38° 24′ S., long. 130° 45′ E., when the time shown by chron. correct for G.M.T. was $10^h\ 13^m$; the Sun's bearing by compass at rising was N. 64° 30′ E. Required the true amplitude, and error of the compass; and supposing the variation to be 6° 15′ W., required the deviation of the compass for the position of the ship's head.

9. August 7th, about $9^h\ 15^m$ A.M. at ship, in lat. at noon 39° 31′ 40″ N.; long. by account 31° W.; the obs. alt. of Sun's L.L. was 53° 53′ 10″; ind. err. + 3′, height of eye, 29 ft.; time by chron. $12^h\ 0^m\ 9^s$ which was slow 10^s for G.M. noon, on January 9th, and on April 29th was correct; run since obs. W. 25 miles. Required the long. at sight, also brought up to noon. The sun's compass bearing was S. 30° E., var. 28° 15′ W., find the true azimuth, comp. error, and deviation.

10. September 2nd, about $3^h\ 30^m$ P.M. at ship, in lat. 51° 25′ S., long. 174° 25′ E.; when the time shown by chron. was $3^h\ 54^m\ 42^s$, the Sun's bearing by compass N. 81° 30′ W., the error of the chron. on G.M.T. was $15^m\ 12^s$ slow. Required the true azimuth and error of the compass by the Tables; and supposing the variation to be 30° 15′ E., required the deviation for the position of the ship's head.

11. August 25th, about $3^h\ 5^m$ A.M. at ship, lat. by account 26° 16′ S., long. 35° 15′ W.; the alt. of Achernar bearing south was 57° 27′ 50″; ind. err. + 2′ 11″; height of eye, 26 ft.; time by chron. $5^h\ 17^m\ 32^s$, which had been found to be $21^m\ 7^s$ fast of G.M.T. Required the lat. by reduction to the meridian.

12. September 23rd, A.M. at ship. If at sea and uncertain of my position when the chron. showed $23^d\ 9^h\ 51^m\ 13^s$ G.M.T.; the alt. of the Sun's L.L. 32° 50′ 30″; again P.M. on the same day, when the chron. showed $23^d\ 13^h\ 51^m\ 9^s$ G.M.T.; the alt. of the Sun's L.L. 32° 54′ 10″; ind. err. − 1′ 20″; height of eye, 17 ft.; the ship becalmed heading east in the interval between the observations. Required the line of position when the first alt. was taken, the Sun's true azimuth when the second alt. was taken, and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 51° 15′ and 51° 45′ S.

13. August 14th, in long. 159° 15′ W., find the mean time of transit of α Arietis across the meridian.

14. On August 14th at $4^h\ 15^m$ A.M. M.T.S., find what stars not less bright than the second magnitude are within two hours east of the meridian in lat. 32° 14′ N., long. 159° 15′ W., above the horizon and pole. Give their meridian distances, and state bearing N. or S.; also find the approx. mer. alt. for setting the sextant of α Persei; ind. cor. − 4′ 29″; eye, 40 ft.

15. August 31st, about 6^h P.M. at ship, in lat. 24° 19′ S., long. by account 180° E., when a chron. showed $6^h\ 4^m\ 3^s$, which was slow on June 16th, $1^m\ 53^s{\cdot}5$; and was fast $1^m\ 29^s{\cdot}5$ on Aug. 5th, for G.M.T.; the obs. alt. of Vega was 21° 29′ 10″; ind. cor. − 2′ 15″; eye, 28 ft. Find the long.

16. August 24th, when the M.T.S. was $2^h\ 15^m$ A.M. in lat. equator, long. Greenwich, Rigel bore by compass S. 85° 30′ E. Find the true azimuth, and deviation, the variation being 20° 15′ W.

17. September 12th, when the M.T.S. was $4^h\ 40^m\ 15^s$ A.M., in long. 136° 24′ E., the obs. alt. of Polaris out of the meridian was 39° 15′ 40″; ind. cor. − 3′ 48″; eye, 27 ft. Find the lat.

18. August 7th, in long. 90° 45′ E., the obs. mer. alt. of the Moon's L.L. was 49° 56′ 20″ bearing south, ind. cor. + 2′ 45″; eye, 36 ft. Find the lat.

State where tables giving the approx. time of most of the principal stars passing the meridian are to be found, the numbers of the tables, and whether given in mean or apparent time.

Can the approx. mer. pass. of a planet and its approx. alt. be found in the same way as in the case of a star?

State where the times of the mer. pass. of the principal planets may be found. Is the time given mean or apparent?

19. August 12th, the obs. mer. alt. of the star α Arietis was 38° 47′ 40″ bearing north ; ind. err. − 5′ 25″ ; height of eye, 32 ft. Required the lat.

20.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N. 7° W.		S.	N. 10° E.	
N.E.	N. 27° W.		S.W.	N. 25° E.	
E.	N. 21° W.		W.	N. 30° E.	
S.E.	N.		N.W.	N. 20° E.	

Find correct magnetic bearing ; construct a Napier curve.

With magnetic courses : E. W.S.W. S.E. by S. N.N.W.
Required compass courses.

With compass courses : N.W. N.E. by N. ½ N. S.S.W. E.S.E.
Required magnetic courses.

With ship's head at W.S.W. by compass, the compass bearings of distant objects were W. by S. and N. Find magnetic bearings.

21. On October 11th, at 7ʰ 40ᵐ A.M. M.T.S., being off Fowey by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

22. August 1st, P.M. at ship in lat. 34° 28′ S., when a chron. showed 7ʰ 20ᵐ 35ˢ, estimated error, 6ᵐ 15ˢ fast of G.M.T., approx. long. 66° E. ; the following observations were taken for finding long. and error of chron., Observed alt. of α Pegasi 27° 16′ 20″, ind. cor. + 4′ 13″, eye, 44 ft. ; observed distance between the star and Moon's remote limb, was 80° 14′ 40″, ind. cor. − 2′ 27″.

23. September 23rd, lat. N., long. 82° 30′ E. With the following observations of the Sun's L.L. find the lat. by double alts. at second obs.

A.T.S. 9ʰ 30ᵐ 10ˢ A.M. Obs. alt. 32° 13′ 10″. Ind. cor. − 2′ 8″.
,, 2 30 20 P.M. ,, 32 13 10 Eye, 28 ft.

Sun's bearing at first obs. S. 46° E. ; course and distance during interval, N. 60° E. ; rate 5·8 knots per hour.

24. August 30th, about 9ʰ A.M. M.T.S., a point in lat. 40° 28′ N., long. 25° 24′·5 W., bore N. 27° E. from a ship, distant 9 miles ; the obs. alt. of Sun's L.L. in an artificial horizon was 79° 10′ 20″ ; ind. cor. − 3′ 18″. Time by chron. 10ʰ 30ᵐ, estimated slow 15ᵐ 15ˢ of M.T.G. Find error on M.T.G.

25. Find the first course and distance on a G.C. from Wolf Rock, lat. 49° 57′ N., long. 5° 48′ W., to Charleston lat. 32° 45′ N., long. 79° 52′ W. ; also position of Vertex, lat. of points 5°, 10°, 15°, 20° from Wolf ; and lat. at which the 61st W. mer. cuts the G.C. arc.

26. In a triangle PQR, Q = 90°, PR = 364, P = 28° 45′. Find PQ.

27. In triangle PQR, PQ = 417, PR = 243, P = 58° 26′. Find R.

28. In spherical triangle ABC, A = 36° 38′, B = 101° 42′, C = 90°. Find AB.

29. In spherical triangle, A = 90°, B = 104° 25′, AC = 107° 39′. Find BC.

30. Coefficient B = −15° 36′, C = +9° 24′. Find direction of ship's head whilst building.

Direction of ship's head whilst building S. 49° W., coefficient B = +20° 15′. Find C.

31. September 20th, 11ʰ 48ᵐ P.M. M.T.S. ; in lat. 49° S., long. 168° 15′ W. Name some stars which would be suitable for finding the correction of the compass.

32. Given deviations with ship's head by compass at

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
to be	17° W.	28° W.	24° W.	9° W.	16° E.	32° E.	26° E.	5° E.

find coefficients A, B, C, D, and E, and construct a table of deviations for all points from N to S, through W.

33. With a ship steering N.E. by N. on the port tack, heeling 12°; the heeling error was −15°. Required the error when steering E.S.E. on the port tack, and heeling 10°.

34. Construct a Mercator's chart, scale 1″·5 = 1° long., extending from lat. 58° to 61° N., and 6° to 10° E. long. Graduate a degree of lat. and long. and place on it a magnetic compass showing points, variation 20° W.

PAPER II.

1. Multiply 47290 by 0·0045, by logs.
2. Divide 4765·07 by 32406·5, by logs.
3.

Hrs.	Courses.	Knts.	10ths.	Winds.	Lee-way.	Deviation.	Remarks.
					Pts.		
1	S.E. by E. ¼ E.	10	7	S.S.W.	¾	25° W.	A point in lat. 25° 15′ S., long. 47°20′ W.; bearing by compass N.N.E. ½ E.; ship's head, S.E. by E. ¼ E.; deviation as per log, distant 15 miles. Variation, 20° W.
2		11	3				
3		9	9				
4		11	1				
5		11	—				
6	E. by S. ¾ S.	10	4	S.	1¼	24° W.	
7		9	7				
8		10	5				
9		9	8				
10		9	6				
11	S.E. by S.	9	7	S.W.	1	18° W.	
12		10	2				
1		10	4				
2		11	7				
3	S.W. by S.	9	6	S.S.E.	1½	25° E.	
4		10	2				
5		11	2				
6	S.	9	5	E.S.E.	1¼	5° E.	
7		10	7				A current set N. 14° W. true, 54 miles during the day.
8		9	8				
9	S. by E. ¾ E.	10	4	E.	1	9° W.	
10		11	3				
11		10	9				
12		9	4				

Find the course and distance made good; lat. and long. of the ship.

4. September 23rd, in long. 28° 45′ W., the obs. mer. alt. of the Sun's L.L. bearing north, was 64° 53′ 10″; ind. err. −4′ 26″; height of eye, 37 ft. Required the latitude.

5. From lat. 40° 30′ N., long. 39° 43′ W., the departure made good was 89 miles to the west. Required long. in by parallel sailing.

6. Required the compass course and distance by Mercator's sailing

From A, lat. 52° 50′ S.	Long. 81° 10′ W.
To B, ,, 41 12 N.	,, 127 55 W.
Variation, 18 25 E.	Deviation, 22 15 E.

7. January 30th. Find the M.T.G. of high water at Dungeness.

8. September 29th, about $6^h\ 5^m$ P.M. at ship, in lat. 42° 10′ N., long. 146° 58′ E., when a chron. correct for M.T.G. showed $8^h\ 2^m\ 33^s$, the Sun's bearing by compass was N. 84° 30′ W. Required the true amplitude, and error of the compass; and supposing the variation to be 0, required the deviation of the compass for the position of the ship's head.

9. August 4th, about $7^h\ 15^m$ A.M. at ship, in lat. at noon 15° 22′ N., long. by account 37° 30′ E.; the obs. alt. of sun's L.L. was 22° 31′ 10″; ind. err. −1′ 30″; height of eye, 25 ft.; time shown by chron. $4^h\ 42^m\ 11^s$, which was $9^m\ 44^s{\cdot}2$ slow for G.M. noon on April 20th, and on June 3rd was $10^m\ 15^s$ slow; run since sights, S. 67° E. 46 miles. Required the long. at sight, also brought up to noon; and if the compass bearing be N. 87° 30′ E., find true azimuth and deviation, the variation being 5° W.

10. September 1st, about 4^h P.M. at ship, in lat. 44° 23′ S., long. 145° 17′ E.; when a chron. correct for G.M.T. showed $6^h\ 24^m\ 35^s$, the Sun's bearing by compass W. ½ N. Required the true azimuth and error of the compass by the tables; and supposing the variation to be 10° 15′ E., required the deviation for the position of the ship's head.

11. August 7th, A.M. at ship, lat. by account 33° 15′ N., long. 10° 57′ W.; the alt. of the Sun's U.L. 72° 53′; ind. err. +3′; height of eye, 16 ft.; time by watch $10^h\ 57^m\ 58^s$, which had been found to be $44^m\ 10^s$ slow of A.T.S.; the diff. long. made westward 17¾ miles after the error was determined. Required the lat. by reduction to the meridian, and if the run since observation was S. 11° W., 4 miles, find the lat. at noon.

12. September 29th about $1^h\ 20^m$ P.M. at ship. If in long. by account 125° 30′ W., when the chron. showed $9^h\ 26^m\ 12^s$ G.M.T.; the alt. of the Sun's L.L. 36° 33′ 40″; again P.M. on the same day, when the chron. showed $12^h\ 2^m\ 10^s$ G.M.T.; the alt. of the Sun's L.L. 18° 39′ 20″; ind. err. −2′ 5″; height of eye, 34 ft.; the ship having made 43 miles on a true E. ¼ N. course in the interval between the observations. Required the line of position when the first alt. was taken, the Sun's true azimuth and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 48° and 48° 30′ N.

13. On Sept. 28th. At what apparent ship time will Capella cross the meridian of a place in long. 73° 48′ E.? Also by inspection?

14. What stars of brightness not less than 1·5 will be within 2 hours of the meridian of a place above the horizon and pole in lat. 30° 55′ S., long. 73° 48′ E., on September 28th at $5^h\ 27^m$ A.M. M.T.S.? Give meridian distances, and if ind. cor. be 3′ 47″ to add, eye 35 ft., find approx. mer. alt. of Rigel for placing on sextant.

15. September 7th, about 7^h P.M. at ship, in lat. 35° 20′ N., long. by account 165° W., when a chron. showed $7^h\ 54^m\ 50^s$, which was fast $1^h\ 58^m\ 5^s{\cdot}4$ on June 29th, and was fast $2^h\ 3^m\ 13^s{\cdot}4$ on August 3rd for G.M. noon, the obs. alt. of Altair was 53° 1′ 40″; ind. cor. +1′ 11″; eye, 29 ft. Find the longitude.

Are you acquainted with any maps, diagrams, or any other means for facilitating the recognition of the fixed stars and planets? If so, give a short description of the one you prefer, and how you use it.

16. August 27th at $1^h\ 15^m$ A.M. A.T.S. in lat. 47° 24′ N., long. 68° 19′ W., the compass bearing of Altair was S. 81° W. Find the true azimuth from the "tables," compass error, and deviation, supposing the variation to be 14° 30′ W.

17. September 7th, when the G.M.T. was noon in long. 125° 40′ E., the

obs. alt. of Polaris out of the meridian in an artificial horizon was 58° 37′ 20″, ind. cor. −5′ 40″. Required the latitude.

18. August 30th, in long. 49° 36′ W., the obs. mer. alt. of the Moon's L.L. was 58° 57′ 50″; zenith south of moon; ind. cor. +6′ 24″; eye, 29 ft. Required the latitude.

19. September 19th, the obs. mer. alt. of the star β Argûs was 50° 26′ 40″ bearing south; ind. err. − 7′ 46″; height of eye, 56 ft. Required the latitude.

20.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	S. 43° W.		S.	W.	
N.E.	S. 50° W.		S.W.	S. 83° W.	
E.	S. 65° W.		W.	S. 65° W.	
S.E.	S. 80° W.		N.W.	S. 45° W.	

Find correct magnetic bearing, and construct a curve.

Given magnetic courses: S.E. by E. N.N.E. N.W. ½ W. S. 10° W. Find compass courses.

Given compass courses: N.W. by W. N. by W. N.E. ½ E. S. 50° E. Find magnetic courses.

With ship's head at E. ¼ N. by compass, the comp. bearings of distant objects were S.E. ¾ E. and N.E. by N. Find magnetic bearings.

21. On December 15th, at 11ʰ 36ᵐ P.M. M.T.G., being off Ardrossan by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

22. August 2nd, at 9ʰ 20ᵐ P.M. M.T.S., in lat. 25° 14′ N., long. by account, 100° 37′ 30″ W.; the following observations were taken for determining the error of a chron. which had run down, and set going on G.M.T. and showed 4ʰ; obs. alt. of the Moon's L.L., 39° 8′ 30″; ind. cor. −4′ 12″; height of eye, 17 feet; obs. distance between Spica and the Moon's near limb, 88° 59′ 40″; ind. cor. +7′ 43″.

23. September 29th, in long. 125° 35′ W., lat. N. With following observations, find lat. at time of taking second obs. :—

A.T.S. 1ʰ 13ᵐ 40ˢ P.M. Alt. Sun's L.L. 36° 33′ 40″, bearing S. 23° W.
„ 3 49 40 „ „ „ 18 39 20

The ship's true course, N. 87° E., 16·5 knots per hour during the interval; ind. cor. −2′ 3″; height of eye, 34 feet.

24. September 23rd, 2ʰ 49ᵐ P.M. M.T.S., at a place in lat. 23° 28′ S., long. 158° 35′ E.; the obs. alt. of Sun's U.L., 41° 32′ 10″; ind. cor. −2′ 25″; eye, 34 feet; the chron. shows 4ʰ 17ᵐ 7ˢ. Find its error on M.T.G.

25. Find initial and final courses and distance, easting to be run down on parallel of 55° S., from lat. 42° 53′ S., long. 147° 21′ E., to 33° 2′ S., long. 71° 38′ W. Find latitudes of points where the meridians 160° E., 170° E., 90° W., and 80° W. cut the track.

26. In triangle ABC, A = 90°, C = 68° 40′, AC = 37·2. Find AB.

27. In triangle OST, OS = 379, OT = 325, ST = 85. Find T.

28. In spherical triangle ABC, C = 90°, B = 103° 20′, BC = 75°. Find AB.

29. „ „ PQR, P = 90°, RQ = 98° 56′, R = 72° 45′. Find Q.

30. If coefficient B = +16° 54′, coefficient C = −18° 36′, find direction of ship's head whilst building.

Coefficient C = −20° 30′; direction ship's head building, S. 39° W. Find B.

31. In lat. 45° N., long. 58° W., find what stars are suitable for finding compass error by azimuth on September 7th at $2^h\ 30^m$ A.M.

32. The deviations with ship's head by compass at N., N.E., E., S.E., S., S.W., W., and N.W. being 5° E., 5° E., 2° W., 13° W°., 15° W., 3° W., 13° E., 8° E., find the coefficients A, B, C, D, E, and construct a table of deviations for every alternate point of the compass, starting from North.

33. With the wind at S.E., the ship heads N.E. by E. ½ E., having a heel of 14°, and heeling error −21°. Find what the error will be when heeling 15° with her head S.S.W., wind in same direction.

PAPER III.

1. Multiply 0·0765 by 0·000006201, by logs.
2. Divide 0·00652 by 0·9876, by logs.
3.

Hour.	Courses.	Knts.	10th.	Winds.	Leeway.	Deviation.	Remarks.
					Pts.		
1	W.	7		N.N.W.	—	10° W.	Departure taken from a point in lat. 60° 45′ N., long. 4° 43′ E.; bearing by compass, S.W.; ship's head, W.; deviation as per log, distant 15 miles. Variation, 17° W.
2		7					
3		7					
4		7					
5	N.W.	7	5	N.N.E.	¼	20° W.	
6		7	5				
7		7	5				
8		7	5				
9	W. by N. ¾ N.	9		S.W. by S.	¾	13° W.	
10		9					
11		9					
12		9					
1	W.S.W.	10		S.	½	7° W.	
2		10					
3		10					
4		10					
5	S.W.	10	5	W.N.W.	¼	—	A current set N.N.E. correct magnetic all day at ¾ mile per hour.
6		10	5				
7		10	5				
8		10	5				
9	W. by S.	8		S. by W.	1¼	8° W.	
10		8					
11		8					
12		8					

Find the course and distance made good; lat. and long. of the ship.

4. August 1st, in long. 124° 27′ E., the obs. mer. alt. of the Sun's U.L. bearing north was 81° 27′ 10″; ind. err. −1′ 25″; height of eye, 33 ft. Required the latitude.

5. In lat. 54° 37′ N., the diff. long. made good was 5° 47′. Required the distance sailed by parallel sailing.

6. Required the compass course and distance by Mercator's sailing.

From A, lat.	50°	10′	N.	Long.	13° 25′ W.
To B, ,,	47	50	N.	,,	62 30 W.
Deviation,	20	30	E.	Variation,	25 0 W.

7. November 5th. Find the M.T.G. of high water at Bridlington.

8. August 5th, about $5^h\ 40^m$ A.M., in lat. 31° 14′ S., long. 157° 20′ E., when the chron. showed correct G.M.T., $7^h\ 11^m\ 40^s$, the Sun's bearing by compass was N. 75° 30′ E. Required the true amplitude and error of the compass; and supposing the variation to be 6° 20′ E., required the deviation of the compass for the position of the ship's head.

9. August 17th, about 8^h A.M. at ship, in lat. at noon 35° 48′ N., long. by account 36° W., the obs. alt. of Sun's L.L. was 30° 25′ 50″; ind. err. + 2′ 14″; height of eye, 30 ft.; time by chron. $10^h\ 39^m\ 14^s$, which was fast $20^m\ 12^s$ for G.M. noon, on June 9th and on July 19th was fast $19^m\ 40^s$; run since sights, S. 25° E. 43 miles. Required the long. at sight, also brought up to noon; and if the compass bearing was S. 80° 30′ E., and variation 23° 30′ W., find the true azimuth and deviation.

10. August 4th, about $10^h\ 14^m$ A.M., in lat. 38° 27′ N., long. 84° 20′ W., when a chron. supposed $15^m\ 12^s$ fast of M.T.G. showed $4^h\ 7^m\ 47^s$, the Sun's bearing by compass S. 52° 30′ E. Required the true azimuth and error of the compass by the tables; and supposing the variation to be 7° 30′ E., required the deviation for the position of the ship's head.

11. August 19th, P.M. at ship, lat. by account, 17° 10′ S., long. 177° 30′ E., the alt. of the Sun's L.L. 57° 15′ 20″; ind. err. + 1′ 18″; height of eye, 18 ft.; time by chron. $12^h\ 51^m\ 39^s$, which had been found to be slow $11^h\ 55^m$ of A.T.S.; the course and dist. were S. 54° E., 23′ since the error was determined. Required the lat. by reduction to the meridian; and if the run from noon to obs. on the same course was 3 miles, find the lat. at noon.

12. September 7th, about 9^h A.M. at ship. If in lat. by account 48° 5′ N., long. 149° 30′ W., chron. showed $6^h\ 47^m\ 8^s$ G.M.T.; the alt. of the Sun's U.L. 32° 26′ 50″; again A.M. on the same day, when the chron. showed $9^h\ 10^m\ 9^s$ G.M.T.; the alt. of the Sun's U.L. 47° 20′ 30″; ind. err. − 3′ 13″; height of eye, 35 ft.; the ship having made 23 miles on a true N. 49° E. course in the interval between the observations; required the line of position, the Sun's true azimuth, and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 47° 30′ N. and 48° N.

13. Find the A.T.S. of the meridian passage of α Cygni in long. 96° 45′ E. on September 7th; also by inspection.

14. What stars not less than magnitude 2 are within 1 hour east of the meridian of a place in lat. 20° 30′ S., long. 80° 40′ E., above the horizon and pole at 7^h P.M. M.T.S. on August 7th?

15. Calculate the mer. alt. of α Pavonis for placing on the sextant ind. cor. − 2′ 13″; eye, 27 ft. in lat. 12° 40′ N.

16. September 26th, in lat. 36° 32′ S., long. 125° 40′ W., at $1^h\ 35^m$ A.M. M.T.S.; the compass bearing of Aldebaran was N. 40° E. Find from the tables the true azimuth; and if the variation be 9° E., find the deviation.

17. September 28th, about 4^h A.M. at ship, in lat. 47° 42′ S., long. by account, 84° 15′ E., the obs. alt. of α Hydræ was 17° 15′ 20″; no ind. cor.; eye, 29 ft.; time by chron., $9^h\ 13^m\ 25^s$, which was slow $1^h\ 16^m\ 12^s$ of G.M. noon on September 2nd, and gaining $4^s{\cdot}6$ daily. Find the longitude.

18. August 12th, when the G.M.T. was $12^d\ 11^h\ 36^m\ 31^s$, in long. 23° 49′ W.; the obs. alt. of Polaris out of the meridian was 42° 17′ 50″; ind. cor. − 3′ 11″; eye, 28 ft. Find the latitude.

19. August 17th, in long. 33° 45′ E.; the obs. meridian alt. of the Moon's L.L. bearing north was 80° 37′ 40″; ind. cor. − 1′ 20″; eye, 35 ft. Find the latitude.

20. September 19th, the obs. meridian alt. of the star β Tauri was 53° 25′ 10″ bearing north; ind. err. − 3′ 17″; height of eye, 24 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N. 41° E.		S.	N. 33° W.	
N.E.	N. 30° E.		S.W.	N. 27° W.	
E.	N. 11° E.		W.	N.	
S.E.	N. 12° W.		N.W.	N. 28° E.	

Find correct magnetic bearing; construct a Napier's curve.

With mag. courses: N. by E. $\frac{1}{2}$ E. E. $\frac{3}{4}$ S. S.S.W. N.W. by N. $\frac{1}{4}$ N. Find comp. courses.

Given comp. courses: N. $\frac{1}{2}$ E. E.N.E. S.E. by E. $\frac{1}{4}$ E. S.W. $\frac{3}{4}$ W. Find mag. courses.

With ship's head at E. $\frac{3}{4}$ S. by compass, the comp. bearings of distant objects were E.S.E. and W. by N. $\frac{3}{4}$ N. Find magnetic bearings.

22. On October 13th, at $2^h\ 47^m$ A.M. M.T.S., being off Fleetwood by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

23. August 10th, about $8^h\ 15^m$ A.M., in lat. 37° 14′ N., long. by account 14° W., when a chron. estimated $5^m\ 25^s$ slow of G.M.T. showed $9^h\ 13^m$; the following observations were taken for determining long. and chron. error: obs. alt. of Moon's L.L. 29° 38′ 20″; ind. cor. + 3′ 14″; height of eye, 30 ft.; obs. distance between enlightened limbs of Sun and Moon, 109° 17′ 50″; ind. cor. — 2′ 35″.

24. August 2nd, about 9^h P.M. in lat. 23° 15′ N., approx. long. 62° 20′ W., when a chron. showed $1^h\ 25^m\ 40^s$ correct M.T.G.; the obs. alt. of Antares was 35° 42′ 40″; and at the same time the obs. alt. of Spica was 14° 18′ 20″; ind. cor. — 3′ 18″; eye, 35 ft. Find the true bearings of the stars and ship's position, either by "Sumner's method," assuming lats. 15′ on each side of lat. by account, or by direct spherics.

25. August 5th, at $2^h\ 30^m$ P.M. M.T.S. in lat. 30° 27′ 45″ S., long. 93° 55′ E.; the obs. alt. of Sun's U.L. in artificial horizon was 62° 50′ 50″; ind. cor. — 4′ 44″; eye, 19 ft. If the chron. showed $8^h\ 19^m\ 41^s$, required its error on M.T.G.

26. Find the first and last courses and distance on a G.C. arc from lat. 11° 51′ N., long. 51° 16′ E., to lat. 52° 33′ S., long. 169° 9′ E., position of vertex, and latitudes of intersection of meridians of 60° E., 70° E., 80° E., 90° E., and 160° E. with the track.

27. In triangle ABC, A = 90°, AC = 1950, C = 1° 3′. Find BC.

28. In triangle XYZ, XY = 210, X = 47° 48′, Y = 67° 26′. Find XZ.

29. In spherical triangle, R = 90°, QR = 119° 8′, PR = 168° 20′. Find P and PQ.

30. The deviations with ship's head by compass, on N., N.E., E., S.E., S., S.W., W., and N.W. are 20° W., 16° W., 5° E., 12° E., 12° E., 9° E., 2° E., 4° W. Find coefficients A, B, C, D, and E, and compute deviations for points from E. to W. through north.

31. With ship steering N. 35° E., and heeling 14° to port, the heeling error is —15°. Find the amount when steering N. 70° W., and heeling 18° to starboard.

Paper IV.

1. Multiply 300·07 by 0·0246, by logs.
2. Divide 5623·87 by 48960·02, by logs.
3.

Hour.	Courses.	Knts.	10ths.	Winds.	Lee-way.	Deviation.	Remarks.
					Pts.		
1	W.S.W.	12	8	N.W.	1¾	26° E.	Departure taken from a point in lat. 37° 3′ N., long. 4° 25′ E.; bearing by compass E. by N.; ship's head S.W.; deviation 29° E., distant 20 miles. Variation 21° W.
2		13	1				
3		12	5				
4		11	6				
5		11	—				
6	W.	12	9	N.N.W.	2	19½° E.	
7		13	—				
8		12	8				
9		12	3				
10	W. ½ N.	12	2	N.	1½	17° E.	
11		12	7				
12		12	1				
1	W. by S.	11	9	N.N.W.	¾	23° E.	
2		11	8				
3		11	9				
4		11	8				
5		11	6				
6	W. ½ S.	11	5	N.N.W.	1¼	20° E.	A current set S. 21° E. true, 42 miles during the day.
7		12	9				
8		12	8				
9		12	8				
10	W.N.W.	13	1	N.	1	11° E.	
11		13	6				
12		13	3				

Find the course and distance made good; lat. and long. of the ship.

4. August 4th, in long. 82° 35′ E., the observed meridian altitude of the Sun's L.L. zenith south of Sun was 15° 32′ 30″; ind. err. −3′ 4″; height of eye, 35 feet. Required the latitude.

5. How far due east must a vessel sail on the parallel of 39° 27′ S. to change her longitude 1° 29′?

6. Required the compass course and distance by Mercator's sailing—

From A, lat. 42° 35′ N. Long. 71° 18′ W.
To B, ,, 53 22 N. ,, 12 30 W.
Variation, 4 30 W. Deviation, 19 20 E.

7. July 24th. Find the M.T.S. of high water at Limerick.

8. September 8th P.M. at ship, in lat. 23° 40′ S., long. 65° 50′ E., when the A.T.S. was 5ʰ 50ᵐ 20ˢ, the Sun's bearing by compass was W. ¼ S. Required the true amplitude, and error of the compass; and supposing the variation to be 10° 30′ W., required the deviation of the compass for the position of the ship's head.

9. August 10th, about 8ʰ 15ᵐ A.M. at ship, in lat. at noon 37° 29′ N., long. by account 14° 30′ W., the obs. alt. of the Sun's L.L. was 36° 6′ 40″; ind.

err. 0; height of eye, 30 feet; time by chron. $9^h\ 20^m\ 53^s$, which was slow $1^m\ 12^s$ for G.M. noon on June 25th, and on July 10th was correct; run between obs. and noon, N.E. by E. 27 miles. Required the long. at sight, also brought up to noon; also, if the compass bearing was East and variation 21° 15′ W., find true azimuth and deviation.

10. September 2nd, about $4^h\ 20^m$ P.M. at ship, in lat. 43° 16′ S., long. 14° 5′ W.; when the correct G.M.T. by chron. was $5^h\ 25^m\ 10^s$, the Sun's bearing by compass N. 65° 15′ W. Required the true azimuth and error of the compass by the tables; and supposing the variation to be 14° W., required the deviation for the position of the ship's head.

11. September 25th P.M. at ship, lat. by account 20° 10′ S., long. 7° 29′ E.; the alt. of Algenib bearing N. was 46° 5′ 50″; ind. err. −2′ 8″; height of eye, 19 ft.; time by watch $11^h\ 25^m\ 4^s$, which had been found to be fast of M.T.S. $2^m\ 22^s$; the diff. long. made eastward was 12·7′ after the error was determined. Required the lat. by reduction to the meridian.

12. September 1st, about $0^h\ 33^m$ P.M. at ship. In lat. by account 49° N., long. 180° E., when the chron. showed $12^h\ 42^m\ 57^s$, and was fast $8^m\ 55^s$ of G.M.T.; the alt. of the Sun's L.L. 48° 51′; again P.M. on the same day, when the chron. showed $4^h\ 50^m\ 20^s$; the alt. of the Sun's U.L. 19° 30′ 20″; ind. err. +1′ 10″; height of eye, 17 ft.; the ship having made 6½ knots per hour on a true E.N.E. course in the interval between the observations. Required the line of position, the Sun's true azimuth, and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 20′ on each side of lat. by account.

13. In long. 127° 45′ E., find the mean time of transit of Fomalhaut on Sept. 14th.

14. What stars not less bright than mag. 2·5 are within 1 hour E. and W. of the meridian, above the horizon of a place in lat. 45° N., long. 95° W. on Sept. 8th at 9^h P.M. M.T.S. Give their meridian distances and bearings.

15. Calculate for placing on the sextant the mer. alt. of α Gruis in lat. 30° 24′ S., ind. cor. +1′ 17″; eye, 30 ft.

16. August 16th, midnight mean time at ship in lat. 30° 20′ N., long. 64° 20′ W., the compass bearing of α Arietis was E. ½ S. Find true bearing from the "tables;" and if the variation was 6° W., required the deviation.

17. August 2nd, about $9^h\ 30^m$ P.M. in lat. 25° 14′ N., long. 97° 15′ W., the obs. alt. of Spica was 12° 19′ 20″; ind. cor. −4′ 12″; eye, 17 ft.; when a chron. showed $3^h\ 55^m\ 58^s{\cdot}5$, and had been found slow $3^m\ 31^s{\cdot}5$ on April 29th of G.M. noon, and fast $2^m\ 15^s$ on June 23rd. Find the longitude.

18. September 8th, A.M. at ship, about 3^h in long. 157° E., when the A.T.G. was $4^h\ 36^m\ 26$, the obs. alt. of Polaris out of the meridian was 29° 26′ 40″; ind. cor. +6′ 4″; height of eye, 41 ft. Find the latitude.

19. August 15th, in long. 136° 15′ E., the obs. alt. of the Moon's L.L. bearing north was 46° 19′ 10″; ind. cor. +2′ 28″; height of eye, 34 ft. Required the latitude.

20. August 22nd, the observed meridian alt. of the star β Canis Majoris was 51° 19′ 20″ bearing north; ind. err. −4′ 37″; height of eye, 35 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N. 32° W.		S.	N. 20° E.	
N.E.	N. 20° W.		S.W.	N. 9° E.	
E.	N. 5° E.		W.	N. 15° W.	
S.E.	N. 17° E.		N.W.	N. 31° W.	

Find correct magnetic bearing; construct a curve.

Given mag. courses: S.E. $\frac{1}{2}$ E. N.E. by N. W. by N. $\frac{1}{2}$ N. S. 72° W.
Find comp. courses.

Given comp. courses: E. by N. $\frac{1}{4}$ N. S.E. $\frac{3}{4}$ S. S.S.W. W. 15° N.
Find mag. courses.

With ship's head at E.S.E. by compass, comp. bearings of distant objects were S. 50° W. and S. 35° E. Find magnetic bearings.

22. On September 7th, at 5^h A.M. M.T.S., being off Wicklow by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line before comparing it with the depth marked on the chart.

23. August 11th, at 4^h 6^m A.M. M.T.S., in lat. 40° 17′ N., long. by account 21° 39′ 45″ W., when a chron. which had run down and been set going showed 5^h 30^m, the following observations were taken for finding the long. and error of chron. for G.M.T.: obs. alt. of Moon's L.L., 54° 33′ 50″; ind. cor. −2′ 2″; height of eye, 27 ft.; obs. dist. between Jupiter's centre and the Moon's near limb, 54° 16′ 10″; ind. cor. −3′ 3″.

24. September 28th, about 4^h A.M. at ship, in lat. 47° 40′ S., long. 84½° E. by account; when a chron. correct for M.T.G. showed 10^h 27^m 40^s; the obs. alt. of ε Orionis was 41° 42′ 20″; and at the same time the obs. alt. of Alphard was 17° 15′ 20″; eye, 29 ft. Find the true bearings of the stars, and position of ship, either by "Sumner's method" assuming lats. 47° and 47° 40′ S., or by direct spherics.

25. September 22nd, at 2^h 17^m 27^s P.M. at ship, from which a place in lat. 9° 12′ N., long. 173° 44′ W., bore N. 33° W., distant 6 miles; the obs. alt. of Sun's L.L. was 53° 29′ 20″; ind. cor. −3′ 19″; height of eye, 21 ft. If a chron. showed 1^h 47^m 27^s, find its error on G.M.T.

26. Find the initial course and distance on a Great Circle, and position of vertex in sailing from lat. 34° 36′ S., long. 58° 22′ W., to lat. 33° 56′ S., long. 18° 29′ E.; and lat. of place where meridian of Greenwich cuts the track.

27. AB is a horizontal line 1300 ft. long.; a vertical line is drawn from B upwards, and in it two points P and Q are taken such that BQ = 3 BP, BAP = 10° 30′ 20″. Find BP and BAQ.

28. XY is 2000 ft. long; Y is due east of X; at Y a point Z bears N. 46° 20′ W., at X it bears N. 8° 45′ E. Find XZ.

29. Two places, A and B, are on the same parallel 35° N., and their difference of longitude is 85°. Find the distance between them on the G.C., also lat. of vertex.

30. From a rock 7 ft. above high water, the angle of elevation to the summit of a cliff distant 5320 ft. in a horizontal direction is 1° 30′; height of eye, 5 ft. Find height of cliff above high water.

31. September 12th, in lat. 25° 17′ S., long. 100° 14′ E. at 8^h 30^m P.M. M.T.S., the compass bearing of γ Crucis was S. 28° 30′ W. Find the deviation, the variation being 8° 15′ W.

32. Coefficient B = −18° 30′, coefficient C = +27° 48′. Find direction ship's head.

Ship's head S. 74° W., coefficient B = +5°·7. Find coefficient C.

33. Ship's head by compass N., N.E., E., S.E., S., S.W., W., N.W.
Deviations 10° W., 15° W., 14° W., 8° E., 20° E., 18° E., 3° W., 5° W.
Find coefficients A, B, C, D and E, and thence compute a table of deviations from E. to W. through S.

34. When steering S. 28° E., and heeling to port 15°, the error was −10°. Find its amount when steering west and heeling 12° to starboard.

Paper V.

1. Multiply 1·0009 by 62·7014, by logs.
2. Divide 7·5 by 1000, by logs.
3.

Hrs.	Courses.	Knts.	10ths.	Winds.	Lee-way.	Devia-tion.	Remarks.
					Pts.		
1	N.E. by E.	10		N. by W.	¼	16° E.	Departure taken
2		10					from a point in
3		10					lat. 62° 30′ N.,
4		10					long. 178° 54′ E.;
5	E. by N.	10	5	S.S.E.	1¼	18° E.	bearing by com-
6		10	5				pass W.N.W.;
7		10	5				ship's head
8		10	5				N.N.E.; devia-
9	N.E.	11		E.S.E.	¼	10° E.	tion 5° E., dis-
10		11					tant 20 miles.
11		11					Variation, 15° E.
12		11					
1	E.N.E.	11	5	S.E.	1	17° E.	
2		11	5				
3		11	5				
4		11	5				
5	E.	12		S. by E.	2¼	20° E.	A current set
6		12					E.N.E. correct
7		12					magnetic, 2¼
8		12					knots per hour
9	N.E. by E. ¼ E.	11	5	S.E. by E.	¼	16° E.	for all the day.
10		11	5				
11		11	5				
12		11	5				

Find the course and distance made good; lat. and long. of the ship.

4. August 6th, in long. 19° 17′ W.; the obs. mer. alt. of the Sun's U.L., 78° 16′ 50″, zenith south of object; ind. err., −5′ 11″; height of eye, 37 ft. Required the latitude.

5. In lat. 45° 50′ N. What is the departure made good to cause a change of 800′ of long. by sailing due east?

6. Required the compass course and distance by Mercator's sailing; var. 30° W.; dev. 10½° E.—

From A, lat. 33° 24′ S. long. 20° 15′ E.
To B, ,, 53 40 S. ,, 63 37 W

7. August 15th. Find the M.T.S. of high water at Seal Cove. N.B.

8. August 15th, about 5ʰ P.M. at ship, in lat. 44° 29′ S., long. 129° 35′ E., when a chron. known to be 15ᵐ 50ˢ slow of M.T.G. showed 7ʰ 56ᵐ 50ˢ, the Sun's bearing by compass was S. 88° 30′ W. Required the true amplitude, and error of the compass; and supposing the variation to be 2° 20′ W., required the deviation of the compass for the position of the ship's head.

9. September 29th, about 4ʰ 45ᵐ P.M. at ship, in lat. at noon 46° 23′ N., long. by account 179° 50′ W., the obs. alt. of Sun's L.L. was 11° 12′; ind. err., *nil*; height of eye, 25 ft.; time by chron., 4ʰ 43ᵐ 46ˢ, which was fast 13ᵐ 47ˢ for G.M. noon on August 10th, and losing 3ˢ·3 per day. The run since noon, N. 50° W. 42 miles. Required the long. at sight, also brought up to noon; and if the compass bearing was W. ¼ N., variation 13° 15′ E., find the deviation.

10. August 14th, about 9^h A.M. in lat. 58° 49′ N., long. 32° 56′ W., when a chron. correct for G.M.T. showed $11^h\ 22^m\ 38^s$; the sun's bearing by compass, S. 5° 30′ E. Required the true azimuth and error of the compass by the "Tables;" and supposing the variation to be 50° W., required the deviation for the position of the ship's head.

11. August 19th P.M. at ship, lat. by account 30° 22′ S., long. 170° 48′ E.; the alt. of the Sun's L.L., 45° 47′ 50″; ind. err., + 3′ 55″; height of eye, 19 ft.; time by chron., $1^h\ 16^m\ 58^s$, which had been found to be fast $10^m\ 6^s$ of M.T.G.; the run was S. 48° W. 35 miles after the error was determined. Require the lat. by reduction to the meridian; and if the run since noon was 4 miles on the same course, find the lat. at noon.

12. September 29th, about $0^h\ 30^m$ P.M. at ship, lat. by account 46° 55′ N., long. 179° 50′ W., when the chron. showed $12^h\ 20^m\ 39^s$ G.M.T.; the alt. of the Sun's L.L., 40° 30′ 40″; again P.M. on the same day, when the chron. showed $4^h\ 32^m\ 46^s$ G.M.T.; the alt. of the Sun's L L., 11° 7′; ind. err., + 5′; height of eye, 25 ft.; the ship having made 42 miles on a true N. 45° W. course in the interval between the observations. Required the line of position when the first alt. was taken, the sun's true azimuth and position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 46° and 46° 40′ N.

13. Find the apparent time of transit of Markab across the meridian of a place long. 121° 45′ E. on September 21st; also by inspection.

14. Find what bright stars of at least 2nd mag. are within 1 hour E. and W. of mer., above horizon and pole, on August 14th at $1^h\ 30^m$ A.M. A.T.S. in lat. 29° N., long. 130° E., and state their hour angles.

15. Calculate mer. alt. of α Cephei for placing on sextant in lat. 12° 15′ N.; ind. cor., − 4′ 26″; height of eye, 35 ft.

16. August 2nd, about 9^h P.M. at ship, in lat. 23° 12′ N., long. 62° 30′ W.; the obs. alt. of Antares was 35° 37′ 10″; ind. cor., + 2′ 14″; eye, 35 ft.; when a chron. showed $1^h\ 13^m\ 21^s$, which was slow $15^m\ 47^s$ on May 2nd, and slow $14^m\ 17^s$ on June 11th for G.M. noon. Required the longitude.

17. August 26th, at $3^h\ 42^m$ A.M. A.T.S., in lat. 42° 18′ S., long. 91° 14′ W.; the compass bearing of Markab was N.W. ¼ N., the variation being 18° 45′ E. Find true azimuth from "Tables," and deviation.

18. September 18th, about $5^h\ 15^m$ A.M. at ship, in long. 148° 15′ W., when a chron. which showed G.M.T. indicated $3^h\ 14^m\ 12^s$, the obs. alt. of Polaris out of the meridian was 51° 7′ 20″; ind. cor., − 5′ 37″; eye, 19 ft. Required the latitude.

19. August 10th, in long. 29° 30′ W.; the obs. mer. alt. of the Moon's U.L. was 48° 13′ 20″; zenith north of Moon; ind. cor., − 3′ 16″; height of eye, 32 ft. Find latitude.

20. The obs. mer. alt. of the star β Argûs under the South Pole was 30° 47′ 50″; ind. err., + 5′ 26″; height of eye, 12 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N. 56° E.		S.	N. 56° E.	
N.E.	N. 48° E.		S.W.	N. 63° E.	
E.	N. 45° E.		W.	N. 68° E.	
S.E.	N. 49° E.		N.W.	N. 64° E.	

Find correct magnetic bearing; construct a curve.

Given magnetic courses: E. by N. S.E. by E. W.S.W. N.N.W.
Find compass courses.

Given compass courses: S. 79° W. S. 23° E. N. 34° E. N.W.
Find magnetic courses.

With ship's head at E.N.E. by compass, the compass bearings of distant objects were S. by W. ½ W., and S. 59° E. Find magnetic bearings.

22. On August 6th, at 6h P.M. M.T.S., being off Tenby by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

23. August 17th, about 8h A.M. at ship, in lat. 36° 27′ N., long. by account, 36° W., when a chron. estimated 12m 10s fast on G.M.T. showed 10h 31m 40s; the obs. alt. of the Sun's L.L. was 30° 25′ 50″; ind. cor., +2′ 14″; height of eye, 30 ft.; obs. distance between the enlightened limbs of Sun and Moon, 30° 12′ 40″; ind. cor., +3′ 16″. Find the true and apparent altitudes of the Moon, long. of ship, and error of chron. on G.M.T.

24. September 7th, in lat. N., long. 149° 30′ W., the following observations were made for finding latitude by "Ivory" or "Direct," at second obs.:—

App. times ship nearly.	G.M.T. by chron.	Obs. alts. Sun's L.L.
8h 50m A.M.	6h 47m 7s	32° 26′ 50″
11 15 A.M.	9 10 8	47 20 30

Bearing at first obs., S. 60° ½ E.; run between obs., N. 48½° E., 9·5 knots per hour; ind. cor., −3′ 13″; eye, 35 ft.

25. September 4th, at 4h 30m P.M., when Cape Farewell in lat. 40° 29′ 50″ S., long. 172° 40′ 6″ E., bore from ship west 13 miles distant, the obs. alt. of Sun's U.L. in artificial horizon was 24° 0′ 30″; ind. cor., − 2′ 8″; time shown by chron., 4h 37m. Find its error on M.T.G.

26. Find the initial and final courses from lat. 34° 21′ S., long. 18° 30′ E., to lat. 38° 18′ S., long. 144° 39′ E., by composite sailing; easting to be run on the parallel of 46° S. Also find total distance, and lats. where meridians of 50° E. and 140° E. cut the track.

27. From a boat the angle of elevation of the truck, known to be 165 ft. above the water-level, was found to be 9° 17′. Find distance of observer from ship.

28. A vessel sails S.S.E. ½ E., 137 miles by log, but by observation she is known to be 101 miles S. ½ W. of her former position. Find the set and drift of the current.

29. In a right-angled spherical triangle, hyp. = 105° 20′; one angle is 35° 4′. Find side opposite given angle, and remaining angle.

30. Coefficient B = −3° 12; coefficient C = +8° 48′. Find direction of ship's head on stocks. With ship's head at building S. 14° W., and coefficient C = −1°·47, find B.

31. Using deviations found in question 21, find coefficients A, B, C, D, and E; and construct a table of deviations for quadrants N. to E and S. to W.

32. Steering S.W. by W., heeling 14°, wind N.N.W., obs. heeling error, −15°. Required the error when course is changed to E. by N., heeling 10°.

PAPER VI.

1. Multiply ·00483506 by 100·0706 by logs.
2. Divide 6789 by 78900 by logs.
3.

Hrs.	Courses.	Knts.	10ths.	Winds.	Leeway.	Deviation.	Remarks.
					Pts.		
1	N.E. ¼ E.	14	6	N.W. ¼ W.	1	20° E.	Departure taken from a point in lat. 37° 45′ S., long. 178°32′E.; bearing by compass, E. ¼ N.; ship's head, N.E. ¼ E.; deviation as per log, distant 17 miles.
2		14	8				
3		13	9				
4		14	3				
5		14	4				
6	E.N.E.	14	9	W.N.W.	¼	21° E.	
7		14	5				
8		13	6				
9	E. by N. ¼ N.	13	9	W.	—	22° E.	Variation, 14° E.
10		13	8				
11		14	3				
12		14	—				
1	E. by S. ¼ S.	14	7	S.W.	1¼	19° E.	
2		13	6				
3		14	3				
4		14	4				
5	E.S.E.	14	2	S.S.W.	1	18° E.	A current set N. ¾ E., correct magnetic, 2½ knots per hour for 24 hours.
6		13	8				
7		14	1				
8		13	9				
9	E.N.E.	14	2	S.S.W.	½	14° E.	
10		14	1				
11		13	9				
12		13	8				

Find the course and distance made good; lat. and long. of the ship.

4. September 23rd, in long. 8° 45′ W., the obs. mer. alt. of the Sun's L.L. bearing north was 47° 4′ 20″; ind. err., + 4′ 32″; height of eye, 39 ft. Required the latitude.

5. Along what parallel is a vessel sailing when for a distance sailed due west of 76 miles she alters her long. 1° 30′?

6. Required the compass course and distance by Mercator's sailing—

From A, lat. 55° 13′ S. Long. 43° 30′ E.
To B, ,, 10 19 S. ,, 120 12 E.
Variation, 36 40 W. Deviation, 14 10 W.

7. July 13th. Find the M.T.G. of high water at Quaco.

8. September 18th, in lat. 35° 18′ N., long. 156° 17′ W., when the A.T.S. (civil time) was $5^h\ 54^m$ A.M., the Sun's bearing by compass was E.N.E. Required the true amplitude, and error of the compass; and supposing the variation to be 14° 20′ E., required the deviation of the compass for the position of the ship's head.

9. September 7th, about $8^h\ 50^m$ A.M. at ship, in lat. at noon 48° 37′ N., approx. long. 149° 30′ W., the obs. alt. of Sun's U.L. was 32° 35′ 20″; ind. err., − 4′ 19″; height of eye, 35 ft.; time by chron., $6^h\ 41^m\ 30^s$, which was fast $3^m\ 10^s{\cdot}6$ for G.M. noon on July 19th, and on August 8th was slow $19^s{\cdot}4$; run since obs. N. 50° E. 42 miles. Required the long. at sight, also brought up to noon; and if the compass bearing was N. 84° 30′ E., and variation 22° 15′ E., find true azimuth and deviation.

10. September 12th, about $2^h\ 15^m$ P.M., in lat. 35° 33′ S., long. 43° 27′ E., when a chron. correct for G.M.T. showed $11^h\ 21^m\ 21^s$, the Sun's bearing by compass, N. 8° 30′ W. Required the true azimuth and error of the compass by the "Tables;" and supposing the variation to be 23° 30′ W., required the deviation for the position of the ship's head.

11. September 17th, about $5^h\ 15^m$ A.M. at ship, lat. by account 40° 35′ S., long. 88° 16′ W., the alt. of Aldebaran bearing north was 32° 58′ 40″; ind. err., −4′ 18″; height of eye, 37 ft.; time by watch, $10^h\ 42^m\ 24^s$, which had been found to be $24^m\ 7^s$ slow of M.T.G.; the diff. long. to the westward 17 miles after the error was determined. Required the lat. by reduction to the meridian.

12. September 23rd, about $9^h\ 30^m$ A.M. at ship, in lat. 47° 30′ N., long. 82° 20′ E. by D.R., when the chron. showed $3^h\ 52^m\ 39^s$ G.M.T.; the alt. of the Sun's L.L., 32° 13′ 10″; again P.M. on the same day, when the chron. showed $8^h\ 52^m\ 46^s$ G.M.T.; the alt. of the Sun's L.L., 32° 13′ 10″; ind. err., −2′ 8″; height of eye, 28 ft.; the ship having made 21 miles on a true N. 60° E. course in the interval between the observations. Required the line of position when the first alt. was taken, and the Sun's true azimuth; also the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 20 miles on each side of D.R. lat.

13. Find the mean time of passage of Arcturus across the meridian of 38° 15′ W. on September 28th.

14. September 28th. What bright stars not less than 2nd mag. are within 2 hours of the meridian, above the horizon and pole in lat. 30° S., long. 140° W., at $2^h\ 15^m$ A.M. M.T.S.? State their mer. distances and bearings.

15. Calculate for placing on the sextant the mer. alt. of γ Cygni in lat. 24° 12′ S.; ind. cor., +3′ 27″; eye, 32 ft.

16. September 16th, at $0^h\ 15^m$ A.M. A.T.S., in lat. 38° 47′ N., long. 14° 29′ W., the compass bearing of Altair was West. Find from the "Tables" the true azimuth; and if the variation be 23° 30′ W., find the deviation.

17. August 5th, about $9^h\ 30^m$ P.M. at ship, in lat. 48° 30′ N., long. by account 50° W., the obs. alt. of α Cygni, 67° 14′ 30″; ind. cor., −3′ 13″; eye, 39 ft.; when a chron. showed $11^h\ 57^m\ 25^s$, which had been found to be $45^m\ 32^s$ slow of G.M. noon on June 6th, and $48^m\ 13^s$ slow on June 26th. Find the longitude.

18. August 26th, at $9^h\ 14^m\ 23^s$ P.M. M.T.S., in long. 64° 45′ W., the obs. alt. of the Pole star out of the meridian was 38° 29′ 20″; ind. cor., + 2′ 25″; eye, 37 ft. Find the latitude.

19. August 7th, in long. 48° 45′ W., the obs. mer. alt. of the Moon's L.L. was 32° 17′ 50″ bearing north; ind. cor., − 5′ 5″, height of eye, 48 ft. Required the latitude.

20. August 13th, the obs. mer. alt. of the star α Ursæ Majoris was 62° 15′ 30″ zenith south of star; ind. err., − 4′ 45″; height of eye, 36 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	S. 5° W.		S.	S. 10° E.	
N.E.	S. 6° E.		S.W.	S. 7° W.	
E.	S. 18° E.		W.	S. 18° W.	
S.E.	S. 17° E.		N.W.	S. 21° W.	

Find correct magnetic bearing, and construct a curve.

Given magnetic courses: N. by W. W. $\frac{3}{4}$ S. S.E. $\frac{1}{4}$ E. N. 32° E. Find compass courses.

Given compass courses: E. by S. $\frac{1}{2}$ S. S.W. $\frac{3}{4}$ S. W. $\frac{1}{4}$ N. N. 8° W. Find magnetic courses.

With ship's head at E. by S. by compass, the compass bearings of distant objects were N.W. $\frac{1}{4}$ W. and S.W. $\frac{3}{4}$ W. Find magnetic bearings.

22. On January 10th, at $8^h\ 15^m$ A.M. M.T.G., being off Beachy Head by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

23. August 30th, at $8^h\ 29^m\ 30^s$ P.M. at ship, in lat. 10° 15′ S., long by D.R. 98 °14′ E., when a chron. which had stopped and been set going showed $1^h\ 45^m$, the obs. alt. of the Moon's L.L. was 76° 38′ 40″; obs. alt. of Antares was 49° 33′ 50″; ind. cor. to alts., − 2′ 15″; height of eye, 40 ft.; obs. distance between the Star and Moon's near limb, 44° 44′ 30″; ind. cor., + 2′ 47″. Find longitude and error of chron. on M.T.G.

24. August 19th, in lat. 48° 30′ S., long. 157° E. by D.R., the following obs. were made for finding lat. at second obs. by "Ivory" or "Direct" method:—

Mean ship times.	G.M.T. by chron.	Obs. alt. α^1 Crucis.
$8^h\ 13^m$ P.M.	$9^h\ 42^m\ 40^s$	44° 19′ 30″ bearing S. 40½ W.
12 33	2 2 40	23 38 50

Run between obs. N. 20½° W., 12 knots per hour; ind. cor., − 2′ 12″; eye, 36 ft.

25. September 4th, at 2^h A.M. A.T.S., Cape Lobos in lat. 18° 45′ S., long. 70° 24′ W., bore S. 40° E., distant 17 miles; the obs. alt. of α Gruis in artificial horizon was 79° 34′ 40″; ind. cor., + 4′ 16″. If a chron. showed $6^h\ 40^m$, find its error on M.T.G.

26. Find initial course and distance on a composite track, highest lat. 50° S., and lats. of points where meridians of 120° W. and 180° W. cut the track—

From lat. 33° 3′ S., long. 71° 38′ W.
To ,, 41 16 S., ,, 174 53 E.

27. In the triangle ABC, C = 90°, AC = 15,866 ft., BC = 13,000 ft. Find the angles, and length of line drawn from B to AC, bisecting angle ABC.

28. The elevation of a tower at A is 28° 17′; at B, in the same horizontal plane and 155 ft. nearer the tower, the elevation is 62° 37′. Find height of tower.

29. A vessel sails for 18 hours S.E. by S. 95 miles by the log, in a current setting S. by W. $\frac{1}{2}$ W. 2 knots per hour. What is the course and distance made good?

30. Using deviations from question 21, find coefficients A, B, C, D, and E,

and construct a table showing deviations for each point from N.N.E. to S.E. through east.

31. The Sun's declination being 18° 30′ N., and his true bearing at rising N.E. by E. ¾ E., find the lat. and hour angle at rising.

32. Construct a Mercator's chart, scale 1·8 in. to 1° long, extending from 59° to 61° S., and 28° 30′ W. to 31° 30′ W. A ship sails by compass from lat. 59° 40′ S., long. 30° 30′ W., as follows: S.E. by S., 57 miles (deviation 3° W.); W., 25 miles (dev. 8° 30′ E.); variation, 10° E. Find position of ship, course, and dist. made good.

33. Steering W. ¼ N., on the port tack, heeling 12°; the error obs. was −1°. Find the amount when steering south on the starboard tack, and heeling 15°.

Paper VII.

1. Multiply 13·5426 by 51400, by logs.
2. Divide 7560009 by 387·003, by logs.
3.

Hrs.	Courses.	Knts.	10ths.	Winds.	Lee-way.	Devia-tion.	Remarks.
1	N.W.	10		W.S.W.	¼	10° E.	Departure taken from a point in lat. 50° 10′ S., long. 110° 15′ W.
2		10					
3	N.N.W.	11		W.	½	15° E.	
4		11					
5	W.S.W.	12		N.W.	¾	8° W.	Variation, 20° E.
6		12					
7	N.N.W. ½ W.	11	5	W.	0	15° E.	
8		11	5				
9		11	5				
10		11	5				
11		12					
12		12					
1		12					Position by observation at end of run, lat. 46°10′S., long. 110° 30′ W.
2		12					
3		12					
4		12					
5		12					
6		12					
7		12					
8		12					
9		12					
10		12					
11		12					
12		12					

Find the set and drift of the current.

4. September 30th, in long. 124° 26′ E., the obs. mer. alt. of the Sun's U.L. bearing south was 88° 56′ 40″; ind. err., + 1′ 44″; height of eye, 41 ft. Required the latitude.

5. From lat. 51° 23′ N., long. 9° 36′ W., the distance sailed due west was 250 miles. Required present position.

6. Required the compass course and distance by Mercator's sailing—

From A, lat. 38° 13′ S. Long. 178° 43′ W.
To B, „ 52 10 N. „ 165 15 E.
Variation, 13 0 E. Deviation, 5 30 W.

7. September 28th. Find the M.T.S. of H.W. at Port Carlisle.

8. August 25, in lat. 48° 30′ S., long. 49° 35′ E., when the M.T.S. was $6^h\ 50^m$ A.M., the Sun's bearing by compass was E. by S. ¼ S. Required the true amplitude and error of the compass; and supposing the variation to be 33° 20′ W., required the deviation of the compass for the position of the ship's head.

9. September 24th, about $2^h\ 45^m$ P.M. at ship, in lat. 23° 28′ S., long. 158° 30′ E., the obs. alt. of Sun's U.L. was 41° 32′ 20″; ind. err., − 2′ 35″; height of eye, 34 ft.; time shown by chron., $3^h\ 58^m\ 53^s$, which was slow $21^m\ 41^s$ for G.M. noon on April 11th, and on July 20th was slow $18^m\ 14^s$; the compass bearing was W.N.W. Required the long., compass error, and deviation, the variation being 6° 30′ E.

10. August 24th, about $8^h\ 30^m$ A.M. in lat. 44° 17′ S., long. 15° 45′ W., when a chron. which was $8^m\ 43^s$ slow of G.M.T. showed $9^h\ 29^m\ 32^s$; the Sun's bearing by compass N.E. ¼ E. Required the true azimuth and error of the compass by the "Tables;" and supposing the variation to be 12° 40′ W., required the deviation for the position of the ship's head.

11. September 29th, A.M. at ship, lat. by account 44° 15′ N., long. 47° 42′ W.; the alt. of the Sun's L.L., 43° 0′ 30″; ind. err., − 3′ 3″; height of eye, 32 ft.; time by chron., $2^h\ 33^m\ 56^s$, which had been found to be $3^h\ 2^m\ 38^s$ fast of A.T.S.; the run was N. 34° E. 25 miles, after the error was determined. Required the lat. by reduction to the meridian; and if the run on the same course was 4 miles, find the lat. at noon.

12. August 4th, about $8^h\ 15^m$ A.M. at ship, in lat. 47° 20′ S., long. Greenwich when the chron. showed $8^h\ 20^m\ 51^s$ G.M.T.; the alt. of the Sun's U.L., 9° 9′ 20″; again P.M. on the same day, when the chron. showed $2^h\ 22^m\ 42^s$ G.M.T.; the alt. of the Sun's U.L., 18° 45′; ind. err., − 2′ 23″; height of eye, 23 ft.; the ship having made 48 miles on a true S.W. ¼ W. course in the interval between the observations. Required the line of position, the Sun's true azimuth, and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 46° 50′ and 47° 20′ S.

13. Find the apparent time of the meridian passage of Castor on August 23rd in long. 49° 30′ E.; also by inspection.

14. Calculate the meridian alt. of ϵ Sagittarii for placing on the sextant in lat. 24° 30′ N.; ind. cor., − 5′ 14″; eye, 40 ft.

15. Find what stars of not less than second mag. are within 2 hours east of the meridian of a place lat. 40° N., long. 150° E., above the horizon and pole on August 21st at $5^h\ 50^m$ A.M. M.T.S., and state their meridian distances.

16. August 12th, about 11^h P.M. at ship, lat. 54° 26′ S., long. 160° 25′ E., when a chron. correct for M.T.G. showed $12^h\ 48^m\ 20^s$, the compass bearing of α Ophiuchi was N.W. ¼ W. The variation being 17° E., find from the "Tables" the true bearing, and thence the deviation.

17. August 30th, about $8^h\ 30^m$ P.M. at ship, in lat. 10° 15′ S., long. 98½° E. by D.R., the obs. alt. of Antares was 49° 33′ 50″; ind. cor., − 2′ 15″; eye, 40 ft.; when a chron. showed $11^h\ 34^m\ 24^s$, which was $2^h\ 17^m\ 38^s$ slow on June 21st, and slow $2^h\ 19^m\ 44^s$ on July 21st for G.M.T. Find the longitude.

18. September 25th, at $11^h\ 30^m\ 50^s$ P.M. A.T.S., in long. 129° 19′ E., the obs. alt. of Polaris out of the meridian was 21° 24′ 50″; ind. cor., + 1′ 14″; eye, 38 ft. Find the latitude.

19. August 30th, in long. 14° 55′ E., the obs. mer. alt. of the Moon's upper limb zenith north of Moon was 29° 46′ 40″; ind. cor., + 2′ 29″; eye, 51 ft. Find the latitude.

20. August 10th, the obs. mer. alt. of the star Arcturus was 37° 50′ 20″ bearing south; ind. err., − 4′ 37″; height of eye, 39 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N. 44° W.		S.	S. 79° W.	
N.E.	N. 47° W.		S.W.	S. 89° W.	
E.	N. 80° W.		W.	N. 69° W.	
S.E.	S. 76° W.		N.W.	N. 52° W.	

Find correct magnetic bearing, and construct a curve.

Given magnetic courses : N. 30° W. S. 55° E. S. 66° W. N. 35° E. Find compass courses.

Given compass courses: E. $\frac{3}{4}$ S. S.S.W. $\frac{1}{4}$ W. W. $\frac{1}{4}$ N. N.W. by W. Find magnetic courses.

With ship's head at N. $\frac{1}{2}$ E. by compass, the compass bearings of distant objects were S. 55° W. and N. 32° W. Find magnetic bearings.

22. On September 21st, at 1^h P.M. M.T.S., being off Torbay by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line, before comparing it with the depth marked on the chart.

23. August 12th, A.M. at ship, in lat. 25° 19′ S., when the time by a chron. estimated $10^m\ 30^s$ fast of M.T.G. was $11^d\ 5^h\ 50^m\ 50^s$; the obs. alt. of Saturn was 41° 1′; ind. cor., + 2′ 14″ ; height of eye, 29 ft.; observed distance between Saturn's centre and the Moon's remote limb was 61° 2′ 10″ ; ind. cor., — 3′ 15″. Find the true and apparent alts. of the moon, longitude, and error of chron. for G.M.T.

24. September 29th, P.M. at ship, in lat. N., long. 180° W., with the following observations, find the lat. at second obs. by "Ivory" or "Direct" method :—

Approx. A.T.S.	M.T.G. by chron.	Obs. alt. Sun's L.L.	
$0^h\ 30^m$ P.M.	$0^h\ 20^m\ 39^s$	40° 35′ 40″	No ind. cor.
4 42 P.M.	4 32 46	11 12 0	Height of eye, 25 ft.

Sun's bearing at first obs., S. 10° W. ; run between obs., N. 45° W. 42 miles.

25. September 28th, at $1^h\ 30^m$ A.M., M.T.S. when Cape San Roque, lat. 5° 29′ S., long. 35° 15′ W., bore N. 64° W. distant 16 miles ; the obs. alt. of Capella in artificial horizon was 45° 20′ 30″; ind. cor., +1′ 16″. If a chron. showed $4^h\ 10^m\ 5^s$, find its error on M.T.G.

26. Find the first and last courses on a great circle, from Tarifa, lat. 36° 0′ N., long. 5° 37′ W., to Monte Video, lat. 34° 57′ S., long. 55° 55′ W., position of vertex, distance, the course to steer crossing the equator, and the long. to cross the equator.

27. The summit of a spire is vertically over the middle point of a horizontal square enclosure whose side is 1000 ft. long ; the shadow of the spire reaches a corner of the square when the sun has an altitude of 27° 29′ 48″. Required the height of the summit of the spire above the level of the square.

28. An island bears from a cape S.E. $\frac{1}{2}$ E. 35 miles ; a ship bound to the island meets a tide running to the eastward, and finds herself after some time 28 miles from the island, and 40 miles from the cape. Required the bearing of the island and set of the tide.

29. Calculate angle A and side AC of a spherical triangle, having given C = 90°, AB = 74° 49′, and BC = 30°.

30. Taking the deviations as found in question 21, find coefficients A, B, C, D, and E; and thence calculate a table of deviations for each point from W. to E. through south.

31. Steering N.W. ½ W., heeling 10° to port, the heeling error was found to be −15°. What will be the amount when steering E. ½ N., and heeling 20° to starboard?

32. August 26th, about 5ʰ A.M., in lat. 5° 30′ S., long. 40° 30′ W., the obs. compass bearing of α Persei was North, the variation being 6° 15′ W. Find the deviation, when a chron. correct for G.M.T. showed 7ʰ 45ᵐ.

PAPER VIII.

1. Multiply ·0043 by ·00025, by logs.
2. Divide 9999·99 by 100,000, by logs.
3.

Hrs.	Courses.	Knts.	10ths.	Winds.	Leeway.	Deviation.	Remarks.
					pts.		
1	W.N.W.	12		N. ¼ E.	¼	25° W.	Departure taken from a point in lat. 59° 49′ N., long. 43° 54′ W.; bearing by compass, E. by N.; ship's head, N.W. ½ N.; deviation as per log, distant 30 miles.
2		12					
3		12					
4		12					
5		13					
6		13					
7	W.	13		N.N.W.	¾	30° W.	
8		13					
9		14					
10		14					
11		14					
12		14					
1	N.W. by W. ½ W.	14		S.W. ½ S.	½	21° W.	Variation: first 8 hours, 51° W.; second 8 hours, 49° W.; third 8 hours, 47° W.
2		14					
3		12					
4		12					
5		15					
6		15					
7		15					
8		15					A current set true S. 65° W., 10 miles for the 24 hours.
9		15					
10	N.W. ½ N.	15		N.E. by N.	¼	20° W.	
11		15					
12		15					

Find the course and distance made good; lat. and long. of the ship.

4. September 24th, in long. 172° 15′ E., the obs. mer. alt. of the Sun's L.L. zenith south of Sun was 56° 42′ 10″; ind. err., − 2′ 23″; height of eye, 43 ft. Required the latitude.

5. At noon the lat. is 51° 20′ S.; by morning sights the long. was found to be 85° 40′ W.; run since sights, East 76 miles. Find noon longitude.

6. Required the compass course and distance by Mercator's sailing—

From A, lat. 35° 7′ N.	Long. 141° 6′ E.
To B, ,, 54 53 S.	,, 75 35 W.
Variation, 1 20 E.	Deviation, 18 40 W.

7. September 13th. Find the M.T.S. of high water at Dartmouth.

8. September 28th, about 6^h P.M., in lat. 50° 5′ N., long. 175° 30′ W., when a chron. correct for M.T.G. showed $5^h\ 43^m\ 24^s$, the Sun's bearing by compass was N. 84° 30′ W. Required the true amplitude, and error of the compass; and supposing the variation to be 18° 20′ E., required the deviation of the compass for the position of the ship's head.

9. August 4th, about $8^h\ 20^m$ A.M. at ship, in lat. 47° 24′ S., long. Greenwich, the obs. alt. of Sun's U.L. was 8° 16′ 30′; ind. err., + 3′ 12″; height of eye, 28 ft.; time by chron., $8^h\ 18^m\ 59^s$, which was 3^m fast for G.M. noon on May 31st, and on July 10th was correct. Find the long.; also, if the compass bearing was N. 50° 30′ E. and variation 10° 30′ W., find the deviation.

10. September 22nd, about 5^h P.M., in lat. 57° 11′ N., long. 169° 28′ E., when a chron. correct for M.T.G. showed $5^h\ 42^m\ 23^s$, the Sun's bearing by compass, S. 60° W. Required the true azimuth and error of the compass by the "Tables;" and supposing the variation to be 10° 40′ E., required the deviation for the position of the ship's head.

11. September 21st, about $10^h\ 20^m$ P.M. at ship; lat. by account 43° 30′ N., long. 146° 27′ E.; the alt. of Fomalhaut, 16° 5′ 40″; ind. err., − 4′ 11″; height of eye, 27 ft.; time shown by chron., $12^h\ 12^m\ 54^s$, which had been found to be $16^m\ 44^s$ slow of M.T.G. Required the latitude by reduction to the meridian.

12. October 1st, about $11^h\ 30^m$ A.M. at ship, in lat. 49° 20′ N., long. 179° 50′ E. by D.R., when the chron. showed $11^h\ 21^m\ 24^s$ G.M.T., the alt. of the Sun's L.L., 37° 30′ 50″; again P.M. on the same day, when the chron. showed $3^h\ 4^m\ 26^s$ G.M.T.; the alt. of the Sun's L.L. 23° 6′ 30″; ind. err., − 2′ 9″; height of eye, 32 ft.; the ship having made 50 miles on a true S. 81° W. course in the interval between the observations. Required the line of position when the first alt. was taken, the Sun's true azimuth, and the position of the ship when the second alt. was observed by "Sumner's method" by projection on the chart, assuming the lats. 48° 40′ N. and 49° 20′ N.

13. Find the mean time of transit of Sirius across the meridian of 62° 15′ W. on August 30th.

14. Find what bright stars up to mag. 2·5 are within 1 hour E. and W. of the meridian, above the horizon and pole, in lat. 35° S., long. 160 W., at $4^h\ 30^m$ A.M. A.T.S. on September 23rd. State their hour angles, and bearings when on meridian.

15. Calculate the mer. alt. for placing on sextant of α Ophiuchi in lat. 55° 30′ N.; ind. cor., 5′ 14″ on the arc; eye, 40 ft.

16. September 14th, about 9^h P.M., in lat. 35° 37′ N., long. 74° 20′ W., when a chron. showed $2^h\ 12^m\ 10^s$ G.M.T., the compass bearing of γ Pegasi was E. by N. Find from the "Tables" the true azimuth; and if the variation was 5° 15′ W., find the deviation.

17. August 20th, about $0^h\ 30^m$ A.M., in lat. 48° 20′ S., long. 157° E. by D.R., the obs. alt. of α^1 Crucis W. of mer. was 23° 38′ 50″; ind. cor., − 2′ 8″; eye, 36 ft.; time shown by chron., $2^h\ 11^m\ 9^s$, which had been found fast $7^m\ 29^s$ of G.M. noon on June 5th, and fast $8^m\ 13^s$ on July 30th. Find the longitude.

18. August 30th at $2^h\ 47^m\ 23^s$ A.M. M.T.S., in long. 165° 15′ W., the obs. alt. of Polaris out of the meridian was 30° 2′ 10″; ind. cor., − 2′ 16″; eye, 25 ft. Required the latitude.

19. September 1st, in long. 174° 30′ E., the obs. mer. alt. of the Moon's L.L. was 81° 11′ 24″, zenith north of Moon; ind. cor., − 4′ 15″; eye, 27 ft. Required the latitude.

20. September 20th. The obs. mer. alt. of the star λ Scorpii was 41° 13′ 10″ bearing south; ind. err., − 1′ 49″; height of eye, 25 ft. Required the latitude.

21.

Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.	Ship's head by standard compass.	Bearings of distant object by standard compass.	Deviation required.
N.	N.		S.	N. 21° W.	
N.E.	N. 7° E.		S.W.	N. 17° W.	
E.	N. 10° W.		W.	N. 18° E.	
S.E.	N. 13° W.		N.W.	N. 16° E.	

Find correct magnetic bearing; construct a curve.

Given magnetic courses: N. ¼ E. S.E. by E. ½ E. W.S.W. W. 12° N. Find compass courses.

Given compass courses: E. ¾ N. N.W. ½ W. S. 65° W. S. 32° E. Find magnetic courses.

With ship's head at E. ½ S. by compass, the compass bearings of distant objects were S. by W. ¼ W., and S.E. by E. Find magnetic bearings.

22. On September 5th, at $0^h\ 30^m$ A.M. M.T.G., being off Fowey by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead-line before comparing it with the depth marked on the chart.

23. August 15th, about $1^h\ 25^m$ P.M. at ship, in lat. 23° 27′ N., long. by account, 170° 30′ W., when a chron. estimated 7^m slow of G.M.T. showed $12^h\ 40^m$, the alt. of the Sun's L.L. was 68° 37′ 10″; ind. cor., + 4′ 9″; height of eye, 28 ft.; obs. distance between the enlightened limbs of the Sun and Moon, 47° 22′ 40″; ind. cor., − 3′ 17″. Find the true and apparent alts. of the Moon, longitude, and error of chron. on G.M.T.

24. August 5th, in lat. 48° N., long. 50° W., with following observations, find lat. at second obs. by "Ivory" or "Direct" method :—

M.T.S. nearly.	Time shown by chron.	Obs. alts. of α Cygni.	
5th $9^h\ 30^m$ P.M.	$12^h\ 46^m\ 4^s$	67° 21′ 40″	Ind. cor., + 2′ 12″
6th 1 48 A.M.	5 3 59	68 26 40	Eye, 39 ft.

The chron. was 5^m slow of M.T.G.; bearing at first obs., N. 86° E.; run between obs., S. 13° W., 10 knots per hour.

25. September 9th, at $9^h\ 15^m$ A.M. A.T.S., when Cape Comino, lat. 40° 31′ 30″ N., long. 9° 50′ 30″ E., bore N. 35° E. dist. 14 miles, the obs. alt. of Sun's L.L. in artificial horizon was 77° 51′ 10″; ind. cor., + 3′ 18″; time shown by chron., $8^h\ 43^m$. Find its error on M.T.G.

26. Find the initial course and distance on a composite track, easting run on parallel of 42° S., from lat. 33° 56′ S., long. 18° 29′ E., to lat. 35° 2′ S., long. 117° 54′ E.; and lats. of points in which meridians of 50° E., 90° E., and 100° E. cut the track.

27. If the diff. lat. is 224·6 N., and dep. 157·9 miles E., find course and distance by calculation.

28. In triangle PQR, PQ = 137, PR = 257, R = 30° 40′. Find QR.

29. The angular elevation of a tower at A is 30°, at B in the same horizontal plane, and 100 ft. nearer the tower, the elevation is 60°. Find height of the tower.

30. In spherical triangle PQR, P = 90°, Q = 68° 30′, PQ = 101° 20′. Find R and RQ.

31. In spherical triangle ABC, C = 90°, AC = 43° 45′, BC = 81° 27′. Find A and AB.

32. Construct a Mercator chart, scale 1″·2 = 1° long., extending from 67° to 70° N., and 2° W. to 2° E. A ship sails from 69° 30′ N., 0° 30′ E., the following true courses: S.W. by S. 30′; E. by S. 40′; S.S.W. 56′; W. by N. 25′. Find her latitude and longitude in.

33. Given coefficient B = −15°·7, coefficient C = −14°·5. Find direction of ship's head at building.

34. Using deviations found in question 21, find coefficients A, B, C, D, and E; and construct a table of deviations for every alternate point of the compass, starting at N.

35. Steering N.W. ½ N., heeling 16° on the port tack, the error was observed to be −14°·5. Find its amount when steering east on the starboard tack and heeling 20°.

36. August 17th, at 11ʰ 35ᵐ 20ˢ P.M. M.T.S., in lat. 5° 24′ N., long. 68° 30′ E., the compass bearing of β Gruis was S.S.E. Find the true bearing; and if the variation is 1° 15′ W., find the deviation.

ANSWERS.

CHAPTER III.

Logarithms.

1.	0·477121	0·532627
	1·672098	0·875061
	1·698970	2·213385
	2·000000	2·847579
	2·439333	4·513697
	3·922206	9·698970
	1·758079	8·690196
	2·563173	7·556302
	0·000389	7·080626
	4·876391	7·827886
	1·797278	6·447794
	2·243058	5·876795
	4·361779	8·539087
	2·215643	8·878545
	4·333447	9·564667
	4·370328	

Numbers.

2.	1598	149899·7
	139·9	3400015·6
	50·15	10010023·1
	5·199	230100
	1·010	11000
	48026·9	1121000
	490000	1·001
	1408961	4679·01
	10969874	18880043·5
	63980	239901·1

Multiplication by Logs.

1.	19639	13.	·0000129
2.	23360	14.	·000259
3.	279088	15.	·5
4.	358900	16.	24951·6
5.	709·3934	17.	100000
6.	47·808	18.	20·42
7.	2110·41	19.	·2303
8.	288·651	20.	·001795
9.	3540707·3	21.	1267·618
10.	5·9585	22.	229500
11.	·0000081005	23.	18·16
12.	6·4831	24.	143·21

Division by Logs.

1. 46·3572
2. 293·6
3. 9040
4. 176020·7
5. 4·613
6. 24·7069
7. 1328856·7
8. 1026·903
9. 1·424
10. 142199·67
11. 1·02217
12. ·00001029
13. 1490996·5
14. 44826082·5
15. ·000001
16. 20
17. 1·2187
18. 5·8916
19. 57·454
20. 20967·1
21. 20·399
22. ·032725
23. 7·9876
24. 957·94

Involution by Logs.

1. 547·56
2. 6·602
3. ·000005797
4. ·000000001411
5. ·000002191

Evolution by Logs.

1. 17·5
2. 3·6
3. ·30012
4. 230
5. ·08

Additional Exercises.

1. 143·8, 27·74, 360·8
2. 891·9 and 9754·5
3. 172974·6, ·03528
4. 1·439

CHAPTER IV.

Right-angled Plane Triangles.

1. A = 44° 47′ 10″; B = 45° 12′ 50″; BC = 354·3.
2. P = 28° 48′ 30″; R = 61° 11′ 30″; PR = 423·4.
3. B = 10° 25′ 30″; P = 79° 34′ 30″; OB = 683·5.
4. AC = 443·4; BC = 937·4.
5. AB = 1095; AC = 617.
6. A = 52° 6′ 11″; B = 37° 53′ 49″; AC = 90·3.
7. PR = 5227; QR = 2864.
8. A = 54° 15′ 30″; B = 35° 44′ 30″; AB = 1208·5.
9. Height, 91·55 ft.
10. Course, S. 42° 22′ W.; dist. 320·1 miles.
11. Height, 15 ft. 7 in.
12. First dist. 9·08 miles; second dist. 5·75 miles.
13. 6° 15½′.
14. 113·8 ft.

Oblique-angled Plane Triangles.

1. A = 47° 7′ 8″; B = 14° 25′ 32″; C = 118° 27′ 20″.
2. A = 42° 16′ 41″; B = 23° 38′ 38″; C = 114° 4′ 41″.
3. A = 83° 25′ 54″; B = 55° 52′ 48″; C = 40° 41′ 18″.
4. C = 67° 41′ 0″; b = 164; c = 157·5.
5. A = 64° 27′ 0″; a = 1078·7; c = 1172·8.
6. B = 32° 10′ 0″; a = 20·62; b = 11·08.
7. A = 96° 16′ 25″; B = 44° 3′ 35″; c = 4466.
8. A = 45° 31′ 53″; C = 65° 42′ 7″; b = 348·7.
9. B = 46° 50′ 2″; C = 35° 51′ 58″; a = 82·95.
10. B = 42° 50′ 18″; C = 84° 43′ 42″; c = 1086.
11. A = 19° 4′ 29″; B = 121° 51′ 31″; a = 180·5; or 82° 47′ 31″; or 58° 8′ 29″; or 547·8.
12. B = 90° 30′ 11″; C = 39° 53′ 49″; b = 1398·4.
13. B = 35° 4′ 5″; C = 123° 32′ 55″; c = 1342; or 144° 55′ 55″; or 13° 41′ 5″; or 380·9.
14. A = 48° 11′ 22″; B = 58° 24′ 42″; C = 73° 23′ 56″.
15. A = 27° 7′ 36″; B = 22° 19′ 54″; C = 130° 32′ 30″.
16. A = 44° 24′ 54″; B = 57° 7′ 20″; C = 78° 27′ 46″.
17. C = 11° 40′ 0″; a = 6291; c = 4044.
18. B = 61° 25′ 0″; a = 206·3; b = 184·7.
19. B = 88° 30′ 1″; C = 33° 30′ 59″; a = 201·2.
20. A = 48° 41′ 9″; C = 73° 1′ 51″; b = 2015.
21. A = 46° 35′ 31″; B = 16° 13′ 31″; c = 1591·8.
22. B = 62° 31′ 25″; C = 102° 17′ 35″; c = 5919; or 117° 28′ 35″; or 47° 20′ 25″; or 4455.
23. B = 59° 57′ 53″; C = 84° 47′ 7″; c = 345·1; or 120° 2′ 7″; or 24° 42′ 53″; or 144·9.
24. 212 ft.
25. 134·7 ft.

Miscellaneous Problems.

1. 56° 18′ 36″ and 33° 41′ 24″.
2. 610·5 yards.
3. N. 36° 12′ 15″ W.
4. 71·36 ft.
5. Height 74 ft. Width 47·7 ft.
6. 367·9 ft.
7. 4° 57½′.
8. 1628·7 ft.
9. S. 53° 11′ 24″ W.
10. S. 80° W.
11. 115 ft.
12. S. 52″, 1′ 36″ E.
13. 578·5 ft., 818·5 ft., 54° 39′ 28″.
14. 64° 26′ 30″. 29·66 yards.

CHAPTER V.

Right-angled Spherical Triangles.

1. A = 54° 15′ 52″; B = 70° 5′ 36″; AB = 74° 53′ 52″.
2. BC = 71° 20′ 56″; AC = 38° 18′ 16″; AB = 75° 27′ 57″.
3. A = 62° 17′; B = 79° 8′ 34″; AC = 77° 42′ 53″.
4. B = 77° 2′ 15″; BC = 50° 17′ 53″; AC = 73° 20′ 39″.
5. A = 45° 40′ 14″; AC = 39° 6′ 20″; AB = 49° 18′ 54″.
6. B = 139° 26′ 36″, or 40° 33′ 24″; AC = 145° 5′ 45″, or 34° 54′ 15″; AB = 118° 21′ 6″, or 61° 38′ 54″.
7. C = 70° 55′ 17″; A = 101° 55′ 44″; AC = 94° 11′ 25″.
8. C = 110° 3′ 55″; A = 98° 13′ 51″; AC = 86° 58′ 17″.
9. AB = 117° 2′ 58″; BC = 65° 5′ 37″; AC = 101° 2′ 29″.
10. AB = 92° 52′ 13″; BC = 118° 45′ 49″; AC = 88° 37′ 9″
11. C = 78° 10′ 28″; A = 144° 52′ 34″: BC = 146° 41′.

12. C = 96° 57′ 4″; A = 101° 19′ 54″; AB = 97° 5′ 24″.
13. B = 55° 2′ 52″; C = 96° 0′ 20″; AC = 54° 49′ 36″.
14. B = 144° 52′ 47″; AC = 148° 4′ 7″; AB = 62° 23′ 2″.
15. B = 129° 14′ 9″; AC = 129° 43′ 36″; AB = 100° 40′ 35″.
16. C = 95° 41′ 50″; AC = 76° 8′ 49″; BC = 91° 24′ 9″.
17. B = 106° 24′ 22″; AB = 94° 18′ 33″; BC = 88° 47′.
18. B = 49° 37′, or 130° 23′; AC = 46° 57′ 45″, or 123° 2′ 15″; BC = 106° 21′ 15″, or 73° 38′ 45″.

Oblique-angled Spherical Triangles.

1. A = 82° 42′ 30″; B = 90° 9′ 16″; C = 104° 19′ 36″.
2. A = 153° 7′ 14″; B = 41° 22′ 24″; C = 43° 50′ 32″.
3. A = 116° 19′ 7″; B = 104° 58′ 43″; AB = 137° 29′ 6″.
4. BC = 91° 29′ 30″.
5. A = 119° 15′; C = 73° 59′.
6. A = 91° 30′ 10″.
7. 24° 20′ 38″.
8. 249° 51′ 15″. 29° 37′ 12″ N.
10. 41° 0′ 20″ N. 42° 36′ 50″ W.

CHAPTER VI.

Parallel Sailing.

1. 174′·8.
2. 116′·7.
3. 373′·1.
4. 70° 31′ 44″.
5. 12′·01.
6. 57° 52′ N. 41° 52′ S. Diff. lat. 5984 miles.
7. 34° 44′ W.
8. 45° 20′ N. Dep. 210·9.
9. 13·09.
10. 38° 8′ S.
11. 1266.
12. 60°.
13. 136° 49′ 24″ E.
14. 44° 21′ 18″ E.
15. 96·24.
16. 50° 44′.
17. 92·99.
18. 41° 24½′.

Days' Works.

1. N. 31° W., 49′; N. 34° W., 61′; N. 17° E., 63′; N. 60° W., 55′; N. 15° W., 57′; S. 27° W., 62′; S. 25° E., 57′; S. 30° E., 4′; D. lat. 125′·0, dep. 105′·3; lat. in 1° 31′ N.; long. in 2° 2′ W. Course, N. 40° W., dist. 163′.

2. N. 37° W., 7′; S. 81° W., 99′·6; S. 79° W., 49′·4; S. 80° W., 47′·2; S. 78° W., 96′·8; S. 56° E., 18′; S. 58° E., 18′; S. 60° E., 18′; D. lat. 76′·4, dep. 246′·5. Course, S. 73° W.; dist. 258; lat. in 53° 57′·6 N.; long. in 15° 25′·5 W.

3. N. 41° W., 19′; N. 38° E., 37′; S. 87° E., 25′; S. 78° E., 23′; S. 88° W., 21′; S. 9° W., 34′; N. 62° E., 40′; S. 22° E., 15′; D. lat. 8′·0, dep. 72′·4. Course, N. 83½° E., 73′; lat. in 18° 51′ S.; long. in 178° 36′ W.

4. S. 14° E., 60′; S. 31° W., 53′; N. 82° W., 51′; S. 64° W., 58′; N. 44° E., 49′; N. 26° E., 65′; S. 84° W., 21′; N. 75° W., 39′; D. lat. 20′·4, dep. 111′·5. Course, S. 80° W., 113′; lat. in 58° 50′ S.; long. in 68° 28′ W.

5. S. 23° W., 15′; N. 49° W., 12′; S. 37° E., 13′; S. 16° W., 25′; N. 34° E., 20′; N. 78° W., 15′; N. 15° E., 27′; D.R. lat. 30° 4′·5 N.; long. 132° 41¼ E. Set N. 46° E.; drift 30′.

6. S. 33° E., 8′; S. 6° W., 49′; N. 24° E., 53′; S. 65° E., 49′; S. 30° E., 44′; S. 25° W., 54′; S. 66° E., 52′; D.R. lat. 52° 6′·1 N.; long. 178° 26′·9 W. Set S. 62½° E.; drift 15′.

Mercator Sailing.

No.	True course.	Compass course.	Distance.
	° ′ ″	° ′ ″	
1	S. 85 55 41 W	N. 54 19 19 W.	2141 miles.
2	N. 3 8 15 E.	N. 16 21 15 E.	672 ,,
3	S. 12 5 34 W.	S. 2 5 34 W.	5673 ,,
4	N. 19 27 0 W.	N. 33 57 0 W.	900·4 ,,
5	S. 71 5 33 E.	S. 48 5 33 E.	3009 ,,
6	S. 60 27 26 E. or W.	S. 49 27 36 E. or S. 71 27 26 W.	10951·7 ,,
7	S. 54 6 30 W.	S. 31 6 30 W.	6261 ,,
8	N. 53 43 0 W.	N. 50 23 0 W.	5428 ,,
9	N. 89 41 19 W.	N. 44 0 19 W.	1292 ,,
10	N. 2 5 0 E.	N. 1 5 0 W.	3810 ,,
11	N. 71 56 0 W.	N. 53 52 0 W.	909·3 ,,
12	N. 89 16 43 E. or W.	S. 83 33 17 E. or N. 82 6 43 W.	10801 ,,

Miscellaneous Exercises.

1. 244·7 miles.
2. 39° 16″.
3. Lat. 35° 20′ N., long. 30° 17′·5 W.
4. 36·2 hours.
5. N. 7° 54′ W., 44·8 miles.
6. 6ʰ 33ᵐ.
7. Cliff, 131·6 ft.; lighthouse, 107·2 ft.
8. 315·3 ft.
9. S. 60° 9′ W., 13·36 miles.
10. 116·82 miles.
11. N. 56° 13′ E., 116·9 miles.
12. S. 7° 50′ W., 5·6 miles.
13. 6ʰ 45ᵐ.
14. 12·34 miles.
15. S. 63° 20′ E.; 7·38 miles.
16. N. 71° 15½′ E.; 7·27 miles.

CHAPTER VII.

TIDES.

No.	A.M.	P.M.
	h. m.	h. m.
1	6 48	7 15 M.T.G.
	10 31	10 58 ,,
	0 13	0 49 ,,
	11 53	— ,,
	11 35	11 57 ,,
2	4 42	5 0 M.T.S.
	5 18	5 36 ,,
	11 8	11 28 ,,
	11 47	— ,,
	1 8	1 46 ,,
	2 20	2 50 ,,
	Midnight	0 23 ,,
	0 32	0 54 ,,
	11 45	— ,,
	0 4	0 26 ,,
3	9 1	9 32 M.T.G.
	4 39	5 10 M.T.S.
	1 28	1 49 M.T.G.
	1 6	1 27 M.T.S.
4	11 34	11 54 M.T.G.
	{ 0 1	{ 0 18 ,,
	{ 1 38	{ 1 55 ,,

CHAPTER VIII.

	Interval.	Angle.
	h. m.	°
1	5 48 R.	174
2	1 43 F.	48
3	5 24 R.	162
4	2 48 F.	86
5	4 53 F.	117
6	7 11 F.	175

	M.T.L.	H.D.R.
	ft. ins.	ft. ins.
1	10 0	2 11
2	5 11	6 6
3	8 1	6 10
4	11 2 R.	5 8
	11 0 F.	5 10
5	12 5 R.	12 6
	12 2 F.	12 9
6	10 11 R.	9 6
	11 2 F.	9 3
7	8 7 R.	3 9
	8 5 F.	3 11
8	11 0 R.	11 0

No.	Angle.	H.D.R.	Red. to M.L.W.S.
	°	ft. ins.	ft. ins.
1	61	10 4	− 15 6
2	154	3 3	− 5 3
3	144	9 10	− 1 6
4	168	10 4	+ 1 2
5	166	3 11	− 8 2
6	29	5 7	− 14 3
7	47	9 3	− 17 4
8	105	0 11	− 1 10
9	31	7 6	− 13 10
10	176	1 5	− 4 1
11	158	5 8	− 2 8
12	145	5 9	− 7 4
13	180	14 3	+ 1 11
14	170	7 7	+ 0 5
15	151	13 8	− 2 11
16	11	2 9	− 7 10
17	176	12 5	− 2 3
18	45	4 6	− 7 8
19	120	2 3	− 4 7

20. 26 ft. 10 ins.
21. $5^h\ 8^m$ p.m.
22. 22 ft. 8 ins.
23. depth 34 ft. 9 ins.; under 9 ft. 9 ins.
24. $7^h\ 33^m$ a.m.

CHAPTER IX.

Time Equivalents.

$11^m\ 58^s$; $11^h\ 56^m\ 3^s$; $1^h\ 0^m\ 1^s$; $4^h\ 31^m\ 31^s{\cdot}6$; $1^m\ 9^s{\cdot}3$; $41^m\ 18^s{\cdot}6$; $10^h\ 20^m\ 46^s{\cdot}4$; $6^h\ 12^m\ 3^s$; $7^h\ 50^m\ 34^s$; $9^h\ 39^m\ 36^s$; $11^h\ 46^m\ 38^s{\cdot}3$; $13^s{\cdot}53$; $3^m\ 59^s$; 1^s; 1^m; $3^m\ 12^s$; $11^h\ 12^m\ 36^s$; $4^h\ 17^m\ 15^s$.

Arc Equivalents.

18° 7′ 30″; 154° 58′ 45″; 120° 2′ 30″; 104° 49′ 30″; 179° 59′ 30″; 45° 4′ 45″; 89° 50′ 30″; 135° 24′ 15″; 3° 48′ 45″; 36° 15′; 2° 38′ 45″; 11′ 15″; 36° 11′; 165° 4′ 45″; 168° 22′ 30″; 75° 1′ 15″; 127° 13′; 150° 3′; 3′; 25′.

Difference of Times.

$41^m\ 20^s$ less; $1^h\ 11^m$ more; $7^h\ 21^m\ 8^s$ more; $11^h\ 30^m\ 24^s$ less; $6^h\ 41^m\ 6^s$ more; $6^h\ 0^m\ 19^s$ less; 29^s more; $11^h\ 59^m\ 47^s$ less; $4^m\ 9^s$ less; $7^h\ 40^m\ 2^s$ more; 11^s more; $11^h\ 0^m\ 1^s$ less.

Longitudes.

1. 96° 25′ 45″ E.
2. 106 50 15 W.
3. 110 5 0 W.
4. 122 27 30 E.
5. 78 48 45 W.
6. 117 58 30 E.
7. 0 34 45 E.
8. 9° 7′ 30″ E.
9. 0 35 0 W.
10. 174 1 45 W.
11. 22 9 45 W.
12. 167 0 0 E.
13. 80 17 30 W.
14. 179 2 0 W.

Greenwich Times.

1. $16^d\ 7^h\ 9^m\ 36^s$.
2. 1 2 13 41·3.
3. 12 20 50 36.
4. 30 6 4 56.
5. 31 21 36 41.
6. 17 12 42 26.
7. $28^d\ 5^h\ 48^m\ 17^s$.
8. 20 21 9 34.
9. 31 15 50 34.
10. 10 0 16 34.
11. 22 7 58 15.
12. 29 16 25 20.

Ship Times.

1. October 15th, $5^h\ 29^m\ 14^s$ A.M.
2. May 6th, 11 43 46 A.M.
3. August 31st, 7 3 33 P.M.
4. June 3rd, $5^h\ 42^m\ 18^s$ A.M.
5. October 19th, 11 37 47 P.M.
6. May 1st, 4 49 6 P.M.

Chronometer Rates and G.M.T.

1. Rate 2ˢ·7 losing.
2. ,, 5·3 gaining.
3. ,, 10·05 gaining.
4. ,, ·5 gaining.
5. ,, ·5 losing.
6. ,, 5·4 gaining.
7. ,, ·9 gaining.
8. ,, 10·05 losing.
9. G.M.T. Nov. $26^d\ 13^h\ 45^m\ 45^s$.
10. ,, Oct. 31 10 17 47.
11. ,, Mar. 20 8 18 49.
12. ,, Dec. 10 12 57 36.
13. ,, July 3 23 58 22.
14. ,, Nov. 26 19 23 52.
15. ,, Dec. 14 5 23 24.
16. ,, May 1 0 24 10.
17. ,, April 15 19 17 16.

CHAPTER X.

Sun's Elements.

1. Dec. 17° 37′ 36″·2 N. Eq. of Time $6^m\ 1^s·73$ R.A. $8^h\ 51^m\ 37^s·01$.
2. ,, 16 30 16·8 N. ,, 5 37·39 ,, 9 7 39·28.
3. ,, 14 23 25·7 N. ,, 4 32·33 ,, 9 34 56·38.
4. ,, 3 30 9·9 N. ,, 4 22 ,, 11 27 33·02.
5. ,, 0 4 55·7 N. ,, 7 28·01 ,, 11 59 14·6.
6. ,, 0 0 14·1 S. ,, 7 32·62 ,, 12 0 2·2.
7. ,, 0 3 47·9 S. ,, 7 35·80 ,, 12 0 35·1.
8. ,, 0 11 12·3 S. ,, 7 42·39 ,, 12 1 43·4.
9. ,, 2 27 10 S. ,, 9 40·69 ,, 12 22 40·04.
10. A.T.S. Aug. $20^d\ 20^h\ 18^m\ 13^s·6$.
11. M.T.S. Sept. 29 21 34 7·4.
12. M.T.G. Sept. 24 9 34 13.

Sidereal Time or R.A.M.S.

1. $9^h\ 50^m\ 41^s{\cdot}32$.
2. $10\ 23\ 41{\cdot}51$.
3. $10^h\ 57^m\ 10^s{\cdot}48$.
4. $12\ 29\ 48{\cdot}66$.

Moon's S.D. and H.P.

1. S.D. $15'\ 33''$ H.P. $56'\ 28''{\cdot}6$.
2. S.D. $15\ 24{\cdot}9$ H.P. $55\ 49{\cdot}4$.
3. S.D. $16\ 20{\cdot}5$ H.P. $59\ 4{\cdot}7$.
4. S.D. $16\ 10{\cdot}5$ H.P. $58\ 24{\cdot}4$.

Moon's Meridian Passage.

1. M.T.G. Aug. $1^d\ 21^h\ 34^m{\cdot}3$.
2. M.T.G. Aug. $28\ 14\ 8{\cdot}92$.
3. M.T.G. Aug. $2\ 1\ 53{\cdot}7$.
4. M.T.G. Aug. $27\ 0\ 16{\cdot}9$.
5. M.T.G. Aug. $3^d\ 23^h\ 44^m{\cdot}8$.
6. M.T.G. Aug. $14\ 10\ 25{\cdot}5$.
7. M.T.G. Aug. $8\ 14\ 5{\cdot}0$.
8. M.T.G. Aug. $6\ 10\ 24{\cdot}3$.

Moon's R.A. and Dec.

1. R.A. $22^h\ 30^m\ 9^s{\cdot}7$ Dec. $11°\ 39'\ 9''{\cdot}8$ S.
2. R.A. $0\ 45\ 20$ Dec. $0\ 2\ 2{\cdot}1$ S.
3. R.A. $6\ 9\ 4{\cdot}27$ Dec. $20\ 3\ 45{\cdot}9$ N.
4. R.A. $12\ 45\ 42{\cdot}13$ Dec. $0\ 1\ 34{\cdot}8$ S.

Planet's Elements.

1. R.A. Jupiter $6^h\ 9^m\ 38^s{\cdot}94$ Dec. $23°\ 7'\ 26''{\cdot}6$ N. Mer. Pass. $10^d\ 4^h\ 56^m{\cdot}27$.
2. „ Saturn $23\ 1\ 59{\cdot}32$ „ $8\ 24\ 59{\cdot}6$ S. „ $11\ 3\ 45{\cdot}3$.

CHAPTER XI.

Sidereal Time or R.A.M.

1. $0^h\ 2^m\ 8^s{\cdot}66$.
2. $15\ 58\ 34{\cdot}45$.
3. $10\ 48\ 51{\cdot}2$.
4. $7^h\ 36^m\ 46^s{\cdot}18$.
5. $8\ 0\ 5{\cdot}93$.
6. $17\ 41\ 11{\cdot}29$.

Hour Angles.

1. E.H.A. $2^h\ 6^m\ 6^s{\cdot}41$.
2. W.H.A. $1\ 59\ 16{\cdot}02$.
3. W.H.A. $3\ 7\ 29{\cdot}19$.
4. E.H.A. $0^h\ 0^m\ 12^s{\cdot}84$.
5. M.T.S. Aug. $17^d\ 20^h\ 32^m\ 1^s{\cdot}49$.
6. M.T.S. Sept. $14\ 6\ 35\ 15{\cdot}75$.

Star's Meridian Passage.

1. A.T.S. $3^h\ 3^m\ 25^s$ A.M.
2. M.T.S. $3\ 35\ 23$ A.M.
3. M.T.S. $4\ 52\ 20$ A.M.
4. A.T.S. $4\ 8\ 33$ A.M.
5. M.T.S. $7\ 3\ 46$ A.M.
6. A.T.S. $7\ 30\ 9$ A.M.
7. M.T.S. $7\ 27\ 17$ A.M.
8. A.T.S. $8\ 28\ 22$ A.M.
9. M.T.S. $7^h\ 49^m\ 35^s$ A.M.
10. A.T.S. $11\ 53\ 49$ A.M.
11. M.T.S. $8\ 58\ 56$ A.M.
12. A.T.S. $2\ 33\ 53$ P.M.
13. M.T.S. $5\ 8\ 5$ P.M.
14. A.T.S. $6\ 56\ 6$ P.M.
15. M.T.S. $7\ 33\ 23$ P.M.

BRIGHT STARS NEAR MERIDIAN.

1. Altair 43m 1° W. and S.
   γ Cygni 10 11 W. ,, N.
   Deneb 9 1 E. ,, N.
   α Cephei 47 7 E. ,, N.
2. Fomalhaut 56m 8° W. and S.
3. Vega 20m 2° W. and N.
   Altair 52 25 E. ,, N.
4. α Cephei 1h 24m 17° W. and N.
   ε Pegasi 1 1 4 W. ,, N.
   α Gruis 0 38 18 W. ,, S.
   β Gruis 0 3 34 W. ,, S.
   Fomalhaut 0 11 50 E. ,, S.
   Markab 0 19 28 E. ,, N.
   Alpheratz 1 22 55 E. ,, N.
   β Cassiopeiæ 1 23 32 E. ,, N.
   α Phœnicis 1 41 1 E. ,, S.
   β Ceti 1 58 15 E. ,, S.
5. Capella 0h 38m 7° W. and N.
   Rigel 0 37 51 W. ,, S.
   Bellatrix 0 27 48 W. ,, S.
   β Tauri 0 27 31 W. ,, S.
   ε Orionis 0 16 26 W. ,, S.
6. Sirius 0h 11m 21° E. and N.
7. 11h 22m 11°.
8. + 13m 14·6°.
9. + 16m 25·9°.

CHAPTER XII.

ALTITUDES.

1.	50° 9′ 35″				
2.	20 57 44				
3.	19 56 46				
4.	32 35 36				
5.	47 2 57				
6.	12 20 15				
7.	56 20 43	S.D. 15′ 0″	H.P. 54′ 13″	Cor. 29′ 47″	
8.	15 38 28	,, 14 57	,, 54 26	,, 49 4	
9.	29 13 31				
10.	47 19 24				
11.	23 8 22				
12.	53 13 51				
13.	39 20 11				
14.	47 49 27				
15.	52 1 47	S.D. 15′ 56″	H.P. 57′ 33″	Cor. 35′ 6″	

CHAPTER XIII.

Latitude by Sun's Meridian Altitude.

No.	Declination.	True altitude.	Latitude.
	° ′ ″	° ′ ″	° ′ ″
1	17 39 10 N.	34 22 44	37 58 6 S.
2	17 8 20 N.	50 16 30	56 51 50 N.
3	16 39 55 N.	19 43 40	53 36 25 S.
4	16 8 3 N.	69 28 41	4 23 16 S.
5	15 27 39 N.	63 42 38	10 49 43 S.
6	14 53 16 N.	49 43 13	25 23 31 S.
7	14 6 41 N.	78 50 4	8 6 45 N.
8	13 37 54 N.	79 47 14	3 25 8 N.
9	12 50 19 N.	52 38 40	50 11 39 N.
10	2 28 54 S.	88 48 35	1 17 29 S.
11	0 7 55 N.	49 10 53	40 57 2 N.
12	0 4 45 N.	43 27 24	46 27 51 S.
13	1 21 36 S.	58 21 2	30 17 22 N.
14	2 9 50 S.	36 10 5	55 59 45 S.
15	0 1 35 S.	67 19 57	22 38 28 N.
16	0 17 37 N.	51 59 25	38 18 12 N.
17	0 13 55 S.	22 34 39	67 11 26 N.
18	0 9 48 S.	30 10 10	59 59 38 S.

Latitude by Star's Meridian Altitude.

1. 10° 15′ 12″ N.
2. 13 27 39 S.
3. 27 58 26 N.
4. 43 3 40 N.
5. 49 43 18 N.
6. 50 0 45 N.
7. 35° 58′ 14″ N.
8. 44 22 3 S.
9. 37 51 6 N.
10. 19 18 20 S.
11. 45 18 37 N.

Latitude by Moon's Meridian Altitude.

No.	M.T.G.	S.D.	H.P.	Declination.	True alt.	Latitude.
	d. h. m.	′ ″	′ ″	° ′ ″	° ′ ″	° ′ ″
1	21 4 38·4	15 58	57 49	7 37 1 N.	34 46 16	47 36 43 S.
2	22 8 50·6	16 10	58 22	2 10 58 N.	54 8 55	38 2 3 N.
3	30 5 5·0	16 17	58 42	19 40 28 S.	61 47 40	47 52 48 S.
4	31 6 20·2	16 4	58 19	17 42 1 S.	25 44 55	46 33 4 N.
5	31 18 38·0	16 8	58 11	16 19 18 S.	69 32 47	4 7 55 N.
6	3 23 57·2	16 16	58 46	17 29 13 S.	39 16 31	68 12 42 S.
7	9 15 1·3	15 11	54 56	4 47 13 N.	43 25 36	51 21 37 N.
8	10 10 44·9	15 3	54 34	8 4 24 N.	47 5 35	50 58 50 N.
9	11 9 11·5	15 4	54 21	11 29 47 N.	54 37 2	23 53 11 S.
10	14 5 5·5	15 5	54 24	18 50 58 N.	75 33 28	4 24 26 N.
11	16 10 59·1	15 19	55 16	20 11 19 N.	74 21 44	4 33 3 N.
12	24 22 32·7	16 25	59 8	9 54 37 S.	84 40 30	4 35 7 S.

Latitude by Meridian Altitude below Pole.

1. M.T.G. $10^d\ 17^h\ 1^m{\cdot}1$. Dec. 8° 23′ 56″ S. Lat. 43° 16′ 44″ S.
2. 48° 23′ 59″ S.
3. 54° 6′ 9″ N.
4. M.T.G. $28^d\ 10^h\ 31^m{\cdot}9$. S.D. 16′ 12″. H.P. 59′ 1″. Dec. 19° 57′ 26″ S. True Alt. 7° 53′ 4″. Lat. 77° 55′ 38″ S.
5. Dec. 18° 0′ 19″ N. True Alt. 5° 53′ 11″. Lat. 77° 52′ 52″ N.
6. 50° 44′ 17″ S.

Star's calculated Altitudes.

1. 39° 48′	6. 32° 15′
2. 42 6	7. 61 44
3. 19 38	8. 37 57
4. 57 0	9. 51 4
5. 67 50	10. 25 1

CHAPTER XIV.

Latitude by Ex-Meridian Altitude of the Sun.

No.	Hour angle.	Nat. No.	Arc I.	Arc II.	Latitude sights.	Latitude at noon.
	m. s.		° ′ ″	° ′ ″	° ′	° ′ ″
1	24 13 W.	4213	18 22 46	55 41 42	37 19 S.	37 16 36 S.
2	33 31 E.	9161	17 0 56	9 16 21	26 17 N.	—
3	27 36 W.	6927	16 9 4	21 46 7	5 37 S.	—
4	17 42 E.	1725	15 4 6	38 7 50	53 12 N.	—
5	7 13 W.	342	—	—	44 53 S.	—
6	41 4 E.	15638	12 17 42	14 51 4	2 33¼ S.	2 39 4 S.
7	18 36 W.	2503	10 40 49	28 39 8	39 20 N.	39 22 30 N.
8	20 43 E.	2932	8 28 25	51 53 20	43 25 S.	43 27 6 S.
9	30 42 W.	6173	2 13 34	48 37 35	46 24 N.	46 29 12 N.
10	23 42 E.	5134	0 5 53	15 41 42	15 36 N.	—
11	10 8 W.	611	0 2 41	51 11 6	51 14 N.	51 17 54 N.
12	19 12 W.	2675	—	—	40 20 S.	40 25 0 S.

Latitude by Ex-Meridian Altitude of a Star.

No.	Hour angle.	Nat. No.	Arc I.	Arc II.	Latitude.
	m. s.		° ′ ″	° ′ ″	° ′ ″
1	7 56·5	365	38 42 45	77 34 45	38 52 0 S.
2	29 35·0	5126	8 41 27	42 45 50	51 27 17 N.
3	21 9·7	2318	57 8 54	57 8 55	0 0 1 N.
4	21 42·6	2861	30 13 56	72 40 57	42 27 1 N.
5	19 13·7	2950	28 39 23	11 25 19	17 14 4 N.
6	47 43·0	11403	58 16 35	49 34 31	8 42 4 S.
7	23 18·4	4037	23 7 31	54 59 32	31 52 1 S.
8	14 32·4	1152	16 21 7	37 3 44	53 24 51 N.
9	28 43·0	5000	46 7 42	69 43 43	23 36 1 S.
10	25 8·7	3365	8 21 34	47 12 24	55 33 58 S.
11	6 15·0	264	44 57 15	41 53 0	3 4 16 N.
12	32 53·7	6532	47 42 46	67 51 30	20 8 44 N.

Latitude by Pole Star.

No.	R.A.M.	First cor.	Second cor.	Third cor.	Latitude.
	h m. s.	° ′ ″	′ ″	′ ″	° ′ ″
1	4 10 55·9	−0 53 56	0 15	0 53	35 44 59
2	22 36 2	−0 53 17	0 22	1 16	45 17 10
3	12 55 19	+1 11 23	0 0	0 54	30 4 31
4	16 59 12	+0 42 49	0 40	1 10	54 15 16
5	7 25 1	0 0 0	0 19	0 31	23 4 59
6	19 28 29	−0 1 6	0 28	1 27	32 22 41
7	0 1 21	−1 7 15	0 7	1 20	49 49 19
8	7 44 9	+0 5 59	0 51	0 28	48 22 55
9	18 9 40	+0 23 13	0 24	1 15	31 1 12

CHAPTER XV.

Compass Error by Amplitude.

No.	Declination.	True amplitude.	Compass error.	Deviation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	17 55 0 N.	W. 24 35 30 N.	34 28 30 W.	5 58 30 W.
2	1 45 17 S.	E. 2 45 16 S.	28 15 16 E.	4 0 16 E.
3	16 2 20 N.	E. 19 25 53 N.	5 22 8 W.	16 42 8 W.
4	2 33 52 S.	W. 8 51 28 S.	1 46 2 E.	31 31 2 E.
5	17 31 14 N.	E. 22 13 15 N.	5 20 45 W.	6 9 15 E.
6	1 13 23 S.	W. 1 54 46 S.	18 35 14 E.	4 4 46 W.
7	16 34 56 N.	E. 17 43 39 N.	17 43 39 W.	24 58 39 W.
8	6 2 21 N.	W. 12 14 4 N.	12 14 4 E.	0 5 56 W.
9	17 39 44 N.	E. 46 42 45 N.	46 6 0 E.	23 26 0 E.
10	0 13 43 N.	E. 0 13 43 N.	10 13 43 W.	10 13 43 W.
11	0 35 35 S.	W. 0 36 33 S.	14 6 33 W.	12 6 33 W.
12	0 0 0	West	2 0 0 E.	4 30 0 W.
13	9 9 23 N.	E. 9 9 23 N.	18 58 7 E.	9 28 7 E.
14	0 0 0	East	0 0 0	1 15 0 W.

15. $4^h\ 41^m\ 30^s$ A.M.
16. $1^h\ 11^m\ 17^s$.
17. 2° E.
18. 45°.
19. 41° 49′.
20. 24° 42′. $2^h\ 40^m\ 50^s$.

CHAPTER XVI.

Compass Error by Alt. Azimuth.

No.	Polar distance.	True altitude.	True azimuth.	Compass error.	Deviation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	107 31 58	16 51 58	N. 49 39 6 E.	0 9 6 E.	18 9 6 E.
2	77 7 20	26 7 13	S. 84 10 50 W.	5 49 10 W.	3 40 50 E.
3	81 4 35	36 2 26	S. 50 53 8 E.	22 23 8 W.	6 43 8 W.
4	81 26 0	34 3 54	S. 65 53 0 E.	15 15 30 W.	9 44 30 E.
5	90 16 20	21 56 18	N. 61 9 54 W.	0 40 6 E.	0 40 6 E
6	87 35 42	25 14 42	N. 61 8 10 W.	6 21 50 E.	10 8 10 W.

Compass Error by Time Azimuth of Sun.

No.	A.T.S.	Corrections.	True azimuth.	Deviation.
	h. m. s.	′	° ′	° ′
1	2 48 0 P.M.	+16	N. 132 45 W.	
2	3 19 0 P.M.	+14−25	N. 112 35 W.	
3	9 14 0 A.M.	+25+7+2	S. 141 25 E.	
4	3 32 0 P.M.	+21+5	N. 54 43 W.	3 9 W.
5	7 20 0 A.M.	−14	N. 73 51 E.	12 47 E.
6	7 57 22 A.M.	+4+5+17	N. 88 36 E.	19 27 E.
7	3 38 31 P.M.	−33+10+14	N. 61 24 W.	0 26 E.
8	8 41 26 A.M.	+19−27+11	S. 65 47 E.	9 51 E.
9	4 38 18 P.M.	−17+6	S. 89 49 W.	9 11 W.
10	3 50 0 P.M.	−18+37+5	S. 121 34 W.	10 19 E.
11	8 34 6 A.M.	+17−18+18	N. 93 24 E.	19 55 W.
12	4 19 35 P.M.	+4−1+15	N. 91 54 W.	16 18 E.
13	9 41 21 A.M.	+19+16+11	S. 143 55 E.	35 40 W.
14	10 39 9 A.M.	82° 59′+72° 45′	N. 155 44 E.	56 43 W.

Compass Error by Time Azimuth of Stars.

No.	Hour angle.	Corrections.	True azimuth.	Deviation.
	h. m. s.		° ′	° ′
1	3 1 39 E.	−17′−2′ +59′	N. 97 37 E.	22 22 E.
2	4 53 9 E.	−12 −25+20	S. 92 51 E.	14 42 W.
3	3 26 4 W.	−28 −27+24	N. 114 36 W.	35 26 W.
4	1 48 14 W.	89° 59′+68° 58′	S. 158 57 W.	15 43 W.
5	5 37 7 E.	−11−19+8	N. 88 5 E.	23 43 E.
6	3 23 41 W.	+5−27+13	N. 113 30 W.	26 0 E.
7	3 29 46 E.	−21+12+8	S. 128 24 E.	21 36 E.
8	3 23 47 W.	94° 5′+38° 37	N. 132 42 W.	0 5 W.
9	3 35 29 E.	91 57 +39 34	S. 131 31 E.	10 25 E.
10	9 16 22 W.	24 30 − 3 47	S. 20 43 W.	7 1 E.
11	4 30 20 W.	111 1 +43 22	S. 154 23 W.	23 10 W.
12	8 41 20 W.	48 49 − 1 12	S. 47 37 W.	2 26 W.

14. N. 48° 20′ E.
15. Dec. 19° 19′ N.; Lat. 51° 30′ N.

CHAPTER XVII.

Longitude by Sun Chronometer.

No.	M.T.G.	Pol. dist.	Eq. Time.	True altitude.	Hour angle.	Longitude at sights.	Longitude at noon.
	d. h. m. s.	° ′ ″	m. s.	° ′ ″	h. m. s.	° ′ ″	° ′ ″
1	3 14 37 35	107 31 58	+6 0	16 51 58	3 19 35	92 12 30 E.	92 30 40 E.
2	19 9 10 31	77 7 22	+3 34	26 7 13	4 25 34	70 20 45 W.	70 53 26 W.
3	30 21 35 58	81 4 35	+0 29	36 2 26	2 37 42	3 17 30 W.	3 54 28 W.
4	31 21 22 37	81 26 0	+0 11	34 3 54	3 19 30	10 29 0 W.	10 56 37 W.
5	22 18 29 38	90 16 20	−7 18	21 56 18	3 37 24	135 7 0 E.	—
6	29 15 20 26	87 35 42	−9 38	25 14 42	3 29 48	179 56 0 E.	179 28 30 W.

LONGITUDE BY STAR CHRONOMETER.

No.	M.T.G.	R.A.M.S.	True altitude.	Hour angle.	Longitude.
	d. h. m. s.	h. m. s.	° ′ ″	h. m. s.	° ′ ″
1	9 9 8 13	11 12 0	59 24 30	2 9 2 W.	110 45 30 E.
2	23 21 37 52	10 7 2	12 1 18	3 36 40 E.	74 55 15 W.
3	1 7 47 31	8 38 1	42 2 0	2 22 59 E.	14 25 15 E.
4	11 10 56 5	11 20 11	26 8 42	3 42 30 W.	0 0 0
5	20 15 27 42	11 56 24	87 12 48	3 48 43 E.	30 46 15 W.
6	23 9 11 21	10 4 59	27 14 27	3 50 37 E.	90 46 0 E.
7	7 9 42 18	11 4 12	22 42 6	3 25 33 E.	110 35 0 E.
8	30 21 6 21	10 34 32	15 44 30	2 29 43 E.	169 32 15 W.
9	2 6 54 54	10 44 2	25 13 55	4 36 0 W.	50 10 45 E.
10	14 2 2 6	9 28 19	21 59 16	4 59 0 E.	179 43 30 W.
11	28 6 58 18	10 24 20	34 50 51	2 51 28 W.	4 56 45 W.
12	27 19 24 11	12 24 39	68 25 1	2 5 9 E.	161 3 45 E.

Chronometer Azimuth.

No.	M.T.G.	Pol. dist.	Eq. time.	True altitude.	Hour angle.	Obs. longitude.	Noon longitude.	True azimuth.	Comp. error.	Deviation.
	d. h. m. s.	° ′ ″	m. s.	° ′ ″	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	31 14 7 20	81 19 24	+ 0 17	13 29 3	5 5 39	135 21 0 W.	136 2 0 W.	S. 98 50 38 W.	6 1 38 E.	0 21 38 E.
2	30 20 18 49	87 7 30	−10 2	16 55 19	4 41 24	17 33 45 W.	—	N. 79 27 50 E.	41 28 25 W.	29 13 25 W.
3	31 15 54 42	98 38 58	+ 0 15	14 53 46	3 30 20	173 58 15 E.	175 12 47 E.	N. 54 20 30 W.	3 50 30 W.	20 0 30 W.
4	22 15 4 0	90 19 38	− 7 15	10 40 40	5 8 23	55 5 30 E.	54 6 42 E.	N. 82 42 12 E.	7 17 48 W.	8 22 12 E.
5	31 15 25 20	98 39 25	+ 0 16	65 5 7	0 58 30	143 21 30 E.	—	N. 36 21 0 W.	3 39 0 E.	2 6 0 W.
6	22 1 28 26	89 27 5	− 7 3	30 38 48	3 8 35	71 1 0 W.	—	S. 58 25 56 E.	10 37 11 W.	0 52 49 E.
7	22 15 45 53	90 19 0	− 7 15	43 8 6	1 43 41	147 38 15 E.	148 3 40 E.	N. 36 48 0 W.	8 12 0 E.	2 33 0 E.
8	4 20 14 54	107 12 25	+ 5 53	31 5 47	2 23 51	93 42 30 E.	93 20 0 E.	N. 40 55 36 W.	1 33 6 W.	12 46 54 E.
9	1 0 20 40	81 28 39	+ 0 9	28 48 47	4 4 44	56 3 15 E.	—	S. 98 32 16 W.	7 2 16 E.	11 17 16 E.
10	1 11 8 49	81 38 24	0 0	18 7 8	4 46 43	95 31 30 W.	95 6 18 W.	S. 98 48 0 W.	11 33 0 E.	9 3 0 E.
11	20 7 36 32	102 34 18	+ 3 22	29 51 34	3 40 53	168 30 45 W.	169 1 30 W.	N. 67 34 30 E.	0 55 30 W.	10 10 30 W.
12	23 13 50 5	90 2 31	− 7 35	53 3 9	2 23 48	173 28 0 W.	173 40 18 W.	S. 77 36 30 W.	7 17 45 E.	1 22 15 W.

CHAPTER XVIII.

SUMNER'S METHOD.

1. *First observation:* T.A. 36° 26′ 20″; P.D. 72° 29′ 5″; eq. time, + 6ᵐ 0ˢ; H.A.'s, 3ʰ 37ᵐ 17ˢ and 3ʰ 36ᵐ 25ˢ; longs., (*a*), 64° 14′ 30″ E.; (*b*), 64° 27′ 30″ E.
Second observation: T.A. 43° 47′ 38″; P.D. 72° 33′ 18″; eq. time, + 5ᵐ 58ˢ; H.A.'s, 2ʰ 49ᵐ 12ˢ and 2ʰ 47ᵐ 37ˢ; longs., (*c*), 64° 36′ 15″ E.; (*d*), 64° 12′ 30″ E.
First line: {N. 15½° E. / S. 15½° W.}; second azimuth, S. 63° W.; lat. 49° 27′ N.; long. 63° 52′ E.

2. *First observation:* T.A. 38° 26′ 23″; P.D. 73° 54′ 28″; eq. time, + 5ᵐ 26ˢ; H.A.'s, 3ʰ 15ᵐ 40ˢ and 3ʰ 13ᵐ 56ˢ; longs., (*a*), 4° 43′ 15″ W.; (*b*) 4° 17′ 15″ W.
Second observation: T.A. 40° 19′ 48″; P.D. 73° 58′ 52″; eq. time, + 5ᵐ 24ˢ; H.A.'s, 3ʰ 2ᵐ 35ˢ and 3ʰ 0ᵐ 34ˢ; longs., (*c*), 2° 0′ 15″ W.; (*d*), 2° 30′ 30″ W.
First line: {N. 23° E. / S. 23° W.}; azimuth, S. 67° E.; second line: {N. 25° W. / S. 25° E.}; lat. 50° 0′ N.; long. 2° 30′ 30″ W.

3. *First observation:* T.A. 24° 9′ 42″; P.D. 78° 35′ 56″; eq. time, + 2ᵐ 30ˢ; H.A.'s, 4ʰ 23ᵐ 33ˢ and 4ʰ 22ᵐ 57ˢ; longs., (*a*), 0° 18′ 15″ E.; (*b*), 0° 27′ 15″ E.
Second observation: T.A. 48° 46′ 26″; P.D. 78° 38′ 51″; eq. time, + 2ᵐ 28ˢ; H.A.'s, 1ʰ 11ᵐ 42ˢ and 1ʰ 5ᵐ 19ˢ; longs., (*c*), 2° 43′ 45″ W.; (*d*), 1° 8′ 0″ W.
First line: {N. 11° E. / S. 11° W.}; azimuth, S. 79° E.; lat. 50° 28′ N.; long. 0° 41′ 30″ W.

4. *First observation:* T.A. 48° 31′ 31″; P.D. 81° 18′ 3″; eq. time, + 0ᵐ 18ˢ; H.A.'s, 0ʰ 52ᵐ 40ˢ and 0ʰ 39ᵐ 41ˢ; longs., (*a*), 175° 15′ 45″ W.; (*b*), 178° 30′ 30″ W.
Second observation: T.A. 19° 9′ 3″; P.D. 81° 21′ 45″; eq. time, + 0ᵐ 15ˢ; H.A.'s, 4ʰ 43ᵐ 0ˢ and 4ʰ 42ᵐ 21ˢ; longs., (*c*), 179° 32′ 15″ W.; (*d*), 179° 42′ 0″ W.
First line: {N. 72° W. / S. 72° E.}; second azimuth, S. 81° W.; lat. 49° 17′ N.; long. 179° 40′ W.

5. *First observation:* T.A. 27° 6′ 30″; P.D. 90° 10′ 21″; eq. time, − 7ᵐ 23ˢ; H.A.'s, 3ʰ 14ᵐ 28ˢ and 3ʰ 11ᵐ 57ˢ; longs., (*a*), 59° 56′ 30″ W.; (*b*), 59° 18′ 45″ W.
Second observation: T.A. 32° 27′ 30″; P.D. 90° 4′ 38″; eq. time, − 7ᵐ 28ˢ; H.A.'s, 2ʰ 36ᵐ 18ˢ and 2ʰ 32ᵐ 46ˢ; longs., (*c*), 60° 17′ 15″ W.; (*d*), 61° 10′ 15″ W.
First line: {N. 33° W. / S. 33° E.}; azimuth, N. 57° E.; lat. 46° 18′ S.; long. 60° 27′ W.

6. *First observation:* T.A. 18° 27′ 6″; P.D. 98° 27′ 58″; eq. time, + 0ᵐ 6ˢ; H.A.'s, 3ʰ 13ᵐ 54ˢ and 3ʰ 11ᵐ 24ˢ; longs., (*a*), 109° 34′ 15″ W.; (*b*), 108° 56′ 45″ W.
Second observation: T.A. 26° 4′ 10″; P.D. 98° 23′ 10″; eq. time, + 0ᵐ 2ˢ; H.A.'s, 2ʰ 6ᵐ 27ˢ and 2ʰ 2ᵐ 0ˢ; longs., (*a*), 109° 33′ 30″ W.; (*d*), 110° 40′ 15″ W.
First line: {N. 41° W. / S. 41° E.}; azimuth, N. 49° E.; lat. 49° 21′ S.; long. 109° 45′ W.

7. *First observation:* T.A. 21° 4′ 7″; P.D. 71° 38′ 7″; eq. time, + 6ᵐ 12ˢ; H.A.'s, 5ʰ 15ᵐ 16ˢ and 5ʰ 15ᵐ 29ˢ; longs., (*a*), 167° 54′ 45″ E.; (*b*), 167° 51′ 30″ E.
Second observation: T.A. 57° 12′ 25″; P.D. 71° 40′ 48″; eq. time, + 6ᵐ 12ˢ; H.A.'s, 0ʰ 45ᵐ 14ˢ and 0ʰ 35ᵐ 36ˢ; longs., (*c*), 168° 15′ 15″ E.; (*d*), 170° 39′ 45″ E.
First line: {N. 6° W. / S. 6° E.}; second azimuth, S. 73° E.; lat. 49° 44′ N. long. 166° 13′ E.

8. *First observation:* T.A. 11° 1′ 58″; P.D. 91° 52′ 56″; eq. time, − 9ᵐ 12ˢ; H.A.'s, 4ʰ 46ᵐ 19ˢ and 4ʰ 45ᵐ 0ˢ; longs., (*a*), 179° 27′ 30″ E.; (*b*), 179° 47′ 15″ E.
Second observation: T.A. 26° 59′ 56″; P.D. 92° 0′ 41″; eq. time, − 9ᵐ 18ˢ; H.A.'s, 3ʰ 1ᵐ 6ˢ and 2ʰ 57ᵐ 16ˢ; longs., (*a*), 177° 7′ 30″ E.; (*d*), 176° 10′ 0″ E.
First line: {N. 15° E. / S. 15° W.}; azimuth, S. 75° W.; lat. 46° 40′ N.; long. 177 31′ E.

CHAPTER XIX.

Chart.

1. (*a*) S. 17° E., 30° E., 18° W.; 35½ miles.
   (*b*) Lat. 52° 35½′ N.; long. 4° 17′ W.; set N. 70° E.; dist. 5½ miles.
   (*c*) Lat. 52° 20½′ N.; long. 4° 17½′ W.; dist. 7¾ miles.
   (*d*) S. 5° E.; dist. 20 miles; she steams 40 miles.
   (*e*) Lat. 52° 24½′ N.. long. 4° 31′ W.
2. (*a*) Comp. Co. S. 49° E.; dev. 15° E.; var. 18° W.; dist. 15½ miles.
   „ N. 85° E.; dev. 7° W.; var. 18° W.; dist. 23 miles.
   „ N. 58° E.; dev. 21° W.; var. 18° W.; dist. 31½ miles.
   (*b*) Lat. 52° 6½′ N.; long. 6° 24½′ W.; set S. 80° W.; dist. 3½ miles.
   (*c*) Lat. 52° 23½ N.; long. 5° 51¼′ W.; dist. 10½ miles.
   (*d*) Comp. Co. S. 42° E.; N. 87° E.; N. 49° E.; dist. 52 miles.
   (*e*) Lat. 51° 56¼′ N.; long. 6° 44½′ W.
3. (*a*) Comp. Co. N. 23° E.; dev. 30° W.; var. 18° W.; dist. 17¾ miles.
   „ N. 63° E.; dev. 18° W.; dist. 37½ miles.
   (*b*) Lat. 51° 45¾′ N.; long. 5° 26½ W.; set N. 14° W.; dist. 3 miles.
   (*c*) Lat. 52° 17′ N.; long. 5° 6′ W.; dist 8½ miles.
   (*d*) Comp. Co. N. 71° E.; dist. made good 22 miles.
   Total dist. steamed 32½ miles.

CHAPTER XX.

Napier's Curve.

1. Correct magnetic bearing: N. 66° E.
   Deviations: 17° W. 28° W. 24° W. 9° W. 16° E. 32° E. 26° E. 5° E.
   Compass courses: N. 76° E. S. 80° E. S. 28° E. N. 16° E.
   Magnetic courses: N. 44° W. S. 16° W. N. 79½° E. N. 9° W.
   Deviation: 31° E.
   Bearings: S. 53° W. N. 76° E.
2. Correct magnetic bearing: N. 82½° W.
   Deviations: 12½° E. 18½° E. 9½° E. 6½° W. 9½° W. 14½° W. 8½° W. 3½° W.
   Compass courses: N. 71° E. S. 69° E. S. 20° W. N. 42° W.
   Correct magnetic bearings: N. 12½° E. S. 44° E. S. 37° W. N. 35° W.
   Deviation: 0.
   Correct bearings: S. 80° W. N. 40° E.

3. Correct magnetic bearing: S. 14° W.
   Deviations: 10° E. 15° E. 14° E. 8° W. 20° W. 18° W. 3° E. 5° E.
   Compass courses: N. 16° W. S. 75° W. S. 75½° E. N. 51° E.
   Correct magnetic courses: N. 56° W. N. 56° E. S. 27° E. N. 33° W.
   Deviation: 4° E.
   Correct bearings: S. 57° W. N. 58° W.
4. Correct magnetic bearing: N. 8½° W.
   Deviations: 23½° E. 18½° E. 14½° W. 10½° W. 12½° W. 12½° W. 1½° E. 6½° E.
   Compass courses: S. 57° W. N. 18½° W. S. 73° E. S, 88° E.
   Correct magnetic courses: S. 80° W. N. 40° E. S. 45° E. S. 1° E.
   Deviation: 16° E.
   Bearings: N. 52° W. S. 39° W.
5. Correct magnetic bearing: N. 83¾° E.
   Deviations: 21° 45′ E. 17° 45′ E. 0° 45′ E. 19° 15′ W. 21° 15′ W. 15° 15′ W. 1° 15′ W. 16° 45′ E.
   Compass courses: N. 29° E. S. 86° E. S. 25° W. N. 26° W.
   Correct magnetic courses: N. 58¾° E. S. 61° E. N. 87¼° W. N. 21° W.
   Deviation: 22° E. Bearings: S. 3¼° E. S. 73½° E.
6. Correct magnetic bearing: S. 13½° E.
   Deviations: 18¾° E. 8½° W. 29¼° W. 34¾° W. 20° 5′ W. 12¾° E. 29¾° E. 31° 20′ E.
   Compass courses: N. 2½° E. S. 60½° E. S. 26° W. N. 81½° W.
   Correct magnetic courses: N. 32° E. N. 50° E. S. 46½° E. N. 11° W.
   Deviation: 35° W.
   Correct bearings: N. 15° 38′ E. N. 49° 4′ W.
7. (*a*) Magnetic bearing, N. 56° E.
   (*b*) N. ¼ E., S. 80° E., S. 66° W., N. 7° W.
   (*c*) S. 19° W., N. 59° W., N., S. 71° E.
   (*d*) E. 11° N., N. 3° E.
8. (*a*) N. 73½° W.
   (*b*) S. 37° E., S. 18° W., N. 65° W., N. 78° E.
   (*c*) N.W. by W., W. by S., S. 28° W., N. 80° E.
   (*d*) N. 74½° E., N. 10½° W.
9. (*a*) N. 6° W.
   (*b*) W.S.W., N. 65° W., N. 7° E., S. 75° E.
   (*c*) N. 26° E., N. 51° E., N. 87° E., S. 21° W.
   (*d*) S. 78° W., S. 5° W.
10. (*a*) S. 65° W.
    (*b*) S. 40° E., N., N. 63° W., S. 21° W.
    (*c*) N. 40° W., N. 10° E., N. 64° E., S. 63° E.
    (*d*) S. 51° E., N. 36° E.
11. (*a*) S. 11° E.
    (*b*) S. 84° W., N. 48° E., S. 58° E., S. 11° E.
    (*c*) N. 85° E., N. 13° E., N. 61° W., S. 14° W.
    (*d*) N. 5° E., N. 75° E.
12. (*a*) N. 2½° W.
    (*b*) S. 76° E., S. 35° E., N. 58° W., N. 50° E.
    (*c*) N. by W., S. 76° W., N. 27° E., S. 82½° E.
    (*d*) S. 32½° W., S. 46° E.

CHAPTER XXI.

Great Circle Sailing.

1. First course, S. 84° 44′ 50″ W.; last course, N. 63° 10′ 40″ W.; lat. of vertex, 34° 43′ S., long. 9° 15′ E.; distance, 3269 miles.

Points.	Longitude.	Latitude.
Long. from V. ° ′	° ′	° ′
9 15	18 30 E.	34 22 S.
5	14 15 E.	34 37 S.
0	9 15 E.	34 43 S.
5	4 15 E.	34 37 S.
10	0 45 W.	34 19 S.
15	5 45 W.	33 47½ S.
20	10 45 W.	33 4 S.
25	15 45 W.	32 7½ S.
30	20 45 W.	30 57½ S.
35	25 45 W.	29 34½ S.
40	30 45 W.	27 57½ S.
45	35 45 W.	26 6½ S.
50	40 45 W.	24 0½ S.
52 44	43 9 W.	22 55 S.

2. Initial course, N. 56° 54′ W.; final course, S. 54° 18′ W.; distance, 4469 miles; lat. and long. of vertex, 48° 34′ N., 169° 14′ W.

Points 1. 40° 57½′ N., 129° 14′ W.
,, 2. 46 47½ N., 149 14 W.
,, 3. 48 34 N., 169 14 W.
,, 4. 46 47½ N., 170 46 E.
,, 5. 40 57½ N., 150 46 E.

3. Courses, N. 55° 43′ E., and N. 60° 15′ E.; dist. 6441 miles; vertex, lat. 46° 40′ N., long. 79° 29′ W.

Points 1. 27° 55½′ S., 160° 31′ E.
,, 2. 19 56 S., 170 31 E.
,, 3. 10 26 S., 179 29 W.
,, 4. 0 0 169 29 W.
,, 5. 10 26 N., 159 29 W.
,, 6. 19 56 N., 149 29 W.
,, 7. 27 55½ N., 139 29 W.
,, 8. 34 16½ N., 129 29 W.

4. Courses, N. 48° 47½′ E., and S. 45° 23½′ E.; dist. 9904 miles; vertex, lat. 45° 52½′ N., long. 179° 14′ W.

Points 1. 27° 16′ N., 120° 46′ E.
,, 2. 38 18 N., 140 46 E.
,, 3. 44 5½ N., 160 46 E.
,, 4. 45 52½ N., 179 14 W.
,, 5. 44 5½ N., 159 14 W.
,, 6. 38 18 N., 139 14 W.
,, 7. 27 16 N., 119 14 W.
,, 8. 10 9 N., 99 14 W.
,, 9. 10 9 S., 79 14 W.

5. Courses, S. 67° 35½′ W., and S. 35° 34½ W.; dist. 3452 miles; vertex, lat. 55° 29′ N., long. 21° 21½′ E.

Points	1.	51°	33′	N.,	8° 38′ W.
,,	2.	49	59	N.,	13 38 W.
,,	3.	48	5	N.,	18 38 W.
,,	4.	45	48	N.,	23 38 W.
,,	5.	43	4	N.,	28 38 W.
,,	6.	39	50	N.,	33 38 W.
,,	7.	36	1½	N.,	38 38 W.
,,	8.	31	34½	N.,	43 38 W.
,,	9.	26	26½	N.,	48 38 W.
,,	10.	20	37½	N.,	53 38 W.
,,	11.	14	10	N.,	58 38 W.

6. Courses, N. 68° 45′ E., and S. 68° 45′ E.; dist. 2494·6 miles; vertex, lat. 47° 36½′ N., long. 180° E.

Points	1.	44°	47½′	N.,	155° E.
,,	2.	45	49½	N.,	160 E.
,,	3.	46	37	N.,	165 E.
,,	4.	47	10	N.,	170 E.
,,	5.	47	30	N.,	175 E.
,,	6.	Vertex.			
,,	7.	47	30	N.,	175 W., etc.

7. Courses, S. 66° W., and S. 66° W.; dist. 7544·6 miles; vertex, lat. 44° 19½′ N., long. 64° 49½′ W.

Points	1.	29°	9′	N.,	120° W.
,,	2.	25	54½	N.,	125 W.
,,	3.	22	18	N.,	130 W.
,,	4.	18	19½	N.,	135 W.
,,	5.	14	2	N.,	140 W.
,,	6.	9	28	N.,	145 W.
,,	7.	4	42	N.,	150 W.
,,	8.	0	0		154 49½′ W.

8. Long. 32° 39½′ W.
9. Lat. 45° 52½′ N.
10. 50° S., 110° E.
11. 119 miles north.
12. A. Lat. 46° 40′ N.; long. 53° 3′ W.
    B. Lat. 51° 25′ N.; long. 9° 29′ W.

COMPOSITE TRACKS.

No.	First course.	Last course.	Distance.	Longitude arrival.	Longitude departure.
	° ′	° ′		° ′	° ′
1	S. 65 27½ E.	N. 59 6½ E.	7832·4	25 40½ W.	68 32½ E.
2	S. 53 5½ W.	N. 68 44½ W.	5600·6	19 1½ W.	21 55½ W.
3	N. 52 5 E.	S. 75 26 E.	4096·1	167 1 W.	143 54 W.

4. Courses, S. 59° 45′ W., and West; longs. 118° 24′ W., and 172° 50′ E.; dist. 2988 + 2266·2 = 5254·2 miles.

Points	1.	33°	46′	S.,	73° W.
,,	2.	35	57	S.,	78 W.
,,	3.	37	49	S.,	83 W.
,,	4.	39	24	S.,	88 W.

Points 5. 40 42 S., 93 W.
,, 6. 41 45 S., 98 W.
,, 7. 42 33 S., 103 W.
,, 8. 43 7½ S., 108 W.
,, 9. 43 28½ S., 113 W.
,, 10. 43 36 S., 118 W., etc.

CHAPTER XXII.

Latitude by Double Altitudes.

1. H.E.T. $2^h\ 52^m\ 57^s$; mid. dec. 14° 38′ 13″ N.; cor. for run, + 11′ 6″; first alt. 30° 38′ 37″; second alt. 26° 43′ 34″; XY = 83° 1′ 34″; X_1 = 103° 19′ 32″; X_2 = 63° 0′ 32″. Cor. for lat. + 12″. Lat. 30° 21′ 30″ S.

2. H.E.T. $2^h\ 23^m\ 55^s{\cdot}5$; mid. dec. 13° 12′ 25″ N.; cor. for run, − 21′ 48″; first alt. 50° 22′ 0″; second alt. 42° 57′ 0″; XY = 69° 46′ 30″; X_1 = 80° 37′ 38″; X_2 = 46° 5′ 30″. Cor. for lat. = 21″. Lat. 43° 27′ 45″ N.

3. H.E.T. $2^h\ 15^m\ 56^s{\cdot}7$; dec. 38° 41′ 45″ N.; cor. for run, 0; first alt. 75° 44′ 55″; second alt. 34° 50′ 11″; XY = 51° 43′ 52″; Y_1 = 67° 8′ 50″; Y_2 = 17° 14′ 56″. Lat. 50° 20′ N.

4. H.E.T. $1^h\ 30^m\ 20^s{\cdot}3$; dec. 16° 19′ 15″ N.; cor. for run, − 12′ 54″; first alt. 17° 53′ 0″; second alt. 45° 37′ 12″; XY = 43° 15′ 18″; Y_1 = 96° 40′ 1″; Y_2 = 41° 9′. Lat. 25° 31′ 8″ S.

5. R.A.M.S. $9^h\ 43^m\ 39^s$; T.A. α Arietis, 27° 6′ 20″; longs., (*a*) 179° 57′ W.; (*b*) 179° 59′ E. Bearing, N. 85° 6′ E. T.A. Altair, 47° 1′ 41″; longs., (*c*) 179° 4′ 30″ W.; (*d*) 179° 35′ 15″ E. Bearing, S. 28° 50′ W. XY = 90° 12′ 41″; X_1 = 80° 32′ 23″; X_2 = 34° 32′ 3″. Lat. 48° 21′ 0″ N; long. 180° W.

6. H.E.T. $1^h\ 53^m\ 2^s{\cdot}5$; mid. dec. 7° 47′ 35″ N.; run, 0; first alt. 18° 4′ 22″; second alt. 47° 2′ 15″; XY = 55° 57′ 8″; X_1 = 85° 53′ 6″; X_2 = 44° 53′. Cor. for lat. − 2′ 30″. Lat. 48° 51′ N.

7. H.E.T. $2^h\ 45^m\ 28^s{\cdot}6$; run, + 49′ 24″; first alt. 15° 50′ 58″; second alt. 28° 8′ 24″; XY = 69° 44″; X_1 = 113° 50′ 32″; X_2 = 65° 18′ 15″. Lat. 24° 27′ N.

8. H.E.T. $1^h\ 52^m\ 34^s$; mid. dec. 3° 22′ 34″ N.; run, + 45′; first alt. 39° 34′ 56″; second alt. 14° 18′ 3″; XY = 56° 10′ 39″; Y_1 = 91° 51′ 40″; Y_2 = 51° 37′ 54″. Lat. 46° 23′ 48″ S.

9. H.E.T. $3^h\ 50^m\ 47^s{\cdot}8$; run, − 1° 3′ 0″; first alt. 54° 47′ 7″; second alt. 49° 9′ 37″; XY = 53° 40′ 42″; Y_1 = 36° 47′ 22″; Y_2 = 45° 33′ 48″. Lat. 43° 19′ 6″ S.

10. R.A.M.S. $10^h\ 15^m\ 59^s$; T.A. Aldebaran, 35° 3′ 22″; longs., (*a*) 156° 7′ 30″ E.; (*b*) 156° 22′ E. Bearing, S. 72° 9′ E. T.A. Markab, 47° 42′ 15; longs., (*c*) 157° 39′ 45″ E.; (*d*) 156° 52′ E. Bearing, S. 43° W. XY = 79° 0′ 30″; X_1 = 77° 43′ 59″; X_2 = 38° 21′ 14″. Lat. 50° 14′ N.; long. 156° 29′ E.

11. H.E.T. $2^h\ 35^m\ 4^s{\cdot}5$; mid. dec. 5° 16′ 18″ N.; run, − 26′ 12″; first alt. 51° 36′ 17″; second alt. 26° 34′ 54″. XY = 77° 8′ 54″; Y_1 = 94° 16′ 13″; Y_2 = 38° 18′ 8″. Cor. for lat. − 1′ 7″. Lat. 27° 14′ S.

12. H.E.T. $1^h\ 45^m\ 23^s{\cdot}8$; run, + 49′ 0″; first alt. 63° 0′ 37″; second alt. 18° 37′ 27″. XY = 52° 3′ 28″; Y_1 = 94° 14′ 43″; Y_2 = 21° 37′ 35″. Lat. 13° 25′ S.

13. R.A.M.S. $11^h\ 35^m 30^s$; T. A. Rigel, 49° 19′ 1″; longs., (*a*) 125° 4′ 30″ E.; (*b*) 127° 25′ 15″ E. Bearing, N. 23° E. T.A. Fomalhaut, 27° 21′ 6″; longs., (*c*) 127° 23′ 15″ E.; (*d*) 127° 39′ 45″ E. Bearing, S. 74° 30′ W. XY = 89° 35′ 57″; Y_1; 80° 36′ 55″; Y_2 = 31° 45′ 47″. Lat. 47° 24′ 30″ S.; long. 127° 41′ 30″ E.

14. H.E.T. $2^h\ 24^m\ 44^s$; mid. dec. 1° 47′ 48″ S.; first alt. 34° 36′ 15″; run, − 16′ 24″; second alt. 16° 22′ 59″; XY = 72° 19′ 30″; Y_1 = 91° 15′ 47″; Y_2 = 58° 9′ 30″. Cor. for lat. − 1′ 25″. Lat. 52° 35′ N.

15. H.E.T. $4^h\ 5^m\ 45^s{\cdot}2$; first alt. 66° 3′ 3″; run, − 57′ 36″; second alt. 20° 19′ 39″. XY = 76° 53′ 45″; Y_1 = 37′ 37″; Y_2 = 23° 52′ 18″. Lat. 34° 12′ 32″ N.

CHAPTER XXIII.

Error of Chronometer by Single Altitude.

1. Fast M.T.G. $7^m\ 35^s$; slow A.T.S. $5^h\ 55^m\ 13^s$.
2. Slow $15^m\ 39^s$.
3. Slow 31^s.
4. Slow $5^m\ 6^s$.
5. Fast $17^m\ 38^s$.
6. Slow $8^m\ 23^s$.
7. Fast 46^s.
8. Slow A.T.S. $3^h\ 43^m\ 43^s$; fast M.G.T. 18^s.

CHAPTER XXIV.

Calculation of Altitudes.

1. E.H.A. $0^h\ 45^m\ 25^s$; dec. 6° 10′ 42″ N.; T.A. 46° 54′ 43″; A.A. 46° 55′ 30″.
2. W.H.A. $4^h\ 42^m\ 25^s$; dec. 2° 25′ 30″ S.; T.A. 11° 18′ 30″; A.A. 11° 23′ 6″.
3. W.H.A. $1^h\ 34^m\ 34^s{\cdot}6$; T.A. 35° 32′ 16″; A.A. 35° 33′ 36″.
4. E.H.A. $4^h\ 54^m\ 27^s{\cdot}3$; T.A. 17° 6′ 1″; A.A. 17° 9′ 4″.
5. W.H.A. $3^h\ 42^m\ 21^s{\cdot}3$; H.P. 54′ 51″; dec. 5° 51′ 28″ N.; T.A. 30° 37′ 13″; A.A. 29° 51′ 19″.
6. W.H.A. $3^h\ 13^m\ 52^s{\cdot}6$; dec. 8° 24′ 48″; T.A. 40° 56′ 54″; A.A. 40° 57′ 58″.
7. E.H.A. $0^h\ 36^m\ 28^s$; H.P. 58′ 50″; dec. 19° 49′ 51″ S.; T.A. 76° 59′ 48″; A.A. 76° 46′ 33″.
8. E.H.A. $3^h\ 56^m\ 40^s{\cdot}9$; dec. 23° 7′ 40″ N.; T.A. 30° 20′ 56″; A.A. 30° 22′ 32″.

CHAPTER XXV.

Longitude by Lunar Distance.

G.M.T.

1. $6^d\ 17^h\ 27^m\ 36^s{\cdot}5$.
2. $7^d\ 20^h\ 27^m\ 13^s$.
3. $12^d\ 11^h\ 7^m\ 1^s{\cdot}5$.
4. $22^d\ 21^h\ 58^m\ 2^s$.
5. $28^d\ 7^h\ 0^m\ 25^s$.

True Distance.

1. Z = 119° 37′ 16″; B = 25° 26′ 45″; true dist. = 96° 18′ 32″.
2. Z = 122° 9′ 50″; B = 24° 56′ 56″; true dist. 107° 17′ 39″.
3. Z = 125° 55′ 28″; B = 21° 16′ 40″; true dist. 100° 39′ 17″.
4. Z = 96° 14′ 22″; B = 21° 59′ 24″; true dist. 55° 37′ 4″.

Longitudes and Errors.

1. *Sun's elements:* Dec. 14° 58′ 51″ N.; R.A. 9ʰ 27ᵐ 39ˢ; R.A.M.S. 9ʰ 22ᵐ 46ˢ·3; A.A. 34° 22′ 13″; T.A. 34° 20′ 57″; E.H.A. 1ʰ 36ᵐ 32ˢ.
Moon's elements: R.A. 4ʰ 0ᵐ 1ˢ·3; dec. 15° 30′ 16″ N.; H.P. 54′ 9″·1; S.D. 14′ 51″·6; W.H.A. 3ʰ 51ᵐ 6ˢ; T.A. 15° 6′ 45″; A.A. 14° 17′ 56″; app. dist. 78° 53′ 30″; B = 40° 41′ 23″; Z = 86° 11′ 1″; true dist. 78° 27′ 16″; error, 6ᵐ fast; long. 93° 31′ 45″ E.
2. *Sun's elements:* R.A. 10ʰ 26ᵐ 44ˢ·8; dec. 9° 44′ 29″ N.; R.A.M.S. 10ʰ 25ᵐ 34ˢ; W.H.A. 3ʰ 34ᵐ 17ˢ·2; T.A. 37° 5′ 42″; A.A. 37° 6′ 51″.
Moon's elements: R.A. 18ʰ 9ᵐ 38ˢ·7; dec. 20° 6′ 2″ S.; S.D. 16′ 13″; H.P. 59′ 8″·3; E.H.A. 4ʰ 8ᵐ 36ˢ·7; A.A. 14° 6′ 50″; T.A. 15° 0′ 27″; app. dist. 118° 6′ 57″; B = 16° 7′ 49″; Z = 143° 5′ 37″; true dist. 117° 22′ 44ˢ; error, 9ᵐ 18ˢ slow; long. 163° 32′ 0″ W.
3. *Star's elements:* R.A. 4ʰ 30ᵐ 31ˢ·5; dec. 16° 19′ 15″ N.; R.A.M.S. 9ʰ 8ᵐ 54ˢ·3; E.H.A. 3ʰ 53ᵐ 49ˢ; T.A. 34° 40′ 39″; A.A. 34° 42′ 0″.
Moon's elements: R.A. 1ʰ 16ᵐ 27ˢ·6; dec. 2° 50′ 3″ N.; S.D. 15′ 17″·7; H.P. 55′ 14″·5; A.A. 53° 17′ 50″; T.A. 53° 50′ 9″; E.H.A. 0ʰ 39ᵐ 45ˢ; app. dist. 49° 35′ 21″; B = 35° 30′ 39″; Z = 67° 1′ 7″; true dist. 49° 32′ 54″; error, 3ˢ slow; long. 174° 36′ 53″ E.
4. *Star's elements:* R.A. 19ʰ 46ᵐ 11ˢ·8; dec. 8° 37′ 11″ N.; R.A.M.S. 10ʰ 0ᵐ 12ˢ·9; A.A. 28° 17′ 54″; T.A. 28° 16′ 8″; E.H.A. 3ʰ 21ᵐ 18ˢ·9.
Moon's elements: R.A. 12ʰ 11ᵐ 39ˢ·9; dec. 3° 6′ 37″ N.; S.D. 16′ 1″·7; H.P. 58′ 19″; W.H.A. 4ʰ 13ᵐ 13ˢ; T.A. 21° 15′ 24″; A.A. 20° 23′ 17″; app. dist. 113° 24′ 57″; B = 21° 11′ 29″; Z = 132° 58′ 10″; true dist. 112° 48′ 56″; error, 10ᵐ 14ˢ·5 fast; long. 33° 36′ 7″ E.
5. *Planet's elements:* R.A. 23ʰ 1ᵐ 40ˢ; dec. 8° 27′ 13″ S.; R.A.M.S. 9ʰ 22ᵐ 50ˢ·3; W.H.A. 0ʰ 15ᵐ 59ˢ·3; T.A. 50° 54′ 27″; A.A. 50° 55′ 12″.
Moon's elements: R.A. 4ʰ 0ᵐ 49ˢ; dec. 15° 32′ 59″ N.; S.D. 14′ 54″; H.P. 54′ 11″; A.A. 23° 21′ 26″; T.A. 24° 8′ 58″; E.H.A. 4ʰ 43ᵐ 10ˢ; app. dist. 78° 26′ 58″; B = 28° 56′ 47″; Z = 100° 42′ 28″; true dist. 77° 50′ 26″; error, 3ˢ fast; long. 40° 50′ W.
6. *Sun's elements:* R.A. 10ʰ 9ᵐ 42ˢ·7; dec. 11° 21′ 4″ N.; R.A.M.S. 10ʰ 7ᵐ 16ˢ; A.A. 47° 5′ 38″; T.A. 47° 4′ 51″; E.H.A. 38ᵐ 45ˢ·7.
Moon's elements: R.A. 13ʰ 45ᵐ 23ˢ·2; dec. 5° 28′ 21″ S.; S.D. 16′ 8″·3; H.P. 58′ 49″; A.A. 9° 52′ 3″; T.A. 10° 44′ 41″; E.H.A. 4ʰ 14ᵐ 26ˢ·2; app. dist. 56° 54′ 0″; B = 47° 32′ 25″; Z = 51° 10′ 2″; true dist. 56° 13′ 1″; error, 14ᵐ 35ˢ fast; long. 4° 34′ E.
7. *Star's elements:* R.A. 16ʰ 23ᵐ 38ˢ·5; dec. 26° 13′ 26″ S.; R.A.M.S. 10ʰ 28ᵐ 46ˢ·8; A.A. 56° 36′ 29″; T.A. 56° 35′ 52″; W.H.A. 2ʰ 7ᵐ 23ˢ·5.
Moon's elements: R.A. 18ʰ 58ᵐ 41ˢ·5; dec. 20° 17′ 45″ S.; S.D. 16′ 21″·5; H.P. 58′ 54″; E.H.A. 0ʰ 27ᵐ 39ˢ·7; T.A. 61° 2′ 31″; A.A. 60° 34′ 6″; app. dist. 36° 10′ 9″; B = 24° 33′ 36″; Z = 72° 46′ 12″; true dist. 35° 57′ 44″; error, 11ᵐ 41ˢ fast; long. 30° 16′ W.
8. *Planet's elements:* R.A. 6ʰ 8ᵐ 5ˢ·6; dec. 23° 7′ 40″ N.; R.A.M.S. 9ʰ 7ᵐ 21ˢ·7; A.A. 30° 22′ 32″; T.A. 30° 20′ 56″; E.H.A. 3ʰ 56ᵐ 39ˢ.
Moon's elements: R.A. 0ʰ 58ᵐ 15ˢ·5; dec. 1° 9′ 37″ N.; S.D. 15′ 23″; H.P. 55′ 33″; W.H.A. 1ʰ 13ᵐ 9ˢ; T.A. 71° 17′ 45″; A.A. 71° 9′ 0″; app. dist. 78° 16′ 58″; B = 3° 4′ 32″; Z = 168° 17′ 36″; true dist 78° 1′ 6″; error, 4ˢ slow; long. 20° 26′ 52″ W.

9. *Star's elements:* R.A. $23^h\ 0^m\ 4^s{\cdot}7$; dec. 14° 41′ 58″ N.; R.A.M.S. $10^h\ 36^m\ 45^s{\cdot}6$; A.A. 67° 48′ 54″; T.A. 67° 48′ 31″; W.H.A. $1^h\ 7^m\ 15^s$.
Moon's elements: R.A. $20^h\ 57^m\ 35^s$; dec. 17° 14′ 51″ S.; S.D. 16′ 6″; H.P. 58′ 21″; W.H.A. $3^h\ 9^m\ 44^s{\cdot}5$; T.A. 40° 14′ 27″; A.A. 39° 30′ 36″; app. dist. 44° 36′ 57″; B = 26° 54′ 48″; Z = 65° 5′ 14″; true dist. 43° 58′ 42″; error, $12^m\ 24^s$ slow; long. 43° 9′ 30″ E.

CHAPTER XXVI.

Mercator's Charts.

1. Lat. 57° 31½′ N., long. 27° 5½′ W.
2. Lat. 64° 10½′ S., long. 121° 22′ E., magnetic bearing S. 22° W.
3. Lat. 55° 51½′ N., long. 30° 24′ W.
   True course, N. 47° E.; distance, 110 miles.
4. Obs. lat. 54° 29′ N., long. 5° 21½′ W.
   D.R. lat. 54° 22½′ N., long. 5° 15′ W.
   Current, N. 28° W. 7¾ miles.
5. Lat. 69° 19′ N., long. 10° 1¾′ E.; course, S. 61° E.; distance, 39·5 miles.
6. Lat. 50° 50′ S., long. 24° 45′ E.
   1st sounding, lat. 51° 40′ S., long. 21° 55′ E.
   2nd sounding, lat. 49° 3′ S., long. 24° 53′ E.
7. Bearing and distance from A of 1st S. 26° E., 6·85 miles.
   2nd S. 57½ E., 5·8 miles.
   3rd S. 49° E., 2·1 miles.
8. N. 84° E. (mag.), ·75 miles.
9. Lat. 55° 33′ N., long. 6° 3½′ W., dist. 30 miles.
   3·4 knots. Mag. co. N. 86° E., dist. 35½ miles.

CHAPTER XXIX.

SYLLABUS.

Coefficients.

31. 1. N. 37° W. 2. N. 61° E. 3. S. 31° E. 4. − 11°·6. 5. + 14°·1. 6. + 5°·5.

Deviations from Tables.

62. 1. A, 0; B, +5½; C, −28½; D, −4½; E, −1.
−8 20, −12 7, −15 50, −19 32, −23, −25 58, −28 12, −29 30, −29 20, −28 6, −25 12, −20 46, −15 2.

2. A, −0 45; B, 0; C, +23 30; D, −2; E, −0 45.
0, −3 51, −7 49, −11 38, −15 22, −18 40, −21½, −23 30, −25, −25 18, −24 24, −22 28, −19 22, −15 26, −10 37, −5 27, 0.

3. A, = −1¾; B, = −12½; C, = +26; D, = −¾; E, = +1¾.
−5 7, −10 51, −16, −20 25, −23 58, −26 34, −28 13, −28 57, −28 17, −27 47, −26.

4. A, +1; B, −4½; C, −15½; D, 0; E, +1½.
−13, −11 56, −10 32, −8 49, −6 37, −4 27, −1 50, +1 1, +4, +7 2, +10 2, +12 47, +15 9, +16 57, +18 6, +18 28, +18.

5. B, +21; C, +3; D, +3.
−8, −14, −18, −20, −21, −21½, −21, −20, −18, −16, −14. −11.

Stars in Good Position.

70. 1. E β Argûs, α Argûs, Rigel.
W. α Centauri, Altair.
2. E. Aldebaran, Capella.
W. Vega, Altair.
3. E.
W. Altair.
4. E. Rigel.
W. α Crucis, α Centauri, Marcab.
5. E. Aldebaran, α Orionis.
W. Vega, Altair.
6. None.

Heeling Error.

92. 1. −3°. 2. 7° 15′. 3. + 11° 48′. 4. + 10° 18′. 5. −25° 30′. 6. + 5° 54′. 7. −11° 15′. 8. 0. 9. −14° 18′. 10. −3° 30′. 11. + 9° 12′. 12. −5° 56′.

CHAPTER XXX.

Miscellaneous Problems.

1. £2142.
2. £7066·5.
3. £3174 9*s*.
4. $33,157·025. £19,994.
5. 13*s*. per gall.; 17¾*d*. per lb.
6. 15,624 tons.
7. 13·6.
8. 2 feet.
9. ·8.
10. 49.
11. 29 feet.
12. 967·5 francs.
13. £42 10*s*. 6*d*.
14. 4 feet.
15. 78·35 shillings.
16. 92 tons.
17. 30·6.
18. 977 feet.
19. ·91.
20. 12,600 sq. ft.
21. 2406·25 lbs.
22. 62 tons.
23. 80·3 tons.
24. 3·6 tons, 14·4 tons, 4·9 tons.
26. 1¾ tons, 3¼ tons, 6 tons.
27. 8 cwt., 2½ tons, 10 tons.
28. 4.
29. 8 tons nearly.
30. 7½ tons.
31. 4½″ rope; 2¾″ wire.
32. 5″ rope; 3″ wire.
33. 162 tons.
34. 562 grains.
35. 525 grains.
36. 566 grains.
37. 1·76 nautical miles.

38. 186,324 miles.
39. 8·44 knots.
40. 8 %.
41. 10 %.
42. 43·2 tons ; 12·8 tons.
43. 202·6 tons; 73·9 tons.
44. 10·62 kts.; 14·73 kts.
45. 13·4 kts. ; 16·9 kts.
46. 2034·5 tons.
47. 540 tons.
48. 484·8 tons.
49. 6836 ; 56·4 tons ; 101·4 tons.
50. 1000 tons.
51. 8·3 tons.
52. 9 knots.
53. 6·84 knots.
54. 19·03 tons ; 30·28 tons.
55. Increase 12 tons.
56. 69 tons.

Paper I.

1. ·49298 log = 9·692831.
2. 85·406 log = 1·931488.
3. S. 27° W., 12′ ; S. 29° E., 28′ ; S. 50° W., 29′ ; S. 34° W., 30′ ; N. 41° E., 20′ ; S. 79° W., 35′ ; N. 49° E., 27′ ; D.R. lat. 49° 51′·4 N. ; long. 0° 36′·6 W. Set S. 14° W. ; drift, 22 miles.
4. Dec. 17° 44′ 18″ N.; T.A. 58° 21′ 26″ ; lat. 49° 22′ 52″ N.
5. Long. 45° 12′·4 E.
6. D. lat. 2472, M.D.L. 2569, D. long. 876 ; true comp. S. 18° 49½′ W. ; comp. course, S. 56° 34½′ W. ; dist. 2612 miles.
7. $4^h\ 24^m$ A.M. ; $4^h\ 42^m$ P.M.
8. Dec. 17° 19′ 5″ N. ; true amp. E. 22° 19½′ N. ; dev. 9° 25½′ E.
9. M.T.G. Aug. $7^d\ 0^h\ 0^m\ 0^s$; Dec. 16° 37′ 13″ N. ; eq. time, $+5^m\ 40^s$; T.A. 54° 6′ 6″ ; H.A. $2^h\ 7^m\ 17^s$; st. long. 30° 24′ 15″ W. ; noon long. 30° 56′ 40″ W. ; true azi. S. 59° 30′ E. ; dev. 1° 15′ W.
10. A.T.S. $3^h\ 47^m\ 38^s$ P.M. ; dec. 8° 17′ 0″ N. ; cor., $-46+10+5$; true azi. S. 121° 43′ W. ; dev. 7° 2′ W.
11. H.A. $48^m\ 35^s$; R.A.M.S. $10^h\ 10^m\ 12^s$; T.A. 57° 24′ 25″ ; nat. no. 10729. Arc I. 58° 17′ 50″ ; Arc II. 32° 0′ 58″ ; lat. 26° 16′ 52″ S.
12. First dec. 0° 1′ 22″ N. ; T.A. 32° 59′ 45″ ; eq. time, $-7^m\ 31^s$; long. (*a*), 179° 9′ 45″ W. ; (*b*), 178° 1′ 30″ W. ; second dec. 0° 2′ 32″ S. ; T.A. 33° 3′ 25″ ; eq. time, $-7^m\ 35$; long. (*c*), 179° 47′ 45″ E. ; (*d*), 178° 39′ 15″ E.

First line: {N. 57° W. / S. 57° E.} ; second azi. N. 33½° W. ; lat. 51° 2′ S. ; long. 179° 40′ W.

13. $4^h\ 33^m\ 22^s$ A.M.
14. α Persei, N. $1^h\ 34^m\ 9^s$ E. Alt. 72° 53′ 20″.
15. M.T.G. Aug. $30^d\ 18^h\ 0^m\ 49^s$; R.A.M.S. $10^h\ 34^m\ 2^s$; T.A. 21° 19′ 19″ ; E.H.A. $1^h\ 56^m\ 40^s$; long. 179° 26′ 30″ W.
16. E.H.A. $4^h\ 49^m\ 12^s$; cor. $+1-20+0$; true azi. S. 81° 17′ E. ; dev. 24° 28′ E.
17. R.A.M. $3^h\ 59^m\ 53^s$; first cor., − 56′ 10″ ; lat. 38° 9′ 37″ N.
18. M.T.G. $6^d\ 7^h\ 58^m\cdot8$; S.D. 15′ 50″ ; H.P. 57′ 15″ ; dec. 9° 21′ 57″ S. ; A.A. 50° 9′ 2″ ; T.A. 50° 44′ 55″ ; lat. 29° 53′ 8″ N.
19. Lat. 28° 23′ 22″ S.
20. Mag. bearing, N. 3½° E. ; comp. course, N. 59° E., N. 86° W.,

S. 37° E., N. 21° W.; mag. course, N. 61° W., N. 52° E., S. 9° W., S. 53° E.; mag. bearings, S. 54° W., N. 25° W.

21. Red, − 7 ft. 10 ins.

22. *Star's elements:* R.A. 23ʰ 0ᵐ 4ˢ·7; dec. 14° 41′ 58″ N.; R.A.M.S. 8ʰ 37ᵐ 55ˢ·5; A.A. 27° 14′ 3″; T.A. 27° 12′ 12″; E.H.A. 2ʰ 44ᵐ 29ˢ.

Moon's elements: R.A. 18ʰ 8ᵐ 23ˢ·8; dec. 20° 3′ 9″ S.; S.D. 16′ 35″; H.P. 59′ 48″; W.H.A. 2ʰ 7ᵐ 12ˢ; T.A. 58° 27′ 37″; A.A. 57° 56′ 27″; app. dist. 79° 55′ 38″; B = 20° 56′ 4″; Z = 116° 49′ 30″; true dist. 79° 38′ 49″; error 6ᵐ 15ˢ fast; long. 65° 50′ E.

23. First dec. 0° 18′ 53″ N.; run, + 8′; alt. 32° 28′ 27″; second dec. 0° 14′ 1″ N.; alt. 32° 20′ 27″; H.E.T. 2ʰ 30ᵐ 5ˢ; XY = 75° 2′ 26″; X_1 = 89° 50′ 32″; X_2 = 60° 54′ 2″; lat. 47° 50′ 38″ N.

24. Dec. 9° 15′ 51″ N.; T.A. 39° 48′ 21″; H.A. 2ʰ 58ᵐ 16ˢ; error 14ᵐ 31ˢ slow.

25. First course, N. 78° 3′ W.; dist. 3346 miles; vertex lat., 50° 59′ N.; long. 21° 16′ W.; 5° W. of Wolf. lat. 50° 31′ N.; 10° W. of Wolf, 50° 51½′ N.; 15° W, 50° 59½′ N.; 20°, 50° 53½ N.; 61° W., 43° 30½′ N.

26. 319·1.

27. 86° 1′ 21″.

28. 106° 10′ 16.

29. 100° 17′ 28″, or 79° 42′ 32″.

30. N. 31° 4½′ E.; C, − 23° 18′.

31. R.A.M. 23ʰ 45ᵐ 39ˢ; E., Rigel; W. α^1 Crucis, α^2 Centauri, Markab.

32. A + ·25; B − 25°; C − 16°·5; D + 2°; E − ·75. *Deviations, North,* 17° W.; 12° 29′ W.; 7° 22′ W.; 1° 43′ W.; 4° 15′ E.; 10° 19′ E.; 16° 10′ E.; 21° 28′ E.; *West,* 26° E.; 29° 26′ E.; 31° 36′ E.; 32° 21′ E.; 31° 36′ E.; 29° 25′ E.; 25° 58′ E.; 21° 24′ E.; *South,* 16° E.

33. 5° 45′ E.

Paper II.

1. 212·8 log = 2·327982.

2. ·147 log = 9·167437.

3. S. 17° E., 15′; N. 67½° E., 54′; N. 52° E., 50′; S. 83° E., 42′; S. 56° W., 31′; S. 1° E., 30′; S. 37° E., 42′; N. 14° W., 54′; D. lat. 3′·2 N., dep. 122′·6 E.; course, N. 88° E.; dist. 123′; lat. in 25° 12′ S.; long. 45° 4½′ W.

4. Dec. 0° 9′ 13″ N.; T.A. 64° 58′ 19″; lat. 24° 52′ 28″ S.

5. 41° 40′ W.

6. D. lat. 5642; M.D.L. 6465; D. long. 2805; true course, N. 23° 27½′ W.; comp. course, N. 64° 7½′ W.; dist. 6150 miles.

7. 5ʰ 40ᵐ A.M., 6ʰ 23ᵐ P.M.

8. Dec. 2° 5′ 33″ S.; true amp. W. 2° 49′ 25″ S.; dev. 8° 19′ 30″ W.

9. M.T.G. 3ᵈ 16ʰ 53ᵐ 9ˢ; dec. 17° 30′ 27″ N.; eq. time, + 5ᵐ 59ˢ; T.A. 22° 38′ 24″; E.H.A. 4ʰ 42ᵐ 44ˢ; st. long. 37° 31′ 30″ E.; noon long. 38° 15′ 30″ E.; true azi. S. 102° 48′ E.; error 10° 18′ W.; dev. 5° 18′ W.

10. W.H.A. 4ʰ 5ᵐ 30ˢ; dec. 8° 36′ 43″ N.; cor. − 17 + 26 + 6; true azi. S. 116° 50′ W.; dev. 10° 57′ E.

11. E.H.A. 19ᵐ 3ˢ; dec. 16° 36′ 52″ N.; T.A. 72° 36′ 2″; nat. no., 2767. Arc. I. 16° 40′ 9″; Arc. II. 16° 48′ 17″; st. lat. 33° 28′ 26″ N.; noon, 33° 24′ 40″ N.

12. First dec. 2° 18′ 36″ S.; T.A. 36° 40′ 42″; eq. time, − 9ᵐ 33ˢ; long. (*a*), 123° 41′ 30″ W.; (*b*), 125° 20′ 30″ W.; second dec. 2° 21′ 7″ S.; T.A. 18° 44′ 50″; eq. time, − 9ᵐ 35ˢ; long. (*c*), 124° 41′ 45″ W.; (*d*), 125° 4′ 15″ W.

First line: $\left\{\begin{matrix}\text{N. 65° W.}\\ \text{S. 65° E.}\end{matrix}\right\}$; second azi. S. 63° W.; lat. 48° 52′ N.; long. 125° 20′ W.

13. Inspection $4^h\ 46^m$ A.M. ; 28th, $4^h\ 55^m\ 15^s$ A.M.

14. R.A.M. $5^h\ 50^m\ 30^s$; Aldebaran, N. $1^h\ 19^m\ 59^s$ W. ; Capella, N. $40^m\ 45^s$ W. ; Rigel, N. $40^m\ 29^s$ W. ; Canopus, S. $31^m\ 22^s$ E. ; Sirius, N. $50^m\ 30^s$ E. ; Procyon, N. $1^h\ 43^m\ 53^s$ E. ; Pollux, N. $1^h\ 49^m\ 4^s$ E. Approx. alt. 67° 26′.

15. M.T.G. $7^d\ 17^h\ 46^m\ 22^s$; R.A.M.S. $11^h\ 5^m\ 32^s$; E.H.A. $1^h\ 52^m\ 28^s$; long. 164° 32′ 30″ W.

16. W.H.A. $3^h\ 48^m\ 44^s$; cor. − 9 − 27 + 12 ; true azi. N. 109° 26′ W. ; dev. 4° 4′ E.

17. R.A.M. $19^h\ 25^m\ 17^s$; first cor. − 5″ ; second, + 26″ ; third, + 1′ 23″ ; lat. 29° 14′ 50″ N.

18. M.T.G. $30^d\ 12^h\ 50^m{\cdot}1$; S.D. 16′ 15″ ; H.P. 58′ 36″ ; dec. 19° 12′ 4″ S. ; cor. 29′ 29″ ; T.A. 59° 44′ 36″ ; lat. 49° 27′ 28″ S.

19. Lat. 29° 30′ 38″ S.

20. S. 65° W. ; comp. course, S. 40° E. ; N., N. 65° W. ; S. 32° W. ; mag. course, N. 40° W. ; N. 12° E. ; N. 64° E. ; S. 64° E. ; mag. bearings, S. 51° E., and N. 36° E.

21. Red, − 3 ft. 9 ins.

22. *Star's elements:* R.A. $13^h\ 20^m\ 14^s$; dec. 10° 40′ 15″ S. ; R.A.M.S. $8^h\ 43^m\ 19^s$; A.A. 12° 10′ 46″ ; T.A. 12° 6′ 26″ ; W.H.A. $4^h\ 44^m\ 12^s$.

Moon's elements: R.A. $19^h\ 32^m\ 38^s$; dec. 19° 54′ 30″ S. ; S.D. 16′ 26″ ; H.P. 59′ 31″ ; E.H.A. $1^h\ 28^m\ 12^s$; T.A. 40° 1′ 36″ ; A.A. 39° 16′ 42″ ; app. dist. 89° 23′ 49″ ; B = 34° 2′ 53″ ; Z = 99° 21′ 32″ ; true dist. 89° 14′ 47″ ; error, $2^m\ 29^s$ slow ; long. 100° 20′ 45″ W.

23. H.E.T. $1^h\ 18^m\ 0^s$; mid. dec. 2° 19′ 52″ S. ; run − 18′ 48″ ; first alt. 36° 21′ 56″ ; second alt. 18° 44′ 52″ ; XY = 38° 58′ 0″ ; X_1 = 90° 45′ 52″ ; X_2 = 54° 49′ 40″ ; lat. 48° 50′ 20″ N.

24. W.H.A. $2^h\ 56^m\ 1^s{\cdot}5$; eq. time, $-7^m\ 16^s$; error, $2^m\ 41^s{\cdot}8$ fast.

25. First course, S. 51° 31′ E. ; final N. 43° 10′ E. ; dist. 5913 miles ; vertex long. 163° 13′ W., and 134° 33′ W. ; points 48° 50′ S., 160° E. ; 51° 53½′ S., 170° E. ; 45° 30′ S., 90° W. ; 39° 38′ S., 80° W.

26. 95·25.

27. 123° 52′ 10″.

28. 93° 32′ 10″.

29. 116° 34′ 11″.

30. S. 47° 44′ W. ; B = + 25° 18′.

31. E., Castor, Betelgueuse ; W., Vega, Altair.

32. A = + ·25° ; B = − 7°·5 ; C = + 10° ; D = + 1°·75 ; E = − 5°·25. *Deviations, North,* + 5° ; + 4° 9′ ; + 3° 46′ ; + 2° 3′ ; *East,* − 2° ; − 8° 2′ ; − 13° 52′ ; − 16° 45′ ; *South,* − 15° ; − 8° 35′ ; + 0° 14′ ; + 8° 18′ ; *West,* + 13° ; + 13° 30′ ; + 10° 52′ ; + 7° 25′.

33. − 44° 6′.

PAPER III.

1. ·00000047438 log = 3·676123.

2. ·0066019 log = 7·819667.

3. N. 18° E., 15′ ; S. 63° W., 28′ ; N. 85° W., 30′ ; S. 88° W., 36′ ; S. 49° W., 40′ ; S. 25° W., 42′ ; S. 68° W., 32′ ; N. 6° E., 18′ ; D. lat. 55·5 S. ; dep. 161·9 W. ; course, S. 71° W. ; dist. 171 miles ; lat. 59° 49′·5 N. ; long. 0° 43′·6 W.

4. Dec. 18° 16′ 54″ N. ; T.A. 81° 4′ 12″ ; lat. 9° 21′ 6″ N.

5. 201 miles.

6. True course, S. 85° 51′ 48″ W. ; compass course, N. 89° 38′ 12″ W. ; dist. 1941 miles.

7. No. A.M. ; $0^h\ 41^m$ P.M.

8. True amp. E. 20° 24½′ N. ; error, 5° 54½′ W. ; dev. 12° 14½′ W.

9. M.T.G. $16^d\ 22^h\ 19^m\ 57^s$; st. lat. 36° 27′ N. ; P.D. 76° 20′ 3″ ; E.H.A.

$4^h\ 7^m\ 20^s$; st. long. 35° 48′ W.; noon long. 35° 25′ 28″ W.; true azi. S. 84° 30′ E.; dev. 19° 30′ E.

10. Corr. + 20 − 30 + 35; true azi. N. 123° 13′ E.; dev. 11° 47′ W.

11. W.H.A. $47^m\ 57^s$; nat. no. 20290. Arc I. 13° 25′ 58″; Arc II. 30° 35′ 19″. Lat. sights 17° 9′ 19″ S.; noon, 17° 7′ 31″ S.

12. First T.A. 32° 0′ 31″; P.D. 83° 47′ 4″; long. (*a*), 150° 10′ W.; (*b*), 149° 44′ 15″ W.; second T.A. 46° 54′ 48″; P.D. 83° 49′ 18″; long. (*c*), 152° 16′ 45″ W.; (*d*), 150° 10′ 30″ W.

Second line: $\begin{Bmatrix}\text{N. } 70° \text{ E.}\\ \text{S. } 70° \text{ W.}\end{Bmatrix}$; second azi. S. 20° E.; lat. 48° 12′ N.; long. 149° 21′ W.

13. $9^h\ 36^m\ 57^s$ P.M.; by inspection, $9^h\ 31^m$ P.M.

14. R.A.M. $16^h\ 0^m\ 39^s$; Antares, S. $0^h\ 23^m$ E.; α Triang. Aust. S. $0^h\ 38^m$ E.

15. Alt. 20° 27′ 40″.

16. E.H.A. $2^h\ 38^m\ 20^s$; cor. − 26 + 12 + 10; true azi. S. 137° 32′ E.; dev. 6° 32′ W.

17. M.T.G. $27^d\ 10^h\ 27^m\ 40^s$; R.A.M.S. $12^h\ 23^m\ 11^s$; E.H.A. $4^h\ 54^m\ 21^s$; long. 84° 26½′ E.

18. Cor. + 33″, + 41″, + 1′ 17″; lat. 42° 9′ 56″ N.

19. M.T.G. $16^d\ 19^h\ 35^m{\cdot}6$; S.D. 15′ 22″; H.P. 55′ 27″; dec. 19° 57′ 25″ N.; T.A. 80° 54′ 39″; lat. 10° 52′ 4″ N.

20. Lat. 8° 11′ 54″ S.

21. Mag. bearing, N. 4° 45′ E.; comp. course, N. 43° E., S. 81° E., S. 11° E., N. 7° E.; mag. course, N. 30° W., N. 50° E., S. 49° E., S. 81° W.; mag. bearings, S. 68½° E., and N. 71° W.

22. Red, − 14 ft. 8 ins.

23. *Sun's elements:* R.A. $9^h\ 17^m\ 4^s{\cdot}9$; dec. 15° 48′ 11″ N.; R.A.M.S. $9^h\ 11^m\ 46^s{\cdot}7$; E.H.A. $3^h\ 44^m\ 37^s$; T.A. 36° 15′ 56″; A.A. 36° 17′ 6″.

Moon's elements: R.A. $1^h\ 50^m\ 6^s{\cdot}4$; dec. 5° 51′ 28″ N.; S.D. 15′ 7″; H.P. 54′ 51″; A.A. 29° 51′ 19″; T.A. 30° 37′ 13″; W.H.A. $3^h\ 42^m\ 21^s$; app. dist., 109° 46′ 10″; B = 10° 25′ 56″; Z = 154° 49′ 24″; true dist. 109° 4′ 27″; error, $5^m\ 29^s$ slow; long. 14° 27′ W.

24. Antares H.A.'s $1^h\ 36^m\ 20^s$, and $1^h\ 31^m\ 53^s$; long. (*a*), 62° 8′ 45″ W., (*b*), 63° 15′ 30″ W.; Spica H.A.'s $4^h\ 38^m\ 15^s$, and $4^h\ 37^m\ 32^s$; long. (*c*), 62° 31′ W., (*d*), 62° 41′ 45″ W.; bearings S. 26° W., and S. 72° W.; lat. 23° 12′ N.; long. 62° 35′ 15″ W.; $Y_1 = 116°\ 19′\ 47″$; $Y_2 = 53°\ 45′\ 30″$; XY = 45° 54′ 16″; bearings S. 26° 14′ W., and S. 71° 37′ W.; lat. 23° 12′ 6″ N.; long. 62° 35′ 7″ W.

25. W.H.A. $2^h\ 23^m\ 50^s{\cdot}5$; error, $5^m\ 37^s$ fast.

26. First course, S. 36° 48′ E.; final, N. 74° 35′ E.; dist. 6971 miles; vertex lat. 54° 6½° S.; long. 150° E.; points: lats. 0°, 13° 30′ S., 25° 18′ S., 34° 38′ S., 53° 42′ S.

27. 1950·33.

28. 214·37.

29. P = 96° 25′ 50″; P.Q. = 61° 31′ 28″.

30. A = − ·25°; B = + 1°·5; C = − 16°; D = − 3°·75; E = − 3°·75. *Deviations, East,* + 5°; + 0° 8′ E.; − 4° 58′; − 9° 55′; − 14° 14; − 17° 37′; − 19° 47′; − 20° 34′; *North,* − 20°; − 18° 16′; − 15° 37′; − 12° 22′; − 8° 52′; − 5° 30′; − 2° 26′; + 0° 5′; *West,* + 2°.

31. + 8° 3′.

PAPER IV.

1. 7·382 log = 0·868158.

2. ·1149 log = 9·060194.

3. S. 87° W., 20′; S. 53° W., 61′; S. 66° W., 51′; S. 75° W., 37′, S. 72° W., 59′; S. 69° W., 50′; N. 89° W., 40′; S. 21° E., 42′; D. lat.

142·6 S.; dep. 278·7 W.; course, S. 63° W.; dist. 313 miles; lat. 34° 40′ N.; long. 1° 19′ W.

4. Dec. 17° 29′ 20″ N.; lat. 56° 54′ 31″ S.

5. 68·73 miles.

6. True course, N. 74° 36½′ E.; compass course, N. 59° 46½′ E.; dist. 2433 miles.

7. $10^h\ 49^m$ A.M.; $11^h\ 26^m$ P.M.

8. True amp. W. 6° 28′ N.; error, 9° 17′ E.; dev. 19° 47′ E.

9. M.T.G. $9^d\ 21^h\ 18^m\ 25^s$; st. lat. 37° 14′ S.; E.H.A. $3^h\ 44^m\ 36^s$; st. long. 14° 25′ 45″ W.; noon long. 13° 57′ 34″ W.; true azi. S. 82° 20′ 20″ E.; dev. 28° 54′ 40″ E.

10. Cor. $-11+4+2$; true azi. S. 111° 47′ W.; dev. 11° 2′ E.

11. E.H.A. $29^m\ 29^s$; nat. no. 6980. Arc. I. 14° 46′ 41″; Arc. II. 43° 33′ 38″; lat. 28° 46′ 57″ S.

12. First T.A. 49° 3′ 18″; P.D. 81° 18′ 3″; H.A.'s $0^h\ 42^m\ 16^s$, and $0^h\ 24^m\ 3^s$; long. (*a*), 177° 52′ W.; (*b*), 177° 34′ 45″ E.; second T.A. 19° 9′ 3″; P.D. 81° 21′ 45″; H.A.'s $4^h\ 43^m\ 0^s$, and $4^h\ 42^m\ 21^s$; long. (*c*), 179° 32′ 30″ W.; (*d*), 179° 42′ 15″ W.

Second line: $\left\{\begin{matrix}\text{N. } 9° \text{ W.}\\ \text{S. } 9° \text{ E.}\end{matrix}\right\}$; azi. S. 81° W; lat. 49° 11′ N.; long. 179° 40′ W.

13. $11^h\ 21^m\ 47^s$ P.M.

14. Altair, S. $0^h\ 22^m\ 52^s$ W.; γ Cygni, S. $0^h\ 9^m\ 47^s$ E.; Deneb, S. $0^h\ 29′\ 10″$ E.

15. Alt. 73° 3′ 20″.

16. E.H.A. $4^h\ 23^m\ 19^s$; cor. $-19-1+13$; true azi. N. 80° 5′ E.; dev. 9° 33′ W.

17. M.T.G. $2^d\ 15^h\ 49^m\ 27^s$; W.H.A. $4^h\ 44^m\ 10^s{\cdot}5$; R.A.M.S. $8^h\ 43^m\ 17^s$; long. 97° 5′ 15″ W.

18. R.A.M. $2^h\ 5^m\ 56^s$; cor. $-1°\ 10′\ 51″+1′+1′\ 10″$; lat. 28° 14′ 10″ N.

19. M.T.G. $14^d\ 10^h\ 52^m{\cdot}6$; S.D. 15′ 2′; H.P. 54′ 26″; dec. 19° 12′ 35″ N.; T.A. 47° 7′ 32″; lat. 23° 39′ 55″ S.

20. Lat. 56° 46′ 23″ S.

21. Mag. bearing, N. 6° W.; comp. course, S. 27½° E., N. 10° E., N. 84° W., S. 75° W.; mag. course, N. 74° E., S. 61° E., S., N. 60° W.; mag. bearings, S. 30° W., and S. 54° E.

22. Red, − 5 ft. 1 in.

23. *Jupiter's elements:* R.A. $6^h\ 9^m\ 44^s{\cdot}5$; dec. 23° 7′ 26″ N.; R.A.M.S. $9^h\ 15^m\ 6^s{\cdot}2$; E.H.A. $4^h\ 50^m\ 39^s{\cdot}5$; T.A. 27° 34′ 42″; A.A. 27° 36′ 29″.

Moon's elements: R.A. $2^h\ 28^m\ 56^s{\cdot}1$; dec. 9° 8′ 57″ N.; S.D. 15′ 6″; H.P. 54′ 30″; A.A. 54° 41′ 48″; T.A. 55° 12′ 37″; E.H.A. $1^h\ 9^m\ 51^s$; app. dist. 54° 28′ 13″; B = 36° 27′ 21″; Z = 66° 39′ 32″; true dist. 54° 30′ 24″; error, $2^m\ 36^s{\cdot}5$ slow; long. 22° 9′ 15″ W.

24. ϵ *Orionis:* E.H.A.'s $1^h\ 12^m\ 41^s$, and $1^h\ 3^m\ 26^s$; long. (*a*), 81° 58′ 30″ E., (*b*), 84° 17′ 30″ E.

Alphard: E.H.A.'s $4^h\ 54^m\ 50^s$, and $4^h\ 54^m\ 23^s$; long. (*c*), 84° 19′ 30″ E., (*d*), 84° 26′ 15″ E.; bearings, N. 23° E., and N. 84° E.; lat. 47° 43′ S.; long. 84° 27′ E.; XY = 58° 2′ 52″; X_1 = 93° 40′ 57″; X_2 = 51° 12′ 10″; bearings N. 21° 13′ E., and N. 83° 22′ E.; lat. 47° 42′ 48″ S.; long. 84° 26′ 30″ E.

25. Eq. time $-7^m\ 13^s{\cdot}8$; P.D. 89° 39′ 10″; H.A. $2^h\ 21^m\ 54^s$; error, $1^m\ 55^s{\cdot}8$ slow.

26. First course, S. 66° 26½′ E.; dist. 3708 miles; vertex, 41° 1′ S.; long. 20° 51′ W. Point, lat. 39° 6½′ N.; long. 0°.

27. BP = 241·1 ft.; BAQ = 29° 5′ 30″.

28. XZ = 1684 ft.

29. Dist. 4032 miles; lat. 43° 31′ 30″ N.

30. 151·3 ft.

31. W.H.A. $7^h\ 26^m\ 40^s{\cdot}5$; true azi. 44° 56′ − 13° 38′ = S. 31° 18′ W.; dev. 11° 3′ E.

32. N. 56° 22′ E.; C = − 19° 53′.

33. A = − 1°·75; B = − 5°·5; C = − 15°; D = 0; E = + 6°·75; *Deviations, East,* − 14°; − 10° 29′; − 5° 54′; − 0° 36′; + 4° 56′; + 10° 16′; + 14° 49′; + 18° 10′; *South,* + 20°; + 20° 16′; + 19° 2′; + 16° 23′; + 12° 44′; + 8° 35′; + 4° 16′; + 0° 18′; *West,* − 3°.

34. 0.

Paper V.

1. 62·7577 log = 1·797667.

2. 0·0075 log = 7·875091.

3. S. 48° E., 20′; E. 40′; S. 82° E., 42′; N. 64° E., 44′; N. 88° E., 46′; S. 80° E., 48′; E. 46′; N. 83° E., 54′; D. lat. 0, dep. 328·9 E.; course E., dist. 329 miles; lat. 62° 30′ N.; long. 169° 13′·7 W.

4. Dec. 16° 52′ 44″ N.; T.A. 77° 49′ 43″; lat. 4° 42′ 27″ N.

5. 557·4 miles.

6. True course, S. 71° 18′ W.; compass course, N. 89° 12′ W.; dist. 3793 miles.

7. No. A.M.; $0^h\ 4^m$ P.M.

8. Dec. 14° 19′ 10″ N.; true amp. W. 20° 17′ N.; error, 21° 47′ E.; dev. 24° 7′ E.

9. M.T.G. $29^d\ 16^h\ 32^m\ 46^s$; P.D. 92° 25′ 31″; eq. time, $-9^m\ 39^s$; st. lat. 46° 50′; W.H.A. $4^h\ 42^m\ 25^s$; sight long. 180° W.; noon 179° 13′ 40″ W.; true azi. S. 73° 57′ 42″ W.; dev. 32° 6′ W.

10. Cor. + 36 − 19 + 28; true azi. N. 125° 17′ E.; dev. 0° 47′ E.

11. W.H.A. $24^m\ 19^s$; nat. no. 4724. Arc I. 13° 13′ 14″; Arc. II. 43° 38′ 22″; sight lat. 30° 25′ 5″ S.; noon 30° 22′ 23″ S.

12. First P.D. 92° 21′ 23″; eq. time, $-9^m\ 36^s$; H.A.'s $0^h\ 20^m\ 48^s$ and $0^h\ 41^m\ 53^s$; long. (*a*), 177° 38′ 15″ E.; (*b*), 177° 5′ 30″ W.; second P.D. 92° 25′ 28″; eq. time, $-9^m\ 39^s$; W.H.A.'s $4^h\ 42^m\ 43^s$ and $4^h\ 43^m\ 48^s$; long. (*c*), 179° 55′ 30″ W.; (*d*), 179° 39′ 15″ W.

First line: $\left\{\begin{matrix}\text{N. 80° W.}\\ \text{S. 80° E.}\end{matrix}\right\}$; second azi. S. 74° W; lat. 46° 45½′ N.; long. 179° 58′ W.

13. A.T.S. $11^h\ 8^m\ 31^s$ P.M.; inspection, $11^h\ 1^m$ P.M.

14. *α* Gruis S. $57^m\ 20^s$ W.; Fomalhaut, S. $7^m\ 10^s$ W.

15. Alt. 40° 15′ 10″.

16. M.T.G. $2^d\ 13^h\ 25^m\ 40^s$; W.H.A. $1^h\ 34^m\ 35^s$; R.A.M.S. $8^h\ 42^m\ 53^s$; long. 62° 35′ W.

17. W.H.A. $2^h\ 58^m\ 48^s$; cor. − 35 + 24 + 5; true azi. S. 133° 43′ W.; dev. 22° 51′ W.

18. Cor. − 40′ 35″, + 37″, + 50″; lat. 50° 16′ 33″ N.

19. M.T.G. $9^d\ 18^h\ 31^m{\cdot}5$; dec. 5° 23′ 11″ N.; S.D. 15′ 11″; H.P. 54′ 51″; T.A. 48° 25′ 19″; lat. 46° 57′ 52″.

20. Lat. 51° 28′ 29″ S.

21. Mag. bearing, N. 56° E.; compass course, N. 68° E., S. 67° E., S. 79° W., N. 19° W.; mag. course, S. 68° W., S. 18° E., N. 40° E., N. 53° W.; mag. bearings, S. 21° W., and S. 49° E.

22. − 8 ft. 5 ins.

23. *Sun's elements*: R.A. $9^h\ 43^m\ 37^s{\cdot}8$; dec. 13° 39′ 57″ N.; R.A.M.S. $9^h\ 39^m\ 32^s{\cdot}5$; A.A. 30° 38′ 31″; T.A. 30° 37′ 1″; E.H.A. $4^h\ 7^m\ 20^s$.

Moon's elements: R.A. $7^h\ 35^m\ 38^s{\cdot}5$; dec. 19° 51′ 24″ N.; S.D. 15′ 22″; H.P. 55′ 27″; E.H.A. $1^h\ 59^m\ 21^s$; T.A. 59° 6′ 1″; A.A. 58° 37′ 44″; app. dist. 30° 47′ 7″; B = 40° 59′ 0″; Z = 18° 49′ 30″; true dist. 31° 12′ 20″; error, $12^m\ 15^s$ fast; long. 35° 40′ W.

24. H.E.T. $1^h\ 11^m\ 31^s{\cdot}5$; mid. dec. 6° 11′ 49″; cor. run + 7′ 25″; first

alt. 32° 39′ 46″; second alt. 47° 26′ 36″; XY = 35° 32′ 54″; X_1 = 88° 3′ 46″; X_2 = 52° 34′ 2″; lat. correction, − 1′ 47″; lat. 47° 42′ 53″ N.

25. P.D. 97° 32′ 29″; eq. time, − 0ᵐ 42ˢ·7; W.H.A. 4ʰ 30ᵐ 26ˢ; error, 20ᵐ 54ˢ·4 slow.

26. First course, S. 57° 17′ 30″ E.; final, N. 62° 17′ 30″ E.; first long. 67° 12′ E.; second long. 104° 21′ E.; dist. = 2300 + 1549 + 1830 = 5679 miles; points: lat. 44° 41½′ S., long. 50° E., lat. 40° 4¾′ S., 140° E.

27. 1009.

28. N. 70° 7′ 25″ W.; drift 67·55 miles.

29. Side 33° 38′ 50″; angle 100° 30′ 56″.

30. N. 70° E.; B = + 5°·86.

31. A = − ·25°; B = + 11°·5; C = 0; D = + ·5°; E = + 25°; *Deviations, North*, 0 + 2° 25′; + 4° 41′; + 6° 42′; + 8° 23′; + 9° 40′; + 10° 33′; + 10° 59′; *East*, + 11°; *South*, 0; − 2° 4′; − 4° 7′; − 6° 5′; − 7° 53′; − 9° 27′; − 10° 43′; − 11° 34′; *West*, − 12°.

32. − 3° 45′.

Paper VI.

1. ·4839 log = 9·684706.

2. ·08604 log = 8·934729.

3. N. 62° W., 17′; S. 84° E., 72′; S. 75° E., 43′; S. 68° E., 56′; S. 54° E., 57′; S. 47° E., 56′; E. 56′; N. 22° E., 60′; D. lat. 47·7; dep. 315·6; course, S. 81½° E.; dist. 318 miles; lat. 38° 33′ S.; long. 174° 47′ W.

4. Dec. 0° 10′ 30″ N.; T.A. 47° 17′ 56″; lat. 42° 31′ 34″ S.

5. 32° 23′.

6. True course, N. 53° 48′ E.; compass course, S. 75° 22′ E.; dist. 4561 miles.

7. 1ʰ 8ᵐ A.M.; 1ʰ 54ᵐ P.M.

8. Dec. 2° 3′ 32″ N.; true amp. E. 2° 31½ N.; dev. 5° 38′ 37″ E.

9. M.T.G. 7ᵈ 6ʰ 47ᵐ 7ˢ; P.D. 83° 47′ 4″; st. lat. 48° 10′; eq. time, − 1ᵐ 54ˢ; T.A. 32° 7′ 56″; E.H.A. 3ʰ 8ᵐ 28ˢ; st. long, 149° 22′ 15″ W.; noon, 148° 33′ 45″ W.; true azi. S. 59° 20′ 20″ E.; dev. 13° 54′ 40″ E.

10. W.H.A. 2ʰ 18ᵐ 39ˢ; cor. − 41 + 21 + 20; true azi. S. 133° 36′ W.; dev. 14° 24′ W.

11. W.H.A. 23ᵐ 41ˢ; nat. no. 3884. Arc I. 16° 24′ 13″; Arc II. 57° 2′ 9″; lat. 40° 37′ 56″ S.

12. First P.D. 89° 41′ 9″; eq. time, − 7ᵐ 16ˢ; T.A. 32° 20′ 25″; H.A.'s 2ʰ 34ᵐ 37ˢ and 2ʰ 30ᵐ 56ˢ; long. (*a*), 81° 22′ E.; (*b*), 82° 17′ 15″ E.; second P.D. 89° 46′ 1″; eq. time, − 7ᵐ 20ˢ; T.A. 32° 20′ 25″; H.A.'s 2ʰ 34ᵐ 3ˢ and 2ʰ 30ᵐ 21ˢ; long. (*c*), 83° 29′ 15″ E.; (*d*), 82° 33′ 45″ E.

First line: $\left\{\begin{matrix}\text{N. } 43° \text{ E.}\\ \text{S. } 43° \text{ W.}\end{matrix}\right\}$; first azi. S. 47° E; lat. 47° 51′ N.; long. 82° 32′ E.

13. M.T.S. 1ʰ 45ᵐ 15ˢ P.M.

14. Achernar, S. 1ʰ 6ᵐ 7ˢ W.; *a* Persei, N. 37ᵐ 16ˢ E.; Aldebaran, N. 1ʰ 50ᵐ 12ˢ E.

15. Alt. 25° 54′ 40″.

16. W.H.A. 4ʰ 0ᵐ 20ˢ; cor. − 5 − 33 + 25; true azi. N. 102° 36′ W.; dev. 10° 54′ E.

17. M.T.G. 5ᵈ 12ʰ 51ᵐ 4ˢ; E.H.A. 2ʰ 13ᵐ 13ˢ; R.A.M.S. 8ʰ 54ᵐ 37ˢ; long. 50° 10′ W.

18. Cor. − 2′ 12″, + 35″, + 1′ 25″; lat. 38° 23′ 24″ N.

19. M.T.G. 6ᵈ 17ʰ 35ᵐ·6; S.D. 15′ 42″; H.P. 56′ 50″; dec. 7° 41′ 9″ S.; cor. + 46′ 30″; T.A. 33° 8′ 9″; lat. 64° 33′ 0″ S.

20. Lat. 34° 19′ 53″ N.

21. Mag. bearings, South; comp. course, N. 3° W., N. 80° W., S. 66° E.,

N. 31° E.; mag. course, S. 55° E., S. 30° W., S. 77° W., N. 19° W.; mag. bearings, N. 30° W., S. 72° W.

22. Red, — 2 ft. 9 ins.

23. *Star's elements:* R.A. $16^h\ 23^m\ 38^s{\cdot}5$; dec. 26° 13′ 26″ S.; R.A.M.S. $10^h\ 31^m\ 23^s{\cdot}5$; A.A. 49° 25′ 23″; T.A. 49° 24′ 34″; W.H.A. $2^h\ 38^m\ 10^s$.

Moon's elements: R.A. $19^h\ 38^m\ 16^s{\cdot}3$; dec. 19° 49′ 51″ S.; S.D. 16′ 20″; H.P. 58′ 50″; A.A. 76° 46′ 33″; T.A. 76° 59′ 48″; E.H.A. $36^m\ 28^s$; app. dist. 45° 3′ 37″; B = 13° 48′ 1″; Z = 102° 49′ 30″; true dist. 44° 58′ 32″; error, $11^m\ 38^s$ slow; long. 98° 26′ 45″ E.

24. H.E.T. $2^h\ 10^m\ 21^s{\cdot}5$; run cor. − 25′ 12″; first T.A. 43° 45′ 14″; second 23° 28′ 34″; XY = 28° 43′ 33″; X_1 = 60° 25′ 42″; X_2 = 126° 49′ 36″; lat. 48° 20′ S.

25. Ship 18° 32′ S.; 70° 35′·5 W.; W.H.A. $3^h\ 22^m\ 54^s$; error, $37^m\ 40^s$ slow.

26. First course, S. 50° 4½′ W.; final, N. 58° 46½′ W.; longs. 128° 32½ W., 142° 32′ W.; dist. 5050·1 miles; lat. 49° 41′ S.; long. 120° W.; lat. 43° 24½ S.; long. 180° W.

27. B = 50°40′ 12″; A = 39° 19′ 48″; BD = 14383 ft.

28. 115·6 ft.

29. Course, S. 78° 30′ E.; dist. 92 miles.

30. A = + 1°·25; B = + 18°; C = − 7°·5; D = + ·75°; E. = + 1°·25; *Deviations*, + 2° 37′; + 6° 11′; + 9° 26′; + 12° 16′; + 14° 40′; +16° 34′; *East*, + 18°; + 18° 55′; + 19° 20′; + 19° 13′; + 18° 32′.

31. Lat. 42° 5′; H.A. $7^h\ 10^m\ 21^s{\cdot}5$.

32. Lat. 60° 22′ S.; long. 30° 26′ W.; course S. 3° E.; dist. 43 miles.

33. − 25° 30′.

Paper VII.

1. 696090·3 log = 5·842665.

2. 19534·82 log = 4·290809.

3. N. 12° W., 20′; N. 15° E., 22′; S. 71° W., 24′; N. 7° E., 214′; D. lat. 245′·5; dep. 4·9; D.R. lat. 46° 4′·5 S.; long. 110° 7′ 40″ W.; set S. 70° W.; drift 16½ miles.

4. Dec. 2° 24′ 32″ S.; T.A. 88° 36′ 7″. Lat. 1° 0′ 39″ S.

5. 51° 23′ N.; 16° 16′ 30″ W.

6. Compass course, N. 16° 22′ W.; dist. 5489 miles.

7. Midnt.; 0^h 23 P.M.

8. True amp., E. 16° 55′ N.; dev. 2° 21′ E.

9. M.T.G. $23^d\ 16^h\ 14^m\ 51^s$; P.D. 89° 55′ 8″; H.A. $2^h\ 57^m\ 0^s$; long. 158° 38′ E.; true azi. N. 67° 51′ 28″ W.; dev. 6° 51′ 28″ W.

10. A.T.S. $8^h\ 33^m\ 13^s$ A.M.; cor. + 21 + 14 + 5; true azi. S. 126° 15′ E.; dev. 12° 59′ E.

11. E.H.A. $27^m\ 24^s$; nat. no. 5109. Arc. I. 2° 12′ 50″; Arc. II. 46° 29′ 45″. Lat. st. 44° 16′ 55″ N.; noon, 44° 20′ 13″ N.

12. First P.D. 107° 28′ 11″; T.A. 8° 40′ 49″; eq. time, + $5^m\ 59^s$; H.A.'s $3^h\ 41^m\ 54^s$ and $3^h\ 39^m\ 39^s$; long. (*a*), 0° 49′ E.; (*b*), 1° 22′ 15″ E.; second P.D. 107° 24′ 12″; T.A. 18° 19′ 28″; eq. time, + $5^m\ 57^s$; H.A.'s $2^h\ 21^m\ 22^s$ and $2^h\ 17^m\ 9^s$; long. (*c*), 1° 9′ 30″ E.; (*d*), 0° 6′ E.

Second line: $\left\{\begin{matrix}\text{S. } 55° \text{ W.}\\ \text{N. } 55° \text{ E.}\end{matrix}\right\}$; second azi. N. 35° W.; lat. 47° 25′ S.; long. 0° 3′ W.

13. A.T.S. $9^h\ 23^m\ 19^s$ A.M.

14. Alt. 31° 17′ 20″.

15. Aldebaran, S. 47^m 36′ E.; Capella, N. $1^h\ 26^m\ 49^s$ E.; Rigel, S. $1^h\ 27^m\ 5^s$ E.; Bellatrix, S. $1^h\ 37^m\ 9^s$ E.; β Tauri, S. $1^h\ 37^m\ 25^s$ E.; ε Orionis, S. $1^h\ 48^m\ 31^s$ E.

16. W.H.A. $3^h\ 19^m\ 40^s$; cor. − 49 + 18 + 4; true azi. S. 130° 32′ W.; dev. 18° 39′ W.

17. M.T.G. $30^d\ 1^h\ 56^m\ 56^s$; W.H.A. $2^h\ 38^m\ 10^s$; R.A.M.S. $10^h\ 31^m\ 24^s$; long. 98° 22′ 15″ E.

18. Cor. − 1° 4′ 5″, + 2″, + 1′ 25″.; lat. 20° 14′ 2″ N.

19. M.T.G. $30^d\ 8^h\ 21^m{\cdot}9$; S.D. 16′ 11″; H.P. 58′ 41″; dec. 19° 29′ 21″ S.; T.A. 30° 15′ 24″; lat. 40° 15′ 15″ N.

20. Lat. 72° 1′ 56″ N.

21. Mag. bearing, N. 73° 30′ W.; comp. course, N. 2° W., S. 74° E., S. 82° W., N. 55° E.; mag. course, S. 67° E., S. 20° W., S. 86½° W., N. 74° W.; mag. bearings, S. 25½° W., and N. 61° 30′ W.

22. − 9 ft. 5 ins.

23. *Planet's elements:* R.A. $23^h\ 2^m\ 1^s$; dec. 8° 24′ 48″ S.; R.A.M.S. $9^h\ 17^m\ 5^s{\cdot}6$; A.A. 40° 57′ 58″; T.A. 40° 56′ 54″; W.H.A. $3^h\ 13^m\ 54^s$.

Moon's elements: R.A. $2^h\ 52^m\ 18^s{\cdot}1$; dec. 10° 59′ 8″ N.; S.D. 15′ 3″; H.P. 54′ 23″; E.H.A. $36^m\ 23^s$; T.A. 52° 37′ 46″; A.A. 52° 5′ 6″; app. dist. 60° 43′ 53″; B = 27° 38′ 28″; Z = 93° 30′ 2″; true dist. 60° 28′ 13″; error, $10^m\ 34^s{\cdot}5$; long. 169° 38′ 30″ E.

24. H.E.T. $2^h\ 6^m\ 5^s$; mid. dec. 2° 23′ 28″ S.; run cor., − 24′ 6″; first alt. 40° 21′ 10″; second alt. 11° 18′ 30″; XY = 62° 58′ 56″; X_1 = 91° 31′ 18″; X_2 = 98° 18′ 12″; lat. cor., + 2′ 28″; lat. 46° 50′ 30″ N.

25. Ship lat. 5° 36′ S.; long. 35° 0′ 32″ W.; E.H.A. $3^h\ 15^m\ 41^s$; error, $20^m\ 3^s$ fast.

26. First course, S. 39° 16½′ W.; final S. 38° 40½′ W.; vertex lat. 59° 11½′, N.; long. 58° 43½′ E.; dist. 5101 miles; course at equator, S. 31° W.; in long. 31° 16½′ W.

27. 368 ft.

28. Bearing S. 27° 19′ 49″ W.; set E. 3° 49′ 42″ N.

29. A = 31° 12′ 14″; AC = 72° 23′ 48″.

30. A = 0; B = + 5° 5; C = − 28°·5; D = − 4°·5; E = − 1°. *Deviations, West,* − 4° 30′; − 0° 38′; + 3° 22′; + 7° 29′; + 11° 46′; + 16° 6′; + 20° 20′; + 24° 14′; *South,* + 27° 30′; + 29° 49′; + 30° 55′; + 30° 32′; + 28° 32′; + 24° 56′; + 19° 53′; + 13° 35′; *East,* + 6° 30′.

31. + 4° 38′.

32. H.A. 0; dev. 6° 15′ E.

Paper VIII.

1. ·000001075 log = 4·031408.

2. ·1 log = 9·000000.

3. S. 3° W., 30′; S. 34° W., 74′; S. 1° W., 26′; S. 3° W., 56′; S. 54° W., 52′; S. 56° W., 75′; S. 71° W., 45′; S. 65° W., 10′; D. lat. 264·3, dep. 204·9; course, S. 37° 47′ W.; dist. 334·5 miles; lat. 55° 24′ 42″ N.; long. 50° 16′ 30″ W.

4. Dec. 0° 1′ 5″ S.; T.A. 56° 48′ 49″; lat. 33° 12′ 16″ S.

5. Noon long, 83° 38′·4 W.

6. True course, S. 54° 10′ E.; compass course, S. 36° 50′ E.; dist. 9224 miles.

7. $6^h\ 54^m$ A.M.; $7^h\ 15^m$ A.M.

8. Dec. 2° 3′ 16″ S.; true amp. W. 3° 12′ 10″ S.; dev. 27° 2′ 10″ W.

9. M.T.G. $3^d\ 20^h\ 20^m\ 51^s$; P.D. 107° 28′ 12″; eq. time, + $5^m\ 59^s$; T.A. 7° 52′ 33″; E.H.A. $3^h\ 45^m\ 18^s$; long. 0° 2′ 30″ W.; true azi. N. 53° 16′ E.; dev. 13° 16′ E.

10. A.T.S. $5^h\ 7^m\ 10^s$ P.M.; cor. − 41 − 22 + 1; true azi. N. 100° 47′ W.; dev. 8° 33′ E.

11. E.H.A. $39^m\ 8^s$; nat. no. 9125. Arc I. 30° 29′ 10″; Arc. II. 73° 56′ 9″. Lat. 43° 26′ 59″ N.

12. First T.A. 37° 38′ 2″; P.D. 92° 43′ 47″; eq. time, − $9^m\ 55^s$; H.A.'s $0^h\ 46^m\ 6^s$ and $0^h\ 25^m\ 57^s$; long. (*a*), 175° 38′ 45″ E.; (*b*), 179° 19′ W.; second

T.A. 23° 12′ 42″; P.D. 92° 47′ 27″; eq. time, − 9ᵐ 58ˢ; H.A.'s 3ʰ 16ᵐ 57ˢ and 3ʰ 14ᵐ 6ˢ; long. (*c*), 179° 21′ 45″ W.; (*d*), 179° 55′ 30″ E.

First line: {N. 79° E. / S. 79° W.}; second azi. S. 55° W; lat. 49° 16½′ N.; long. 179° 59½′ E.

13. M.T.S. 8ʰ 9ᵐ 53ˢ A.M.

14. Aldebaran, N. 1ᵐ 44ˢ E.; Capella, N. 40ᵐ 58ˢ E.; Rigel, N. 41ᵐ 14ˢ E.; Bellatrix, N. 51ᵐ 18ˢ E.; β Tauri, N. 51ᵐ 34ˢ E.

15. Alt. 47° 20′.

16. E.H.A. 3ʰ 21ᵐ 1ˢ; cor., − 11 − 43 + 31; true azi. N. 101° 43′ E.; dev. 28° 13′ E.

17. M.T.G. 19ᵈ 2ʰ 2ᵐ 40ˢ; W.H.A. 9ʰ 59ᵐ 20ˢ; R.A.M.S. 9ʰ 48ᵐ 2ˢ, long. 157° 30′ E.

18. Cor., − 1° 11′ 58″, + 0, + 1′ 12″; lat. 28° 41′ 36″ N.

19. M.T.G. 31ᵈ 23ʰ 9ᵐ·9; S.D. 16′ 8″; H.P. 58′ 6″; dec. 15° 45′ 11″ S.; cor. + 8′ 37″; T.A. 81° 26′ 50″; lat. 7° 12′ 2″ S.

20. Lat. 11° 52′ 30″ N.

21. Mag. bearing, N. 2½° W.; comp. course, N. 8° E., S. 71° E., S. 79° W., N. 57° W.; mag. course, N. 85° E., N. 72° W., S. 65° W., S. 20° E.; mag. bearings, S. 20° W., and S. 50° E.

22. − 2 ft. 9 ins.

23. *Sun's elements*: R.A. 9ʰ 38ᵐ 24ˢ·2; dec. 14° 6′ 19″ N.; R.A.M.S. 9ʰ 34ᵐ 2ˢ; A.A. 68° 51′ 57″; T.A. 68° 51′ 38″; W.H.A. 1ʰ 20ᵐ 17ˢ.

Moon's elements: R.A. 6ʰ 22ᵐ 19ˢ·5; dec. 20° 12′ 13″ N.; S.D. 15′ 4″; H.P. 54′ 48″; W.H.A. 4ʰ 36ᵐ 21ˢ; T.A. 26° 24′ 26″; A.A. 25° 36′ 59″; app. dist. 47° 50′ 16″; B 32° 52′ 29″; Z = 34° 29′ 14″; true dist. 47° 4′ 21″; error, 7ᵐ 3ˢ slow; long. 170° 36′ W.

24. H.E.T. 2ʰ 9ᵐ 18ˢ·7; run cor., − 12′ 36″; first alt. 67° 4′ 46″; second alt. 68° 22′ 20″; XY = 44° 28′ 55″; X_1 = 65° 54′ 45″; X_2 = 3° 12′ 0″; lat. 48° 29′ 42″ N.

25. P.D. 84° 22′ 30″; eq. time, − 2ᵐ 25ˢ·8; E.H.A. 2ʰ 45ᵐ 48ˢ; error, 9ᵐ 53ˢ·7 fast.

26. First course, S. 63° 36′ E.; longs. 60° 8′ E., and 79° 2′ E.; dist. 4705·6 miles; lat. 41° 33′; long. 50° E.; lat. 41° 28′ 30″ S.; long. 90° E.; lat. 40° 3′ 30″ S.; long. 100° E.

27. N. 35° 6½′ E.; dist. 274·5 miles.

28. QR = 260·9 or 181·2.

29. 86·6 ft.

30. R = 100° 32′ 7″; RQ = 94° 12′ 4″.

31. A = 84° 4′ 12″; AB = 83° 50′ 5″.

32. Lat. 68° 10′ 30″ N.; long. 0° 34′ W.

33. N. 42° 43′ 30″ W.

34. A = + ·75°; B = + 14°; C = − 10°·5; D = + 3°·25; E = + 7°·25. *Deviations, North*: − 2° 30′; + 3° 50′; + 6° 29′; + 6° 50′; *East*, + 7° 30′; + 10° 16′; + 14° 49′; + 18° 38′; *South*, + 18° 30′; + 12° 31′; + 1° 31′; − 10° 59′; *West*, − 20° 30′; − 23° 38′; − 19° 49′; − 11° 29′.

35. 0.

36. E.H.A. 1ʰ 20ᵐ 45ˢ; true azi. 94° 4′ + 69° 30′ = N. 163° 34′ E.; dev. 7° 19′ E.

THE END.

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Ardrishaig	
Ardrossan	Greenock
Ayr	Greenock
Beachy Head	Dover
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Workington	Liverpool

H. W. F. & C. { VH. 43M. (Local mean time). VI. 0 (Standard time). Lat. 50° 22′ N. Long. 4° 10′ W.

SEPTEMBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
F.	1	1 47	0 2	7 45	15 0	2 1	0 8	8 0	15 0
S.	2	2 16	0 9	8 16	14 8	2 32	1 1	8 34	14 6
S.	3	2 48	1 4	8 49	14 3	3 4	1 8	9 7	13 11
M.	4	3 21	2 3	9 25	13 9	3 39	2 5	9 47	13 2
Tu.	5	3 57	3 2	10 8	13 2	4 17	3 4	10 36	12 5
W.	6	4 47	4 3	11 11	12 7	5 26	4 2	11 52	11 9
Th.	7	—	—	6 10	5 1	0 43	12 6	6 58	4 3
F.	8	1 35	12 0	7 48	4 7	2 19	13 4	8 34	3 0
S.	9	3 5	13 0	9 23	3 3	3 44	14 3	10 1	1 5
S.	10	4 16	14 2	10 32	1 10	4 42	15 3	10 57	0 2
M.	11	5 8	15 0	11 22	0 7	5 33	16 3	11 48	*0 7
Tu.	12	—	—	5 55	15 9	0 11	*0 3	6 18	16 7
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Th.	14	1 12	*1 0	7 16	16 0	1 31	*0 5	7 31	16 0
F.	15	1 47	*0 5	7 49	15 7	2 5	0 1	8 6	15 4
S.	16	2 22	0 5	8 22	14 11	2 37	0 11	8 38	14 5
S.	17	2 53	1 6	8 51	14 1	3 5	2 1	9 7	13 4
M.	18	3 21	3 0	9 24	13 3	3 36	3 2	9 45	12 2
Tu.	19	3 57	4 3	10 2	12 3	4 12	4 4	10 31	11 1
W.	20	4 44	5 6	11 3	11 6	5 21	5 3	11 44	10 6
Th.	21	—	—	6 2	6 4	0 30	11 6	6 45	5 2
F.	22	1 13	10 8	7 27	5 10	1 56	12 2	8 10	4 3
S.	23	2 35	11 6	8 51	4 9	3 9	13 1	9 27	3 1
S.	24	3 40	12 7	9 57	3 6	4 5	13 11	10 21	2 1
M.	25	4 26	13 5	10 42	2 6	4 47	14 7	11 2	1 3
Tu.	26	5 6	14 3	11 20	1 5	5 24	15 2	11 38	0 5
W.	27	5 40	15 0	11 55	0 7	5 57	15 7	—	—
Th.	28	0 13	*0 1	6 15	15 4	0 29	0 1	6 30	15 6
F.	29	0 45	*0 1	6 48	15 6	1 1	0 0	7 5	15 6
S.	30	1 19	0 1	7 19	15 4	1 35	0 3	7 36	15 2

OCTOBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	1 52	0 6	7 53	15 1	2 9	0 9	8 11	14 7
M.	2	2 26	1 2	8 30	14 8	2 44	1 3	8 49	13 11
Tu.	3	3 3	2 2	9 8	14 2	3 22	2 1	9 31	13 3
W.	4	3 42	3 3	9 58	13 6	4 7	3 0	10 32	12 3
Th.	5	4 43	4 5	11 10	11 9	5 27	3 10	11 55	11 9
F.	6	—	—	6 13	5 0	0 46	12 11	7 1	3 9
S.	7	1 32	12 3	7 46	4 3	2 15	13 9	8 30	2 7
S.	8	2 54	13 4	9 12	2 11	3 29	14 8	9 46	1 5
M.	9	3 54	14 4	10 11	1 8	4 20	15 5	10 35	0 5
Tu.	10	4 45	15 2	10 59	0 7	5 8	16 0	11 22	*0 4
W.	11	5 28	15 3	11 43	*0 2	5 47	16 2	—	—
Th.	12	0 3	*0 3	6 8	15 11	0 23	*0 6	6 27	15 11
F.	13	0 41	*0 6	6 46	15 9	1 0	*0 8	7 4	15 6
S.	14	1 19	0 1	7 18	15 4	1 34	0 8	7 35	14 10
S.	15	1 51	0 10	7 51	14 5	2 7	1 5	8 8	13 11
M.	16	2 23	2 2	8 24	14 1	2 38	1 11	8 38	13 0
Tu.	17	2 52	3 2	8 52	13 4	3 6	2 11	9 11	12 1
W.	18	3 22	4 4	9 33	12 6	3 42	4 1	9 59	11 2
Th.	19	4 9	5 6	10 31	11 10	4 45	4 11	11 6	10 7
F.	20	5 24	6 2	11 47	11 7	6 4	5 2	—	—
S.	21	0 31	10 10	6 46	5 10	1 9	12 2	7 23	4 6
S.	22	1 48	11 7	8 2	4 10	2 24	12 10	8 40	3 5
M.	23	2 53	12 6	9 11	3 8	3 20	13 8	9 37	2 5
Tu.	24	3 42	13 7	9 59	2 6	4 5	14 4	10 21	1 7
W.	25	4 26	14 5	10 42	1 5	4 46	15 0	11 1	0 8
Th.	26	5 6	15 1	11 20	0 7	5 24	15 4	11 39	0 8
F.	27	5 43	15 5	11 59	0 0	6 1	15 6	—	—
S.	28	0 17	*0 1	6 20	15 7	0 35	*0 2	6 41	15 6
S.	29	0 54	0 0	6 59	15 8	1 14	0 0	7 19	15 2
M.	30	1 33	0 6	7 37	15 6	1 53	0 3	7 59	14 8
Tu.	31	2 15	1 1	8 21	15 0	2 36	0 10	8 43	14 1

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD, or mean time of the meridian of Greenwich. To obtain local mean time *subtract* 17 minutes.

TIDAL DATUM, ETC.

The zero of the predictions, which is also the datum of the Admiralty charts of Devonport, is 8·42 feet below Ordnance datum, and is the level of mean low water spring tides.

TIDAL DIFFERENCES ON DEVONPORT.

Place.	Lat. N.	Long. W.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
St. Mary's . .	49 55	6 19	−1 17	...	+0 4	−0 5	0 0	−0 5
Fowey . . .	50 20	4 38	−0 27	...	−0 5	−0 6	0 0	−0 5
Dartmouth . .	50 21	3 84	+0 30	...	−1 3	−1 5	0 0	−0 4
Tor Bay . . .	50 25	3 32	+0 14	...	−2 1	−2 0	0 0	−1 0
Exmouth . . .	50 37	3 25	+0 41	...	−4 5	−3 8	...	...

H. W. F. & C. { XI$^{H.}$ 12$^{M.}$ (Local mean time). / XI. 7. (Standard time). Lat. 51° 7′ N. Long. 1° 19′ E.

JANUARY.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	1 19	3 9	6 53	15 8	2 0	3 0	7 28	15 4
S.	2	2 31	3 1	7 58	16 3	3 9	2 4	8 30	16 2
M.	3	3 39	2 3	8 57	17 1	4 13	1 5	9 24	17 3
Tu.	4	4 42	1 2	9 51	17 11	5 12	0 8	10 17	17 11
W.	5	5 40	0 6	10 44	18 5	6 8	0 3	11 9	18 7
Th.	6	6 36	*0 2	11 38	18 9	7 2	*0 2	— —	— —
F.	7	0 1	19 0	7 30	*0 6	0 31	18 11	7 53	*0 3
S.	8	0 51	19 2	8 19	*0 8	1 20	18 9	8 40	*0 2
S.	9	1 38	19 0	9 5	*0 6	2 7	18 4	9 23	0 4
M.	10	2 23	18 3	9 46	0 1	2 53	17 4	10 3	1 1
Tu.	11	3 8	17 6	10 28	0 11	3 39	16 3	10 42	2 1
W.	12	3 54	16 6	11 9	2 1	4 29	15 3	11 27	3 2
Th.	13	4 48	15 6	11 59	3 2	5 30	14 4	— —	— —
F.	14	0 24	4 2	5 51	14 9	1 1	3 11	6 36	13 11
S.	15	1 33	4 6	7 5	14 7	2 16	4 0	7 49	14 1
S.	16	3 51	4 3	8 17	14 11	3 30	3 8	8 55	14 8
M.	17	4 2	3 9	9 16	15 5	4 35	3 1	9 44	15 5
Tu.	18	4 59	2 11	10 3	16 0	5 24	2 6	10 24	16 0
W.	19	5 45	2 4	10 42	16 5	6 6	2 2	11 0	16 6
Th.	20	6 25	1 10	11 18	16 9	6 44	1 10	11 34	17 0
F.	21	7 2	1 4	11 53	17 0	7 17	1 6	— —	— —
S.	22	0 7	17 6	7 36	1 0	0 27	17 4	7 50	1 3
S.	23	0 40	17 10	8 9	0 8	1 2	17 6	8 23	1 1
M.	24	1 13	18 1	8 41	0 6	1 37	17 6	8 53	0 11
Tu.	25	1 46	18 2	9 13	0 5	2 13	17 4	9 26	1 1
W.	26	2 22	17 10	9 47	0 9	2 50	16 9	9 59	1 5
Th.	27	3 1	17 2	10 22	1 4	3 29	16 1	10 33	2 1
F.	28	3 48	16 5	11 1	2 3	4 18	15 3	11 20	2 11
S.	29	4 49	15 6	11 57	3 3	5 26	14 7	— —	— —
S.	30	0 26	3 9	6 6	15 0	1 11	3 9	6 49	14 6
M.	31	1 51	3 9	7 33	15 3	2 41	3 8	8 14	15 5

FEBRUARY.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
Tu.	1	3 21	2 10	8 51	16 3	4 4	2 8	9 20	16 8
W.	2	4 37	1 7	9 49	17 3	5 9	1 2	10 12	17 9
Th.	3	5 36	0 6	10 40	18 2	6 2	0 4	10 59	18 7
F.	4	6 28	*0 4	11 29	18 9	6 52	*0 3	11 45	19 3
S.	5	—	—	7 17	*0 11	0 16	19 1	7 37	*0 7
S.	6	0 29	19 6	8 1	*1 2	1 0	19 1	8 19	*0 7
M.	7	1 11	19 6	8 40	*1 1	1 40	18 9	8 57	*0 4
Tu.	8	1 51	19 0	9 17	*0 7	2 18	18 0	9 32	0 4
W.	9	2 30	18 1	9 53	0 4	2 55	16 10	10 6	1 3
Th.	10	3 10	16 11	10 27	1 6	3 33	15 8	10 38	2 6
F.	11	3 52	15 8	11 5	2 10	4 20	14 5	11 23	3 9
S.	12	4 50	14 5	11 55	4 2	5 28	13 5	— —	— —
S.	13	0 27	4 10	6 5	13 8	1 10	4 11	6 50	13 3
M.	14	1 52	5 9	7 33	13 9	2 39	4 9	8 16	13 9
Tu.	15	3 23	4 5	8 54	14 5	4 5	4 0	9 19	14 8
W.	16	4 35	3 4	9 48	15 5	5 5	3 0	10 5	15 9
Th.	17	5 26	2 3	10 28	16 3	5 46	2 3	10 38	16 7
F.	18	6 5	1 6	11 1	16 10	6 22	1 8	11 9	17 4
S.	19	6 40	0 10	11 33	17 5	6 54	1 0	11 40	18 0
S.	20	—	—	7 12	0 2	0 4	17 10	7 26	0 6
M.	21	0 12	18 6	7 43	*0 3	0 35	18 1	7 57	0 2
Tu.	22	0 45	18 9	8 15	*0 5	1 8	18 3	8 28	0 0
W.	23	1 20	18 9	8 48	*0 5	1 42	18 1	9 0	0 1
Th.	24	1 57	18 6	9 21	0 0	2 17	17 6	9 34	0 6
F.	25	2 36	17 8	9 56	0 9	2 57	16 8	10 10	1 3
S.	26	3 19	16 7	10 34	1 9	3 46	15 7	10 54	2 4
S.	27	4 20	15 4	11 25	3 1	4 57	14 7	— —	— —
M.	28	Midt.	3 6	5 45	14 6	0 46	4 0	6 31	14 3
Tu.	29	1 36	3 9	7 26	14 9	2 29	3 8	8 7	15 1

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD, or mean time of the meridian of Greenwich. To obtain local mean time *add* 5 minutes.

TIDAL DATUM, ETC.

The zero of the predictions, which is also the datum of the Admiralty chart of Dover, is 8·42 feet below Ordnance datum, and is 0·15 foot above the level of mean low water spring tides.

TIDAL DIFFERENCES ON DOVER.

Place.	Lat. N.	Long. E.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
Dungeness	50 54	0 58	−0 26	...	+3 7	+4 8	...	...
Deal	51 13	1 25	+0 2	...	−2 2	−1 10	...	...
Ramsgate	51 20	1 25	+0 31	...	−3 1	−1 6	+0 2	−1 4
Rotterdam	51 56	4 30	+6 0	...	...	...	...	...
Beachy head	50 44	0 14	+0 12	...	+1 10	+0 8	...	...
Hastings	50 51	0 35	−0 16	...	+5 9	+3 2	+0 2	+0 7
Rye bay	50 56	0 45	+0 10	...	+3 10	+2 11	...	...

H. W. F. & C. { VI$^{H.}$ 29$^{M.}$ (Local mean time). / VI. 30 (Standard time). Lat. 53° 44′ N. Long. 0° 19′ W.

Week Day.	Month Day.	OCTOBER. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	NOVEMBER. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	DECEMBER. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	8 14	20 11	8 32	20 6	W.	1	9 32	18 9	9 59	18 3	F.	1	10 27	18 8	10 55	18 2
M.	2	8 52	19 11	9 12	19 2	Th.	2	10 30	17 9	11 0	17 4	S.	2	11 23	17 11	11 51	17 8
Tu.	3	9 36	18 7	10 1	18 0	F.	3	11 36	17 1	—	—						
W.	4	10 31	17 5	11 0	16 10	S.	4	0 12	17 0	0 58	17 2	*S.*	3	—	—	0 22	17 7
Th.	5	11 36	16 7	—	—							M.	4	1 0	17 7	1 44	17 7
F.	6	0 20	16 6	1 16	16 8	*S.*	5	1 48	17 5	2 31	17 10	Tu.	5	2 25	17 9	3 1	18 0
S.	7	2 15	17 0	2 58	17 9	M.	6	3 4	18 4	3 37	18 10	W.	6	3 33	18 3	4 2	18 6
						Tu.	7	4 3	19 4	4 24	19 9	Th.	7	4 25	18 10	4 47	19 0
S.	8	3 35	18 5	4 4	19 2	W.	8	4 45	20 1	5 5	20 3	F.	8	5 10	19 2	5 31	19 3
M.	9	4 27	19 10	4 46	20 4	Th.	9	5 26	20 5	5 47	20 6	S.	9	5 52	19 5	6 11	19 5
Tu.	10	5 6	20 9	5 28	21 1	F.	10	6 7	20 7	6 27	20 6						
W.	11	5 48	21 4	6 9	21 7	S.	11	6 47	20 5	7 8	20 4	*S.*	10	6 31	19 6	6 53	19 6
Th.	12	6 30	21 8	6 49	21 7							M.	11	7 14	19 6	7 30	19 6
F.	13	7 8	21 6	7 25	21 4	*S.*	12	7 25	20 2	7 41	19 11	Tu.	12	7 47	19 5	8 4	19 5
S.	14	7 42	21 0	7 58	20 9	M.	13	7 58	19 9	8 13	19 6	W.	13	8 18	19 4	8 36	19 1
						Tu.	14	8 31	18 10	8 51	18 3	Th.	14	8 53	18 8	9 11	18 4
S.	15	8 15	20 4	8 32	19 7	W.	15	9 10	17 10	9 30	17 4	F.	15	9 29	18 0	9 50	17 9
M.	16	8 52	18 10	9 10	18 2	Th.	16	9 52	17 0	10 16	16 8	S.	16	10 9	17 6	10 32	17 3
Tu.	17	9 30	17 6	9 51	16 11	F.	17	10 37	16 4	11 0	16 1						
W.	18	10 15	16 4	10 39	15 10	S.	18	11 31	16 0	—	—	*S.*	17	10 51	17 0	11 11	16 9
Th.	19	11 9	15 6	11 44	15 4							M.	18	11 35	16 7	—	—
F.	20	—	—	0 19	15 3	*S.*	19	0 3	16 0	0 39	16 1	Tu.	19	0 2	16 6	0 33	16 6
S.	21	1 11	15 5	2 0	15 9	M.	20	1 18	16 3	2 2	16 7	W.	20	1 7	16 7	1 49	16 9
						Tu.	21	2 38	17 0	3 8	17 6	Th.	21	2 31	17 0	3 5	17 5
S.	22	2 40	16 2	3 14	16 9	W.	22	3 37	17 11	3 58	18 5	F.	22	3 40	17 10	4 10	18 4
M.	23	3 43	17 4	4 5	17 11	Th.	23	4 19	18 11	4 37	19 4	S.	23	4 33	18 11	4 59	19 5
Tu.	24	4 23	18 6	4 41	19 1	F.	24	4 57	19 8	5 19	20 0						
W.	25	4 57	19 6	5 12	19 10	S.	25	5 41	20 3	6 2	20 6	*S.*	24	5 19	19 9	5 46	20 2
Th.	26	5 29	20 3	5 47	20 6							M.	25	6 12	20 6	6 37	20 11
F.	27	6 6	20 9	6 24	21 0	*S.*	26	6 23	20 9	6 47	20 11	Tu.	26	7 2	21 2	7 28	21 5
S.	28	6 45	21 1	7 6	21 1	M.	27	7 12	21 0	7 33	21 1	W.	27	7 50	21 7	8 13	21 8
						Tu.	28	8 55	21 0	8 16	21 0	Th.	28	8 34	21 8	8 57	21 3
S.	29	7 23	21 1	7 42	21 0	W.	29	8 38	20 7	9 5	20 0	F.	29	9 21	20 9	9 46	20 3
M.	30	8 1	20 11	8 21	20 7	Th.	30	9 30	19 6	9 58	19 0	S.	30	10 11	19 9	10 33	19 3
Tu.	31	8 42	19 11	9 6	19 4												
												S.	31	10 58	18 9	11 21	18 3

Mean tide level 10$^{F.}$ 5$^{I.}$ Approximate { Rise 6$^{H.}$ 0$^{M.}$ / Fall 6 25.

P.M. times and heights are printed in large block type.

The times given are STANDARD, or mean time of the meridian of Greenwich. To obtain local mean time *subtract* 1 minute.

The zero of the predictions, which is also the datum of the Admiralty chart of Hull, is 10·07 feet below Ordnance datum, and is the level of mean low water spring tides.

TIDAL DIFFERENCES ON HULL.

Place.	Lat. N.	Long. W.	Time of H. W.	Height of H. W. Springs.	Height of H. W. Neaps.
	° ′	° ′	H. M.	F. I.	F. I.
Grimsby	53 35	0 4 E.	−0 54	−1 9	−1 6
Spurn point	53 35	0 7 W.	−1 4	−2 2	−1 7
Bridlington	54 5	0 11 W.	−1 50	−4 11	−4 7

H. W. F. & C. { XII^H. 5^M. (Local mean time). XII. 24. (Standard time). Lat. 55° 27′ N. Long. 4° 46′ W.

NOVEMBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
W.	1	3 33	9 10	9 8	1 2	3 57	9 8	9 37	1 6	F. 1	
Th.	2	4 26	9 5	10 15	1 8	4 56	9 2	10 59	1 10	S. 2	
F.	3	5 30	8 11	11 44	2 0	6 10	8 8	—	—		
S.	4	0 26	2 2	6 54	8 7	1 9	2 1	7 36	8 7	S. 3	
S.	5	1 50	2 0	8 19	8 10	2 24	1 8	8 56	9 0	Tu. 5	
M.	6	2 48	1 6	9 26	9 2	3 13	1 4	9 54	9 5	W. 6	
Tu.	7	3 36	1 2	10 16	9 7	3 57	1 0	10 39	9 8	Th. 7	
W.	8	4 18	0 10	10 55	9 10	4 36	0 7	11 22	9 11	F. 8	
Th.	9	4 55	0 5	11 43	9 11	5 15	0 5	—	—	S. 9	
F.	10	0 3	10 0	5 32	0 6	0 22	10 0	5 51	0 6		
S.	11	0 42	10 1	6 10	0 6	1 3	10 1	6 30	0 6	S. 10	
										M. 11	
S.	12	1 20	10 0	6 46	0 6	1 38	9 11	7 3	0 7	Tu. 12	
M.	13	1 58	9 11	7 23	0 8	2 15	9 10	7 41	0 10	W. 13	
Tu.	14	2 34	9 8	8 2	1 0	2 55	9 7	8 24	1 4	Th. 14	
W.	15	3 13	9 6	8 45	1 6	3 30	9 5	9 6	1 10	F. 15	
Th.	16	3 50	9 4	9 30	1 11	4 12	9 1	9 58	2 0	S. 16	
F.	17	4 33	8 10	10 31	2 2	4 55	8 7	10 1	2 5		
S.	18	5 26	8 6	11 41	2 6	6 2	8 5	—	—	S. 17	
										M. 18	
S.	19	0 18	2 6	6 37	8 4	0 52	2 7	7 10	8 4	Tu. 19	
M.	20	1 35	2 5	7 49	8 4	2 2	2 4	8 25	8 4	W. 20	
Tu.	21	2 30	2 1	9 0	8 7	2 53	1 11	9 28	8 10	Th. 21	
W.	22	3 15	1 8	9 48	9 0	3 32	1 6	10 10	9 4	F. 22	
Th.	23	3 51	1 4	10 31	9 6	4 11	1 1	10 53	9 7	S. 23	
F.	24	4 30	0 10	11 15	9 8	4 49	0 7	11 36	9 11		
S.	25	5 9	0 6	11 48	10 1	5 28	0 5	—	—	S. 24	
										M. 25	
S.	26	0 18	10 1	5 48	0 5	0 41	10 2	6 9	0 4	Tu. 26	
M.	27	1 6	10 3	6 32	0 2	1 30	10 3	6 56	0 2	W. 27	
Tu.	28	1 54	10 5	7 19	0 1	2 18	10 4	7 43	0 3	Th. 28	
W.	29	2 41	10 2	8 8	0 5	3 9	10 2	8 33	0 8	F. 29	
Th.	30	3 43	10 1	9 6	0 11	3 58	10 0	9 35	1 1	S. 30	

DECEMBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
F.	1	4 24	9 11	10 8	1 2	4 51	9 7	10 44	1 5		
S.	2	5 18	9 4	11 23	1 8	5 47	9 2	—	—		
S.	3	0 2	1 10	6 20	9 0	0 36	1 11	6 57	8 10		
M.	4	1 12	1 11	7 33	8 8	1 48	1 11	8 12	8 10		
Tu.	5	2 21	1 10	8 52	8 10	2 49	1 8	9 22	9 0		
W.	6	3 12	1 6	9 52	9 1	3 36	1 5	10 17	9 2		
Th.	7	3 58	1 5	10 41	9 4	4 20	1 2	11 5	9 5		
F.	8	4 42	1 0	11 26	9 5	5 0	0 11	11 47	9 6		
S.	9	—	—	5 19	0 11	0 6	9 6	5 37	0 11		
S.	10	0 26	9 8	5 56	0 10	0 43	9 8	6 16	0 10		
M.	11	1 8	9 10	6 35	0 8	1 25	9 10	6 51	0 8		
Tu.	12	1 44	9 10	7 10	0 10	2 3	9 10	7 29	0 10		
W.	13	2 20	9 8	7 45	0 11	2 39	9 7	8 5	1 0		
Th.	14	2 57	9 7	8 25	1 2	3 15	9 7	8 46	1 4		
F.	15	3 32	9 7	9 5	1 6	3 50	9 7	9 26	1 8		
S.	16	4 7	9 5	9 47	1 10	4 27	9 4	10 12	1 10		
S.	17	4 47	9 1	10 40	1 11	5 7	9 0	11 10	2 0		
M.	18	5 29	8 10	11 41	2 2	5 57	8 8	—	—		
Tu.	19	0 14	2 2	6 31	8 6	0 47	2 4	7 4	8 5		
W.	20	1 19	2 4	7 39	8 5	1 54	2 2	8 18	8 6		
Th.	21	2 28	2 1	8 56	8 7	2 58	2 0	9 30	8 10		
F.	22	3 19	1 8	10 0	9 0	3 44	1 6	10 25	9 2		
S.	23	4 6	1 4	10 52	9 5	4 31	1 1	11 14	9 7		
S.	24	4 51	0 10	11 42	9 8	5 15	0 7	—	—		
M.	25	0 7	10 0	5 38	0 5	0 32	10 2	6 2	0 4		
Tu.	26	0 56	10 4	6 24	0 2	1 23	10 6	6 49	0 0		
W.	27	1 47	10 6	7 18	0 0	2 12	10 7	7 37	0 0		
Th.	28	2 37	10 6	8 0	0 1	3 0	10 6	8 27	0 1		
F.	29	3 25	10 6	8 54	0 5	3 49	10 5	9 21	0 7		
S.	30	4 11	10 4	9 47	0 11	4 31	10 1	10 12	1 1		
S.	31	4 54	9 10	10 42	1 2	5 17	9 7	11 15	1 5		

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD, or mean time of the meridian of Greenwich. To obtain local mean time *subtract* 19 minutes.

TIDAL DIFFERENCES ON GREENOCK.

Place.	Lat. N.	Long. W.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
Crinan. . .	56 5	5 33	+ 5 12	...	− 4 3	− 4 3	...	...
Lamlash . .	55 48	4 52	− 0 15	...	− 0 3	− 1 3	...	...
Ardrossan. .	55 38	4 50	− 0 16	...	− 0 3	− 0 9	...	...
Ayr . . .	55 28	4 89	− 0 16	...	− 1 6	− 1 0	...	...
Clydebank .	...	...	...	...	+ 0 44	+ 0 20	...	...

The zero of the predictions, which is also the datum of the Admiralty chart of the River Clyde entrance at Greenock, is 6·08 feet below Ordnance datum, and is 0·24 foot below the level of mean low water spring tides.

H. W. F. & C. { XI^H. 28^M. (Local mean time). / XI. 35. (Standard time). Lat. 53° 24′ N. Long. 3° 0′ W.

SEPTEMBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
F.	1	1 1	28 11	7 56	1 9	1 22	27 9	8 5	2 7
S.	2	1 33	28 5	8 27	2 5	1 58	27 2	8 36	3 3
S.	3	2 8	27 6	8 58	3 7	2 27	26 3	9 10	4 4
M.	4	2 48	26 1	9 31	5 1	3 9	25 0	9 51	5 7
Tu.	5	3 35	24 5	10 16	6 9	4 4	23 8	10 47	7 1
W.	6	4 43	22 11	11 22	8 0	5 20	22 10	—	—
Th.	7	0 6	7 7	6 10	22 5	0 51	8 3	6 57	23 2
F.	8	1 45	7 0	7 49	23 2	2 27	7 4	8 24	24 9
S.	9	3 11	5 3	9 5	25 1	3 41	5 5	9 28	27 0
S.	10	4 15	3 1	10 0	27 3	4 36	3 5	10 15	29 2
M.	11	5 8	1 2	10 44	28 10	5 24	1 9	10 58	30 6
Tu.	12	5 54	0 0	11 26	29 9	6 8	0 9	11 40	31 2
W.	13	—	—	6 37	*0 6	0 6	30 1	6 50	0 4
Th.	14	0 20	31 1	7 16	*0 4	0 44	29 10	7 28	0 7
F.	15	0 58	30 5	7 52	0 6	1 21	29 0	8 4	1 6
S.	16	1 36	29 0	8 26	2 1	1 57	27 6	8 39	3 1
S.	17	2 15	27 2	8 59	4 1	2 33	25 11	9 15	5 0
M.	18	2 56	24 11	9 35	6 3	3 14	23 11	9 58	7 1
Tu.	19	3 43	22 9	10 22	8 4	4 10	22 3	10 56	8 8
W	20	4 49	21 2	11 28	9 8	5 25	21 4	—	—
Th.	21	0 12	9 5	6 14	20 8	0 51	10 1	6 54	21 5
F.	22	1 43	9 0	7 44	21 3	2 20	9 5	8 15	22 6
S.	23	3 5	7 9	8 54	22 9	3 30	8 0	9 13	24 2
S.	24	4 1	6 0	9 44	24 6	4 16	6 4	9 53	25 11
M.	25	4 44	4 5	10 21	26 1	4 55	4 9	10 27	27 5
Tu.	26	5 21	3 0	10 51	27 3	5 30	3 7	10 57	28 6
W.	27	5 53	2 0	11 19	28 1	6 2	2 8	11 25	29 2
Th.	28	6 23	1 6	11 48	28 7	6 33	2 11	11 57	29 6
F.	29	—	—	6 54	1 4	0 19	28 9	7 5	1 10
S.	30	0 32	29 5	7 26	1 7	0 52	28 9	7 38	2 0

OCTOBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	1 8	28 10	7 59	2 4	1 27	28 1	8 13	2 9
M.	2	1 46	27 8	8 33	3 7	2 5	27 0	8 51	4 0
Tu.	3	2 30	26 1	9 10	5 4	2 50	25 6	9 36	5 7
W.	4	3 24	24 3	9 59	7 1	3 50	24 1	10 38	7 0
Th.	5	4 40	22 10	11 14	8 4	5 15	23 4	—	—
F.	6	0 7	7 5	6 13	22 7	0 49	8 5	6 51	23 9
S.	7	1 46	6 7	7 47	23 8	2 20	7 3	8 12	25 5
S.	8	3 5	4 10	8 55	25 7	3 23	5 4	9 9	27 5
M.	9	4 0	2 11	9 44	27 6	4 15	3 5	9 54	29 2
Tu.	10	4 47	1 5	10 24	28 11	5 1	2 0	10 35	30 3
W.	11	5 30	0 6	11 2	29 9	5 43	1 1	11 15	30 7
Th.	12	6 10	0 3	11 39	29 10	6 24	0 11	11 54	30 2
F.	13	—	—	6 48	0 9	0 15	29 6	7 4	1 3
S.	14	0 32	29 5	7 24	1 8	0 50	28 9	7 41	2 2
S.	15	1 9	28 1	7 58	3 0	1 26	27 6	8 16	3 7
M.	16	1 47	26 5	8 30	4 9	2 3	26 1	8 52	5 3
Tu.	17	2 28	24 6	9 4	6 9	2 43	24 4	9 31	7 1
W.	18	3 12	22 8	9 47	8 7	3 34	22 9	10 23	8 6
Th.	19	4 15	21 3	10 48	9 10	4 42	21 10	11 36	9 2
F.	20	—	—	5 32	20 10	0 6	10 3	6 4	21 9
S.	21	0 57	8 10	6 56	21 4	1 27	9 9	7 23	22 7
S.	22	2 18	7 10	8 9	22 8	2 40	8 5	8 26	24 0
M.	23	3 20	6 3	9 2	24 4	3 32	6 9	9 10	25 9
Tu.	24	4 4	4 8	9 43	26 0	4 12	5 1	9 46	27 3
W.	25	4 40	3 3	10 15	27 5	4 50	3 8	10 19	28 6
Th.	26	5 14	2 3	10 46	28 5	5 27	2 7	10 53	29 3
F.	27	5 48	1 8	11 18	29 0	6 4	1 11	11 31	29 5
S.	28	6 24	1 6	11 53	29 3	6 42	1 8	—	—
S.	29	0 11	29 3	7 1	1 11	0 31	29 1	7 21	1 10
M.	30	0 52	28 9	7 38	2 6	1 10	28 8	8 1	2 5
Tu.	31	1 35	27 7	8 16	3 9	1 51	27 8	8 45	3 8

* Signifies *below Datum*.

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD or mean time of the meridian of Greenwich. To obtain local mean time *subtract* 12 minutes.

TIDAL DIFFERENCES ON LIVERPOOL.

Place.	Lat. N.	Long. W.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
Port Carlisle . .	54 57	3 10	+0 48	...	...	...	...	...
Annan foot . .	54 58	3 16	+0 43	...	−0 5	−3 2	...	...
Workington . .	54 39	3 34	−0 1	...	−3 2	−3 2	...	...
Whitehaven . .	54 33	3 36	−0 7	...	−3 2	−4 11	+1 1	+0 10
Fleetwood, Wyre lighthouse	53 57	3 2	−0 12	...	−1 5	−1 8	+0 8	−0 1
Hilbre island . .	53 23	3 18	−0 22	...	−1 2	0 0	−0 2	−0 1
Beaumaris . . .	53 16	4 5	−0 49	...	−5 8	−6 8	...	...

The zero of the predictions, which is also the datum of the Admiralty chart of Liverpool, is 14 feet below Ordnance datum, and is 1·40 feet below the level of mean low water spring tides.

H. W. F. & C. { VI^H. 12^M. (Local mean time). / VI. 32. (Standard time). Lat. 51° 42′ N. Long. 4° 57′ W.

Week Day.	Month Day.	JULY. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	AUGUST. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	SEPTEMBER. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	7 1	20 2	7 20	20 3	Tu.	1	7 45	21 4	8 0	21 5	F.	1	8 17	22 2	8 34	21 10
						W.	2	8 15	21 5	8 31	21 5	S.	2	8 52	21 6	9 9	21 2
S.	2	7 35	20 4	7 51	20 4	Th.	3	8 48	21 4	9 3	21 3						
M.	3	8 7	20 4	8 24	20 4	F.	4	9 23	20 10	9 40	20 6	S.	3	9 27	20 7	9 46	20 0
Tu.	4	8 41	20 4	8 59	20 3	S.	5	9 57	20 1	10 15	19 8	M.	4	10 6	19 4	10 29	18 8
W.	5	9 18	20 2	9 35	19 11							Tu.	5	10 56	18 0	11 24	17 3
Th.	6	9 53	19 9	10 11	19 6	S.	6	10 34	19 3	10 55	18 8	W.	6	11 59	16 11	—	—
F.	7	10 29	19 2	10 48	18 11	M.	7	11 19	18 1	11 44	17 6	Th.	7	0 44	16 10	1 36	17 1
S.	8	11 9	18 7	11 28	18 2	Tu.	8	—	—	0 16	17 3	F.	8	2 20	17 8	3 11	18 7
						W.	9	0 53	17 1	1 41	17 4	S.	9	3 46	19 8	4 21	20 7
S.	9	11 50	17 10	—	—	Th.	10	2 31	17 9	3 14	18 6						
M.	10	0 19	17 8	0 47	17 8	F.	11	3 55	19 6	4 32	20 5	S.	10	4 49	21 6	5 15	22 4
Tu.	11	1 23	17 8	2 7	17 11	S.	12	5 5	21 4	5 34	22 1	M.	11	5 42	23 1	6 5	23 6
W.	12	2 49	18 4	3 27	19 0							Tu.	12	6 26	23 11	6 44	24 1
Th.	13	4 7	19 9	4 41	20 6	S.	13	6 3	22 10	6 28	23 6	W.	13	7 5	24 2	7 25	24 0
F.	14	5 14	21 2	5 45	21 11	M.	14	6 49	23 10	7 13	24 1	Th.	14	7 43	23 8	8 2	23 3
S.	15	6 15	22 6	6 42	23 1	Tu.	15	7 33	24 2	7 53	24 0	F.	15	8 19	22 8	8 38	22 1
						W.	16	8 14	23 9	8 34	23 5	S.	16	8 55	21 5	9 15	20 6
S.	16	7 6	23 6	7 30	23 7	Th.	17	8 52	22 11	9 11	22 4						
M.	17	7 52	23 8	8 16	23 5	F.	18	9 30	21 7	9 47	20 9	S.	17	9 33	19 8	9 50	18 10
Tu.	18	8 40	23 2	9 1	22 11	S.	19	10 7	19 11	10 25	19 0	M.	18	10 10	18 0	10 32	17 1
W.	19	9 24	22 6	9 45	21 11							Tu.	19	10 54	16 4	11 24	15 8
Th.	20	10 3	21 3	10 24	20 7	S.	20	10 42	18 3	11 3	17 4	W.	20	—	—	Noon	15 4
F.	21	10 43	19 11	11 3	19 3	M.	21	11 28	16 6	—	—	Th.	21	0 41	15 3	1 30	15 4
S.	22	11 24	18 5	11 44	17 7	Tu.	22	0 1	15 10	0 36	15 7	F.	22	2 13	15 8	2 54	16 4
						W.	23	1 20	15 5	2 8	15 7	S.	23	3 30	17 0	3 56	17 11
S.	23	—	—	0 9	17 0	Th.	24	2 53	16 0	3 31	16 7						
M.	24	0 41	16 7	1 16	16 3	F.	25	4 3	17 3	4 31	18 0	S.	24	4 22	18 9	4 42	19 5
Tu.	25	1 59	16 2	2 44	16 5	S.	26	4 57	18 8	5 17	19 4	M.	25	5 3	20 2	5 21	20 9
W.	26	3 23	16 10	3 57	17 3							Tu.	26	5 39	21 4	5 56	21 9
Th.	27	4 28	17 9	4 55	18 4	S.	27	5 37	20 0	5 56	20 6	W.	27	6 13	22 2	6 29	22 6
F.	28	5 21	18 10	5 43	19 3	M.	28	6 14	21 0	6 29	21 5	Th.	28	6 44	22 9	7 3	22 10
S.	29	6 5	19 8	6 25	20 0	Tu.	29	6 46	21 10	7 2	22 1	F.	29	7 17	22 10	7 34	22 8
						W.	30	7 18	22 3	7 31	22 5	S.	30	7 52	22 6	8 9	22 2
S	30	6 43	20 5	6 58	20 9	Th.	31	7 46	22 4	8 2	22 4						
M.	31	7 15	21 0	7 30	21 2												

Mean tide level 12^F. 0^I. Approximate { Rise 6^H. 10^M. / Fall 6 15.

P.M. times and heights are printed in large block type.

The times given are STANDARD, or mean time of the meridian of Greenwich. To obtain local mean time *subtract* 20 minutes.

TIDAL DIFFERENCES ON PEMBROKE DOCK.

The Tidal observation station was at H.M. dockyard.

The zero of the predictions, which is also the datum of the Admiralty charts of Milford Haven, is 11·82 feet below Ordnance datum, and is 1·50 feet below the level of mean low water spring tides.

Place.	Lat. N.	Long. W.	Time of H. W.	Height of H. W. Springs.	Height of H. W. Neaps.
	° ′	° ′	H. M.	F. I.	F. I.
Tenby	51 41	4 41	−0 35	+3 0	+2 2
Llanelly	51 40	4 10	+0 1	+2 6	+1 8

H. W. F. & C. { XI^H. 12^M. (Local mean time). / XI. 11. (Standard time). Lat. 53° 18′ N. Long. 6° 7′ W.

SEPTEMBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
F.	1	0 37	11 8	6 24	1 0	0 53	11 7	6 40	1 1
S.	2	1 12	11 6	6 59	1 3	1 32	11 4	7 18	1 5
S.	3	1 51	11 2	7 37	1 7	2 14	10 11	7 58	1 10
M.	4	2 39	10 8	8 22	2 1	3 7	10 5	8 48	2 4
Tu.	5	3 40	10 2	9 18	2 8	4 17	9 11	9 53	2 11
W.	6	4 58	9 9	10 35	3 3	5 40	9 9	11 17	3 5
Th.	7	—	—	6 29	9 9	0 7	3 5	7 15	9 11
F.	8	0 54	3 4	7 53	10 1	1 35	3 0	8 26	10 5
S.	9	2 13	2 8	8 53	10 10	2 43	2 3	9 20	11 2
S.	10	3 7	1 10	9 41	11 6	3 23	1 5	10 2	11 9
M.	11	3 50	1 1	10 23	12 0	4 12	0 9	10 44	12 2
Tu.	12	4 32	0 6	11 1	12 4	4 51	0 4	11 19	12 4
W.	13	5 9	0 3	11 38	12 4	5 26	0 3	11 58	12 4
Th.	14	—	—	5 46	0 4	0 18	12 2	6 5	0 5
F.	15	0 37	12 0	6 24	0 8	0 57	11 10	6 44	0 11
S.	16	1 17	11 6	7 4	1 2	1 37	11 3	7 23	1 6
S.	17	2 0	10 11	7 45	1 10	2 24	10 7	8 7	2 2
M.	18	2 49	10 3	8 30	2 6	3 17	9 11	8 56	2 11
Tu.	19	3 48	9 7	9 25	3 3	4 20	9 4	9 56	3 7
W.	20	5 3	9 3	10 39	3 10	5 44	9 2	11 20	4 0
Th.	21	—	—	6 26	9 2	0 4	4 0	7 10	9 3
F.	22	0 49	4 0	7 44	9 5	1 25	3 9	8 15	9 7
S.	23	2 0	3 6	8 42	9 10	2 30	3 3	9 1	10 2
S.	24	2 51	2 11	9 21	10 6	3 9	2 6	9 38	10 9
M.	25	3 24	2 3	9 53	11 0	3 40	1 11	10 7	11 2
Tu.	26	3 55	1 8	10 21	11 4	4 9	1 5	10 35	11 6
W.	27	4 24	1 2	10 50	11 8	4 39	1 0	11 4	11 9
Th.	28	4 55	0 10	11 19	11 10	5 9	0 9	11 36	11 10
F.	29	5 24	0 9	11 51	11 10	5 39	0 9	—	—
S.	30	0 9	11 10	5 55	0 10	0 27	11 9	6 14	0 11

OCTOBER.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	0 46	11 7	6 33	1 1	1 8	11 5	6 55	1 3
M.	2	1 32	11 3	7 18	1 5	1 53	10 11	7 41	1 9
Tu.	3	2 24	10 8	8 7	2 1	2 55	10 5	8 36	2 4
W.	4	3 31	10 2	9 9	2 8	4 9	9 11	9 46	2 11
Th.	5	4 53	9 10	10 30	3 2	5 41	9 9	11 18	3 4
F.	6	—	—	6 26	9 11	0 5	3 4	7 10	10 0
S.	7	0 49	3 2	7 44	10 4	1 26	2 10	8 12	10 7
S.	8	1 59	2 6	8 37	10 11	2 27	2 2	9 1	11 3
M.	9	2 49	1 9	9 20	11 6	3 6	1 6	9 40	11 8
Tu.	10	3 27	1 2	10 0	11 10	3 47	0 11	10 18	12 0
W.	11	4 7	0 9	10 37	12 1	4 26	0 7	10 57	12 1
Th.	12	4 47	0 6	11 13	12 1	5 3	0 6	11 31	12 0
F.	13	5 20	0 7	11 49	11 11	5 37	0 9	—	—
S.	14	0 8	11 9	5 55	0 11	0 23	11 7	6 15	1 1
S.	15	0 49	11 4	6 36	1 5	1 9	11 1	6 55	1 8
M.	16	1 32	10 9	7 18	2 0	1 55	10 6	7 39	2 3
Tu.	17	2 18	10 3	8 1	2 6	2 45	9 11	8 26	2 10
W.	18	3 15	9 8	8 53	3 1	3 46	9 6	9 22	3 5
Th.	19	4 22	9 4	9 59	3 8	5 4	9 3	10 40	3 10
F.	20	5 40	9 3	11 17	3 11	6 21	9 4	—	—
S.	21	Midt.	3 11	6 59	9 5	0 38	3 9	7 30	9 7
S.	22	1 11	3 7	7 57	9 10	1 41	3 4	8 20	10 1
M.	23	2 7	3 0	8 38	10 4	2 23	2 9	8 56	10 8
Tu.	24	2 45	2 5	9 14	10 11	3 1	2 1	9 31	11 1
W.	25	3 17	1 10	9 45	11 4	3 32	1 7	10 1	11 6
Th.	26	3 48	1 4	10 17	11 7	4 6	1 2	10 35	11 8
F.	27	4 23	1 0	10 51	11 9	4 41	0 10	11 10	11 10
S.	28	5 0	0 9	11 29	11 10	5 17	0 9	11 47	11 10
S.	29	—	—	5 35	0 10	0 8	11 9	5 55	0 11
M.	30	0 31	11 7	6 17	1 0	0 55	11 6	6 42	1 3
Tu.	31	1 20	11 3	7 7	1 6	1 50	11 0	7 35	1 9

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD, or mean time of the meridian of Dublin, or 6° 20′ W. from Greenwich. To obtain local mean time *add* 1 minute.

The zero of the predictions, which is also the datum of the Admiralty chart of Kingstown, is 1·75 feet above Ordnance datum, and is 0·84 foot below the level of mean low water spring tides.

TIDAL DIFFERENCES ON KINGSTOWN.

Place.	Lat. N.	Long. W.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
Wicklow . . .	52 59	6 2	−0 43	...	−2 9	−3 4	...	...
Warren point. .	54 6	6 15	−0 2	...	+4 0	+1 7	+0 2	+2 1
Strangford quay .	54 22	5 33	+1 16	...	−1 5	−1 3	−0 10	−1 8
South rock . .	54 24	5 25	−0 17	...	+1 8	+1 0	−0 10	−1 1
Donaghadee . .	54 38	5 32	−0 2	...	+0 6	+0 5	−0 10	−0 11

H. W. F. & C. { IV$^{H.}$ 35$^{M.}$ (Local mean time). / IV. 46. (Standard time). Lat. 53° 16′ N. Long. 9° 3′ W.

Week Day.	Month Day.	JULY. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	AUGUST. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Week Day.	Month Day.	SEPTEMBER. HIGH WATER. Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	5 15	13 11	5 34	14 1	Tu.	1	6 1	14 10	6 16	14 11	F.	1	6 34	15 5	6 51	15 2
						W.	2	6 32	14 11	6 48	14 11	S.	2	7 10	14 11	7 27	14 7
S.	2	5 50	14 1	6 7	14 1	Th.	3	7 5	14 9	7 21	14 8						
M.	3	6 24	14 1	6 41	14 1	F.	4	7 41	14 5	8 0	14 2	S.	3	7 47	14 3	8 9	13 9
Tu.	4	6 58	14 1	7 17	14 0	S.	5	8 19	13 10	8 41	13 5	M.	4	8 32	13 2	9 0	12 7
W.	5	7 36	13 10	7 55	13 9							Tu.	5	9 31	12 1	10 8	11 9
Th.	6	8 16	13 7	8 37	13 4	S.	6	9 3	13 0	9 27	12 7	W.	6	10 50	11 7	11 42	11 8
F.	7	8 57	13 0	9 20	12 9	M.	7	9 56	12 3	10 29	11 11	Th.	7	—	—	0 30	11 11
S.	8	9 44	12 6	10 8	12 4	Tu.	8	11 8	11 10	11 52	11 10	F.	8	1 12	12 5	1 50	13 0
						W.	9	—	—	0 36	12 1	S.	9	2 20	13 9	2 49	14 4
S.	9	10 37	12 2	11 11	12 2	Th.	10	1 17	12 5	1 53	12 11						
M.	10	11 45	12 3	—	—	F.	11	2 27	13 7	2 59	14 2	S.	10	3 14	15 0	3 36	15 7
Tu.	11	0 19	12 4	0 56	12 7	S.	12	3 29	14 10	3 53	15 4	M.	11	3 59	16 1	4 21	16 5
W.	12	1 32	12 11	2 3	13 4							Tu.	12	4 40	16 9	4 59	16 11
Th.	13	2 37	13 9	3 8	14 3	S.	13	4 19	15 11	4 43	16 5	W.	13	5 19	17 0	5 40	16 10
F.	14	3 36	14 8	4 3	15 2	M.	14	5 4	16 9	5 27	16 11	Th.	14	5 59	16 7	6 19	16 3
S.	15	4 31	15 8	4 56	16 2	Tu.	15	5 48	17 0	6 9	16 11	F.	15	6 36	15 10	6 55	15 4
						W.	16	6 31	16 8	6 51	16 4	S.	16	7 13	14 10	7 34	14 2
S.	16	5 20	16 6	5 45	16 7	Th.	17	7 10	16 0	7 29	15 6						
M.	17	6 8	16 7	6 33	16 5	F.	18	7 49	14 11	8 9	14 4	S.	17	7 55	13 6	8 15	12 10
Tu.	18	6 57	16 3	7 19	15 11	S.	19	8 32	13 8	8 53	12 11	M.	18	8 39	12 1	9 5	11 5
W.	19	7 42	15 8	8 5	15 3							Tu.	19	9 32	10 11	10 12	10 7
Th.	20	8 25	14 9	8 50	14 2	S.	20	9 14	12 3	9 39	11 8	W.	20	10 53	10 5	11 40	10 5
F.	21	9 12	13 7	9 36	13 0	M.	21	10 12	11 1	10 53	10 9	Th.	21	—	—	0 25	10 7
S.	22	10 1	12 5	10 30	12 0	Tu.	22	11 33	10 8	—	—	F.	22	1 2	10 11	1 37	11 4
						W.	23	0 17	10 8	0 59	10 10	S.	23	2 7	11 10	2 28	12 5
S.	23	11 1	11 8	11 37	11 5	Th.	24	1 37	11 1	2 9	11 6						
M.	24	—	—	0 14	11 4	F.	25	2 36	11 11	3 1	12 5	S.	24	2 50	12 11	3 9	13 5
Tu.	25	0 51	11 3	1 29	11 5	S.	26	3 24	12 10	3 40	13 4	M.	25	3 27	13 11	3 42	14 4
W.	26	2 3	11 8	2 31	11 11							Tu.	26	3 57	14 9	4 12	15 1
Th.	27	2 58	12 3	3 22	12 7	S.	27	3 57	13 9	4 13	14 2	W.	27	4 29	15 5	4 43	15 8
F.	28	3 45	12 11	4 3	13 2	M.	28	4 30	14 6	4 44	14 11	Th.	28	4 59	15 11	5 17	16 0
S.	29	4 22	13 6	4 41	13 10	Tu.	29	5 0	15 2	5 16	15 5	F.	29	5 32	16 0	5 50	15 11
						W.	30	5 33	15 7	5 46	15 8	S.	30	6 9	15 8	6 26	15 5
S.	30	4 57	14 2	5 13	14 5	Th.	31	6 2	15 8	6 19	15 7						
M.	31	5 29	14 8	5 45	14 9												

Mean tide level 8$^{F.}$ 2$^{I.}$ Approximate { Rise 6$^{H.}$ 30$^{M.}$ / Fall 5 55.

P.M. times and heights are printed in large block type.

The times given are STANDARD, or mean time of the meridian of Dublin, or 6° 20′ W. from Greenwich. To obtain local mean time *subtract* 11 minutes.

The zero of the predictions, which is also the datum of the Admiralty chart of Galway, is 1·30 feet above Ordnance datum, and is 0·75 foot below the level of mean low-water spring tides.

TIDAL DIFFERENCES ON GALWAY.

Place.	Lat. N.	Long. W.	Time of H. W.	Height of H. W. Springs.	Height of H. W. Neaps.
	° ′	° ′	H. M.	F. I.	F. I.
Foynes island	52 37	9 7	+1 0	−0 2	+0 5
Mellon	52 40	8 50	+1 25	+3 8	+2 8
Limerick	52 38	8 38	+1 34	+3 0	+2 10
Dingle	52 7	10 15	−0 39	−4 9	−3 10
Valentia	51 54	10 20	−0 48	−4 6	−3 3

H. W. F. & C. { XI^H. 21^M. (Local mean time). / XI. 45. (Standard time). Lat. 45° 16′ N. Long. 66° 3′ W.

JULY.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
S.	1	—	—	6 25	4 5	0 23	22 5	6 45	5 10
S.	2	0 33	24 1	7 2	4 5	1 1	22 6	7 20	5 8
M.	3	1 12	24 0	7 39	4 5	1 39	22 7	7 56	5 6
Tu.	4	1 52	23 10	8 17	4 5	2 18	22 8	8 33	5 4
W.	5	2 33	23 6	8 56	4 4	2 59	22 10	9 12	5 1
Th.	6	3 15	23 2	9 36	4 5	3 43	23 0	9 55	5 0
F.	7	4 0	22 10	10 18	4 7	4 30	23 0	10 42	4 11
S.	8	4 49	22 5	11 4	4 10	5 20	23 1	11 33	4 11
S.	9	5 42	22 0	11 55	5 1	6 14	23 4	—	—
M.	10	0 29	4 10	6 39	21 10	0 53	5 2	7 11	23 8
Tu.	11	1 29	4 5	7 40	22 0	1 55	5 0	8 9	24 6
W.	12	2 33	3 8	8 42	22 8	2 58	4 6	9 6	25 6
Th.	13	3 35	2 8	9 42	23 7	3 59	3 7	10 1	26 4
F.	14	4 34	1 8	10 38	24 7	4 56	2 10	10 55	27 1
S.	15	5 29	0 11	11 30	25 6	5 51	2 2	11 48	27 6
S.	16	—	—	6 22	0 7	0 21	25 11	6 44	1 10
M.	17	0 40	27 5	7 14	0 6	1 11	26 0	7 36	1 7
Tu.	18	1 32	27 0	8 5	0 8	2 3	26 0	8 29	1 8
W.	19	2 25	26 4	8 55	1 4	2 57	25 8	9 21	2 2
Th.	20	3 20	25 6	9 45	2 4	3 52	25 4	10 14	3 0
F.	21	4 16	24 4	10 37	3 6	4 46	24 7	11 8	4 0
S.	22	5 13	23 0	11 33	4 10	5 41	23 8	—	—
S.	23	0 5	4 11	6 12	21 11	0 34	6 0	6 38	23 1
M.	24	1 6	5 6	7 13	21 2	1 39	6 7	7 38	22 8
Tu.	25	2 12	5 10	8 15	21 0	2 41	6 8	8 36	22 8
W.	26	3 14	5 6	9 14	21 1	3 37	6 6	9 27	23 0
Th.	27	4 8	5 1	10 4	21 5	4 26	6 1	10 11	23 5
F.	28	4 52	4 8	10 46	21 10	5 7	5 8	10 50	23 8
S.	29	5 29	4 5	11 23	22 2	5 43	5 5	11 28	23 11
S.	30	6 3	4 2	11 59	22 6	6 17	5 1	—	—
M.	31	0 6	24 0	6 36	4 0	0 34	22 10	6 50	4 10

AUGUST.

Week Day.	Month Day.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.	Time. H. M.	Ht. F. I.
Tu.	1	0 43	24 0	7 10	3 11	1 10	23 1	7 24	4 7
W.	2	1 20	23 11	7 45	3 10	1 46	23 5	8 0	4 4
Th.	3	1 58	23 10	8 22	3 10	2 23	23 7	8 39	4 1
F.	4	2 38	23 6	9 2	3 11	3 2	23 8	9 22	4 0
S.	5	3 21	23 1	9 44	4 1	3 45	23 8	10 8	4 1
S.	6	4 8	22 6	10 29	4 7	4 36	23 6	10 58	4 4
M.	7	5 0	21 10	11 20	5 0	5 34	23 5	11 55	4 7
Tu.	8	—	—	6 3	21 4	0 23	5 5	6 38	23 5
W.	9	1 2	4 7	7 14	21 5	1 33	5 4	7 43	23 11
Th.	10	2 14	4 0	8 22	22 2	2 40	4 7	8 46	24 10
F.	11	3 20	3 0	9 23	23 4	3 41	3 6	9 45	25 10
S.	12	4 20	1 10	10 19	24 7	4 38	2 6	10 41	26 8
S.	13	5 13	1 0	11 12	25 7	5 33	1 7	11 34	27 2
M.	14	—	—	6 3	0 5	0 3	26 4	6 26	1 0
Tu.	15	0 25	27 4	6 52	0 5	0 53	26 7	7 16	0 11
W.	16	1 15	26 11	7 40	0 10	1 42	26 8	8 5	1 2
Th.	17	2 6	26 2	8 28	1 7	2 30	26 1	8 53	1 11
F.	18	2 57	25 2	9 17	2 8	3 19	25 5	9 42	2 11
S.	19	3 49	23 11	10 7	4 0	4 9	24 5	10 33	4 0
S.	20	4 42	22 7	10 59	5 4	5 3	23 4	11 28	5 1
M.	21	5 38	21 5	11 55	6 5	6 1	22 6	—	—
Tu.	22	0 29	6 0	6 39	20 7	0 56	7 1	7 2	22 0
W.	23	1 35	6 4	7 44	20 4	2 3	7 2	8 2	22 0
Th.	24	2 39	6 1	8 46	20 7	3 5	6 8	8 59	22 4
F.	25	3 35	5 6	9 37	21 2	3 56	6 1	9 48	22 10
S.	26	4 20	4 10	10 19	21 10	4 36	5 4	10 29	23 5
S.	27	4 58	4 4	10 56	22 6	5 12	4 8	11 8	23 10
M.	28	5 34	3 11	11 31	23 1	5 47	4 4	11 45	24 1
Tu.	29	—	—	6 9	3 8	0 5	23 6	6 21	4 0
W.	30	0 21	24 2	6 43	3 7	0 40	23 11	6 55	3 7
Th.	31	0 56	24 2	7 17	3 6	1 16	24 2	7 31	3 6

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large block type. A comparison of consecutive heights will indicate whether it is high or low water.

The times given are STANDARD, or mean time of the meridian of 60° W. from Greenwich. To obtain local mean time *subtract* 24 minutes.

TIDAL DIFFERENCES ON ST. JOHN.

The zero of the predictions, which is also the datum of the Admiralty chart of St. John, is 55·60 feet below a bench mark near the south-east corner of the Custom House, and is approximately 2½ feet below the level of mean low water spring tides.

Place.	Lat. N.	Long. W.	Time of H. W.	Time of L. W.	Height of H. W. Springs.	Height of H. W. Neaps.	Height of L. W. Springs.	Height of L. W. Neaps.
	° ′	° ′	H. M.	H. M.	F. I.	F. I.	F. I.	F. I.
Seal Cove . .	44 37	66 51	−0 22	...	−5 6	−6 6	...	...
Welchpool . .	44 58	66 58	+0 2	+0 10	−2 0	−1 6	...	...
St. Andrews. .	45 4	67 3	+0 8	+0 18	−0 6	0 0	...	...
Quaco . . .	45 20	65 32	+0 12	...	+4 6	+3 6	...	...

H. W. F. & C. VII$^{H.}$ 48$^{M.}$ Lat. 28° 54′ N. Long. 50° 43′ E.

NOVEMBER.

Week Day.	Month Day.	Time.	Ht.	Time.	Ht.	Time.	Ht.	Time.	Ht.
		H. M.	F. I.	H. M.	F. I.	H. M.	F. I.	H. M.	F. I.
W.	1	5 43	0 9	0 46	2 4	3 22	2 3	10 28	5 9
Th.	2	6 44	0 11	2 35	2 6	4 15	2 6	11 21	5 4
F.	3	—	—	7 51	1 1	3 37	2 10	6 18	2 8
S.	4	0 28	4 9	8 53	1 3	4 10	3 3	8 9	2 8
S.	5	2 2	4 2	9 47	1 5	4 40	3 9	10 0	2 3
M.	6	3 47	3 10	10 34	1 6	5 10	4 3	11 23	1 9
Tu.	7	5 15	3 8	11 14	1 7	5 39	4 9	—	—
W.	8	0 25	1 3	6 21	3 5	11 48	1 8	6 6	5 2
Th.	9	1 16	0 10	7 16	3 3	0 18	1 10	6 34	5 7
F.	10	2 4	0 7	8 4	3 1	0 47	1 11	7 4	5 9
S.	11	2 50	0 5	8 50	2 11	1 15	2 0	7 36	5 11
S.	12	3 34	0 5	9 35	2 9	1 40	2 1	8 10	5 11
M.	13	4 17	0 7	10 22	2 7	2 1	2 2	8 46	5 10
Tu.	14	4 59	0 9	11 14	2 6	2 23	2 4	9 23	5 7
W.	15	5 39	0 11	0 13	2 7	2 49	2 5	10 1	5 4
Th.	16	6 16	1 1	1 16	2 8	3 23	2 7	10 40	4 11
F.	17	6 51	1 3	2 20	2 10	4 22	2 9	11 23	4 6
S.	18	—	—	7 28	1 4	3 6	3 1	6 19	2 11
S.	19	0 23	4 1	8 9	1 5	3 43	3 5	8 11	2 11
M.	20	1 46	3 8	8 52	1 6	4 14	3 10	10 5	2 7
Tu.	21	3 15	3 4	9 35	1 7	4 42	4 3	11 25	2 1
W.	22	4 41	3 2	10 15	1 8	5 10	4 9	—	—
Th.	23	0 21	1 7	5 51	3 0	10 51	1 9	5 39	5 3
F.	24	1 7	1 1	6 51	2 11	11 25	1 9	6 10	5 8
S.	25	1 51	0 8	7 48	2 9	Noon	1 10	6 44	6 1
S.	26	2 35	0 5	8 43	2 7	0 37	1 10	7 21	6 4
M.	27	3 19	0 2	9 37	2 6	1 17	1 11	8 1	6 5
Tu.	28	4 2	0 1	10 31	2 6	2 0	2 0	8 43	6 4
W.	29	4 45	0 2	11 26	2 6	2 47	2 2	9 28	6 1
Th.	30	5 29	0 4	0 21	2 8	3 50	2 3	10 17	5 7

DECEMBER.

Week Day.	Month Day.	Time.	Ht.	Time.	Ht.	Time.	Ht.	Time.	Ht.
		H. M.	F. I.	H. M.	F. I.	H. M.	F. I.	H. M.	F. I.
F.	1	6 14	0 7	1 16	2 11	5 9	2 5	11 11	5
S.	2	—	—	7 0	0 10	2 9	3 4	6 44	2
S.	3	0 14	4 3	7 47	1 2	2 56	3 9	8 26	2
M.	4	1 37	3 7	8 34	1 5	3 38	4 2	10 10	2
Tu.	5	3 31	3 0	9 21	1 8	4 17	4 8	11 34	1
W.	6	5 14	2 9	10 7	1 10	4 54	5 1	—	—
Th.	7	0 41	1 0	6 38	2 8	10 48	2 0	5 30	5
F.	8	1 33	0 7	7 38	2 7	11 26	2 1	6 5	5
S.	9	2 18	0 5	8 31	2 6	Noon	2 1	6 40	5 1
S.	10	2 59	0 3	9 17	2 6	0 33	2 2	7 15	5 1
M.	11	3 37	0 3	9 59	2 6	1 7	2 3	7 50	5 1
Tu.	12	4 12	0 4	10 36	2 7	1 42	2 3	8 25	5
W.	13	4 43	0 6	11 7	2 8	2 19	2 4	9 1	5
Th.	14	5 9	0 7	11 36	2 9	2 59	2 5	9 38	5
F.	15	5 30	0 9	0 7	2 11	3 47	2 6	10 16	4 1
S.	16	5 51	0 11	0 42	3 2	4 45	2 7	10 55	4
S.	17	6 15	1 0	1 22	3 5	5 55	2 8	11 37	3 1
M.	18	—	—	6 43	1 2	2 4	3 8	7 24	2
Tu.	19	0 28	3 4	7 16	1 4	2 47	4 1	9 20	2
W.	20	1 53	2 10	7 54	1 6	3 30	4 7	11 25	2
Th.	21	4 2	2 5	8 38	1 8	4 12	5 0	—	—
F.	22	0 31	1 5	5 45	2 3	9 28	1 9	4 53	5
S.	23	1 18	0 10	6 59	2 3	10 22	1 10	5 35	5 1
S.	24	1 59	0 5	8 0	2 3	11 18	1 11	6 18	6
M.	25	2 37	0 1	8 52	2 4	0 15	1 11	7 3	6
Tu.	26	3 14	*0 1	9 36	2 6	1 13	1 11	7 50	6
W.	27	3 50	*0 2	10 15	2 8	2 12	1 11	8 37	6
Th.	28	4 26	*0 1	10 53	2 11	3 13	1 11	9 25	5
F.	29	5 2	0 2	11 31	3 3	4 16	1 11	10 14	5
S.	30	5 38	0 5	0 10	3 7	5 23	2 0	11 4	4
S.	31	6 14	0 9	0 51	3 11	6 39	2 1	11 57	3

* Signifies *below Datum.*

The tides are placed in order of occurrence, p.m. times and heights being printed in large bloc type. A comparison of consecutive heights will indicate whether it is high or low water.
The times given are LOCAL MEAN TIME.

PERSIAN GULF.

Place.	Position.			H. W. F. & C.		Rise.	
	Latitude.	Longitude.	Long. in Time.	M. T. Place.	G. M. T.	Springs.	Neaps.
	N.	E.	H. M.	H. M.	H. M.	Feet.	Feet.
Kuweit	29 22	48 0	3 12	0 13	9 1	11½	8
Shatt al Arab: Al Basra .	30 30	47 58	3 12	6 0	2 48	9	...
Abu Shahr or Bushire .	29 0	50 43	3 23	7 43	4 20	6	5
Henjam island anchorage .	26 41	55 54	3 44	11 30	7 46	10½	7½